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Log. and Trig. Tables (*Complete Edition*).

A

GRAMMAR SCHOOL ARITHMETIC

BY

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PREFACE.

THIS Arithmetic is designed to give pupils of the grammar-school age an intelligent knowledge of the subject and a moderate power of independent thought.

Whether Arithmetic is studied for mental discipline or for practical mastery over the every-day problems of common life, mechanical processes and routine methods are of no value. Pupils can be trained to logical habits of mind and stimulated to a high degree of intellectual energy by solving problems adapted to their capacities. They become *practical* arithmeticians, not by learning special business forms, but by founding their knowledge on reasoning which they fully comprehend, and by being so thoroughly exercised in logical analysis that they are independent of arbitrary rules.

The book contains a great number of well-graded and progressive problems, made up for youths from ten to fourteen years of age. Definitions and explanations are made as brief and simple as possible. It is not intended that definitions shall be committed to memory, but that they shall be simply discussed by teacher and pupils. Every teacher, of course, will be at liberty to give better definitions, and to make a better presentation of methods, than those given in the book. In short, the chief object in view will be gained if pupils are trained to solve the problems by neat and intelligent methods, and are kept free from set rules and formulas.

A great many number-problems are given in the first pages of the book, so that the necessary facility and accuracy in computing

under the four fundamental rules may be acquired; as want of accuracy and rapidity in mere calculations distracts the attention which should be given to the investigation and correct statement of clothed exercises. The pupil should be required to do only so many of these number-problems as are found to be necessary to give him facility and accuracy in the four fundamental operations; and he should be allowed to omit some of the harder clothed problems until he reviews the book.

The chapter on the Metric System is put at the end of the book because many grammar-school pupils have no time for it, while those who have time can as well learn the system at this stage of their progress as earlier.

The chapter on Miscellaneous Problems is intended as a *review* of the subject-matter of Arithmetic and as a *test* of the learner's knowledge.

The author is under obligations to many teachers who have given valuable suggestions and assistance in the preparation of this work.

G. A. WENTWORTH.

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VOCABULARY.

Abstract number. This phrase is employed to designate numbers used without reference to any particular unit, as 8, 10, 21. But *all numbers are in themselves abstract whether the kind of thing numbered is or is not mentioned.*

Addition. The process of combining two or more numbers so as to form a single number.

Aliquot part. A number which is contained an integral number of times in a given number. Thus, 5, $6\frac{1}{4}$, $12\frac{1}{2}$, $16\frac{2}{3}$, are aliquot parts of 100.

Amount. The sum of two or more numbers. In Interest, the sum of principal and interest.

Analysis. The separation of a question into parts, to be examined each by itself.

Antecedent. The first of the two terms named in a ratio.

Area of a surface. The area of a surface is the number of units of surface it contains; the unit of surface being a square whose side is a unit of length.

Arithmetic. The science that treats of numbers and the methods of using them.

Assets. All the property belonging to an estate, individual, or corporation.

Average. The mean of several unequal numbers, so that, if substituted for each, the aggregate would be the same.

Bank. An establishment for the custody, loaning, and exchange of money; and often for the issue of money.

Bank discount. An allowance received by a bank for the loan of money, paid at the time of lending as interest.

Bonds. Written contracts under seal to pay specified sums of money at specified times, issued by national governments, states, cities, and other corporations.

Cancellation. The striking out of a common factor from the dividend and divisor.

Commission. Compensation for the transaction of business, reckoned at some per cent of the money employed in the transaction.

Common denominator. A denominator common to two or more fractions.

Common factor. A factor common to two or more numbers.

Common multiple. A multiple common to two or more numbers.

Complex fraction. A fraction that has a fraction in one or both of its terms.

Composite number. The product of two or more integral factors, each factor being greater than unity.

Compound denominations. Several denominations used to express parts of one quantity.

Compound interest. When the interest due is left unpaid, and considered as an increase made to the principal, the whole interest, accruing in any time, is called compound interest.

Compound fraction. A fraction of another fraction.

Concrete number. A phrase used to denote numbers applied to specified things; as 6 horses, 8 desks.

Consequent. The second of the two terms named in a ratio.

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Corporation. An association of individuals authorized by law to transact business as a single person.

Couplet. The two terms of a ratio taken together.

Coupon. A certificate of interest attached to a bond, to be cut off when due and presented for payment.

Creditor. A person or firm to whom money is due.

Cube root. One of the three equal factors of a number.

Customs. Duties or taxes imposed by law on merchandise imported, and sometimes on merchandise exported.

Debtor. A person who owes money to another.

Decimal fractions. Fractions of which only the numerators are written, and the denominators are ten or some power of ten.

Decimal point. A dot placed after the *units'* figure to mark its place.

Decimal system. The common system of numbers founded on their relations to *ten*, *ten tens*, etc.

Denominator. The number which shows into how many equal parts a unit is divided.

Difference. The number which, added to a given number, makes a *sum equal to* another given number.

Discount. Allowance made for the payment of money before it becomes due. Also, the difference between the market value and the face value when the market value is *below* the face value.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and required factor (called quotient). In business, the share of profits which belongs to each owner of stock, on his proportion of the capital.

Division. The operation by which, when a product and one of its factors are given, the other factor is found.

Divisor. The number by which a given dividend is to be divided.

Draft. A written order directing one person to pay a specified sum of money to another.

Drawee of a draft. The person to whose order the sum of money named in a draft is to be paid.

Drawer of a draft. The person who signs the draft.

Duties. Taxes required by the government to be paid on goods imported, exported, or put on the market for consumption.

Equation. A statement that two expressions of number are equal.

Equation of payments. The finding of an average time at which several payments may be justly made.

Exchange. A system of paying debts, due to persons living at a distance, by transmitting drafts instead of money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last terms of a proportion.

Evolution. The process of finding the root of a number.

Factors. The factors of a number are a set of numbers whose product is the given number; they are assumed to be integral, except in the extraction of roots. In commerce, agents employed by merchants to transact business.

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometrical forms.

Firm. The name under which a company transact business.

Fractions. One or more of the equal parts into which the unit is divided.

Grace. An allowance of three days, after the date a note becomes due, within which to pay the note.

Gram. The unit of weight in the metric system.

Greatest common measure. The greatest number which is a common factor of two or more given numbers.

Improper fraction. A fraction whose numerator equals or exceeds the denominator.

Index. A figure written at the left and above the radical sign to show what root of the number under the radical sign is required. A fraction written at the right of a number, of which the numerator shows the required power of that number, and the denominator the required root of that power.

Instalment. A payment in part.

Insurance. A guarantee of a specified sum of money in the event of loss of property by fire, storm at sea, or other disaster; or of loss of life.

Integral number. A number which denotes whole things.

Interest. Money paid for the use of money.

Involution. The process of finding a power of a number.

Latitude of a point. The angle made by the vertical line at that point with the plane of the equator.

Least common multiple. The least number which is a common multiple of several given numbers.

Liability. A debt, or obligation to pay.

Line. Length without breadth or thickness. The path of a moving point.

Liter. The unit of capacity in the metric system equal in volume to a cube each edge of which is one-tenth of a meter.

Long division. The method of dividing in which the processes are written in full.

Longitude of a point. The angle between two planes supposed to pass through the centre of the earth and to contain, the one the meridian of that point, and the other the standard meridian.

Loss. The excess of the cost price above the selling price.

Maturity of a note. The date at which a note legally becomes due.

Mean proportional. A number which is both the second and third terms of a proportion.

Means. The terms of a proportion between the extremes.

Meter. The unit of length in the metric system.

Minuend. The given number in subtraction which is equal to the sum of another given number called the subtrahend, and a required number called the difference or remainder.

Mixed number. A number that expresses both entire things and parts of things taken together.

Multiple of a number. The product obtained by taking the given number an integral number of times.

Multiplicand. The number to be multiplied by another.

Multiplication. The operation of finding a number bearing the same ratio to the multiplicand which the multiplier bears to unity.

Multiplier. The number by which the multiplicand is multiplied.

Net proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols.

Note. A written agreement to pay a specified sum of money at a specified time.

Number. The answer to the question, How many?

Numeration. A system of naming numbers.

Obligation. A debt, or liability to pay.

Order of units. A name used to designate the number of things in a group, as *tens*, *hundreds*, *thousands*, etc.

Partial payment. Part payment on a note.

Partnership. An association of two or more persons to carry on business.

Par value. Face or nominal value.

Pendulum. A body suspended by a straight line from a fixed point, and moving freely about that point as a centre.

Percentage. A part of any given number reckoned at some rate per cent.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll tax. A tax levied by the head or poll.

Power. The product of two or more equal factors.

Premium. Money paid for insurance computed at some rate per cent of the value insured. Also the excess of market value above par value.

Present worth. The present value of a debt due at some future day.

Prime number. A number which has no integral factors except itself and one.

Principal. Money drawing interest.

Problem. A question to be solved.

Product. The result obtained by multiplying the multiplicand by the multiplier.

Profit. The excess of selling price above cost.

Proof. The evidence by which the accuracy of any result is established.

Proper fraction. A fraction, the numerator of which is less than the denominator.

Proportion. A statement that two ratios are equal.

Quantity. The answer to the question, How much?

Quotient. The number sought in division.

Rate per cent. Rate by the hundred.

Ratio. The *relative magnitude* of two numbers or of two quantities.

Reciprocal of a number. *One* divided by that number.

Reduction. The process of changing the *unit* in which a quantity is expressed without changing the *value* of the quantity.

Remainder. The number which, added to the subtrahend, gives a sum equal to the minuend.

Root of a number. One of the equal factors of the number.

Rule. The statement of a prescribed method.

Security. Property used to guarantee the payment of any debt.

Share. One of a certain number of equal parts into which the capital of a company is divided.

Short division. The method of dividing in which the operations of multiplying and subtracting are performed mentally.

Solid. A magnitude which has length, breadth, and thickness.

Solution. The process by which the answer to a question is obtained.

Specific gravity of a substance. The ratio of the weight of a given volume of it to that of an equal volume of water.

Square root. One of two equal factors.

Stock. Capital invested in business.

Subtraction. The process of finding a number which added to one of two given numbers will produce the other.

Sum. The number which results from combining two or more numbers by addition.

Surd. An indicated root the value of which cannot be exactly expressed in figures.

Surface. That which has only length and breadth.

Thermometer. An instrument for measuring heat.

Units. The standards by which we count separate objects or measure magnitudes.

Verify. To establish, by trial, the truth of any statement.

Volume of a solid. The volume of a solid is the number of units of volume it contains; the unit of volume being a cube whose edge is a unit of length.

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CHAPTER I.

PRELIMINARY DEFINITIONS.

1. Number. A fundamental idea, like that of number, cannot be defined. A simple, direct answer to the question "How many?" is a *number*.

2. A collection of several similar objects (as a collection of apples) gives the idea of number.

3. Units. In counting separate objects the standards by which we count are called *units*. Thus:

In counting the eggs in a nest, the unit is *an egg*.

In selling eggs by the dozen, the unit is *a dozen eggs*.

In selling bricks by the thousand, the unit is *a thousand bricks*.

4. Measurement. Continuous magnitudes, such as length, surface, space, time, heat, cannot be counted; they *are measured*. Magnitudes, whether continuous or separate, are generally measured, and the *standards* by which they are measured are fixed by law, or by common consent.

5. Units of Measure. The standards by which we measure magnitudes are called *units of measure*. Thus:

An inch, a foot, a yard, a rod, a mile, are units of length.

A square inch, a square foot, are units of surface.

A cubic inch, a cubic foot, are units of volume.

6. Abstract Numbers. Numbers standing alone, as 4, 7, 13, which mean 4 units, 7 units, 13 units, but do not specify the kind of objects counted, or the kind of units of measure taken, are called *abstract numbers*. They signify simply the *number of repetitions of some unit*.

7. Concrete Numbers. Expressions that give the *name* of the objects counted or of the unit of measure employed, and the *number* of such objects, or of such units of measure, are called *concrete numbers*. Thus, 5 horses, 7 feet, 6 pounds, 5 days, are called concrete numbers. Such expressions consist of two parts, the number proper, and the kind of units taken, and should, strictly speaking, be called *quantities*.

8. Arithmetic treats of the simple properties of numbers, and the art of computing by numbers.

NOTATION AND NUMERATION.

9. The first numbers have special names, as follows:

one, two, three, four, five, six, seven, eight, nine, ten.

10. The first nine of these numbers are called **Simple Units**, or *units of the first order*.

11. The group of ten units has received the name of a **Ten**, or *a unit of the second order*; and we count by tens as by units; thus:

one ten, two tens, three tens ... nine tens, ten tens.

12. The group of ten tens has received the name of a **Hundred**, or *a unit of the third order*; and we count by hundreds, as by tens and units; thus:

one hundred, two hundreds ... ten hundreds.

13. A group of ten hundreds is called a **Thousand**, or *a unit of the fourth order.*

14. From ten units of the fourth order is formed *a ten thousand*, or *a unit of the fifth order;* and from ten units of the fifth order is formed *a hundred thousand*, or *a unit of the sixth order.*

15. Units of the *seventh order* are called **Millions**; of the *eighth order, ten millions;* of the *ninth order, hundred millions.* Finally, units of the *tenth order* are called **Billions**; units of the *thirteenth order*, **Trillions**; and so on.

16. The table of units of different orders is as follows:

First order,	*simple units,*	first class.
Second order,	*tens of units,*	
Third order,	*hundreds of units,*	
Fourth order,	*thousands,*	second class.
Fifth order,	*tens of thousands,*	
Sixth order,	*hundreds of thousands,*	
Seventh order,	*millions,*	third class.
Eighth order,	*tens of millions,*	
Ninth order,	*hundreds of millions,*	
Tenth order,	*billions,*	fourth class.
Eleventh order,	*tens of billions,*	
Twelfth order,	*hundreds of billions,*	
Thirteenth order,	*trillions,*	fifth class.
... ...	... ...	
... ...	... ...	

17. The group of the first three orders is called the first class of units, and the group of the three following orders, the second class, and so on.

18. The unit of the second class is equal to a thousand units of the first class, and a unit of the third class is equal to a thousand units of the second class, and so on.

19. To find the value of a number we decompose it into units of the different orders, and state how many groups there are of each kind, commencing with the highest order. Thus, for example, two millions, three thousands, five hundreds, seven tens, and four units.

20. It is clear that the names of all numbers up to a billion are formed by combining the names of the first nine numbers with the words ten, hundred, thousand, million.

21. Usage sanctions the following irregularities:

I. Instead of saying two tens, three tens, four tens, five tens, six tens, seven tens, eight tens, nine tens, we say twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

II. The names of the numbers between ten and twenty are eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

22. The names of the numbers between twenty and a hundred are:

twenty-one, twenty-two, twenty-three ... twenty-nine,
thirty-one, thirty-two, thirty-three ... thirty-nine,
... ...
ninety-one, ninety-two, ninety-three ... ninety-nine.

23. The names of the numbers between a hundred and a thousand are:

hundred one, hundred two ... hundred ninety-nine,
two hundred one ... two hundred ninety-nine,
... ...
nine hundred one ... nine hundred ninety-nine.

24. The **common system of notation** employs ten figures or digits:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The first nine of these figures represent the first nine numbers; the last, which is called *Zero*, *Naught*, or *Cipher*, is used to denote the *absence of units* of the order in which it stands. It is possible to express all numbers by these ten digits *by making the value of each figure increase tenfold for every place that it is moved to the left.*

25. If we have given a number written in figures, the position of each figure counting from the right indicates the order of units that the figure represents. If we divide the number into **periods** of three figures each, the first period on the right will be the period of simple units, the second period will be the period of thousands, the third will be the period of millions, and so on. In each period the first figure on the right expresses the units of that class, the second figure the tens, and the third the hundreds. Thus:

MILLIONS.		THOUSANDS.			UNITS.		
Tens.	*Units.*	*Hundreds.*	*Tens.*	*Units.*	*Hundreds.*	*Tens.*	*Units.*
2	1	3	3	4	3	3	4

Thus, the number 21,334,334 means and is read 21 million, 334 thousand, 334 units. If the number is applied to dollars, it means and is read 21 million, 334 thousand, 334 dollars. The next period is the **billions'** period.

NOTE. The fundamental principle of forming and expressing numbers should be illustrated by making little bundles of wooden toothpicks, ten in each bundle, and then making bundles of hundreds by taking for each hundred ten bundles of ten each. When the pupil has become familiar with forming and expressing numbers consisting of hundreds, tens, and units, he should be shown that the method of forming and expressing numbers of hundreds, tens, and units of thousands is precisely the same, the only difference being that the unit of this period is not a single toothpick, but a pile of ten *bundles of a hundred* each, that is, a thousand.

26. To write a number in figures we write successively the number of units of each order from left to right, beginning at the highest order and taking care to supply by zeros orders of units that may be lacking.

27. To read a number written in figures we divide the number into periods of three figures each from right to left: this done, we begin to read at the left-hand period and read *as if the figures of that period stood alone*, adding the *name* of the period; then the next period to the right is read with the name of that period, and so on.

28. The number 1256 may be read *one thousand two hundred fifty-six*, or it may be read *twelve hundred fifty-six*. The number 5004 may be read *five thousand four*, or it may be read *fifty hundred four*. *The shortest method is the best method of reading any number*. Twelve hundred fifty-six is shorter than one thousand two hundred fifty-six; five thousand four is shorter than fifty hundred four.

29. It will be seen that the value of each figure, in any number expressed in figures, depends on two things:

First, the value attached to the figure without regard to its position.

And, secondly, the value it acquires from the place it holds in the number.

The value of a figure, without regard to its position, is called its **absolute** value; and the value it acquires by its position is called its **local** value.

30. The art of expressing numbers by means of figures is called **Notation**, and the art of expressing in words a number written in figures is called **Numeration.**

31. The unit of money is the dollar. Instead of writing the word *dollars*, this mark $ is used, which is called the

sign for dollars, or the "dollar mark." Thus, if we wish to write five dollars, we write it $5.

It takes ten ten-cent pieces to make a dollar; that is, a ten-cent piece is **one-tenth** of a dollar. It takes ten single cents to be equal in value to a ten-cent piece. If we have one dollar and one ten-cent piece, we write it $1.10. If we have one dollar, one ten-cent piece, and two cents, we write it $1.12.

The dot which is placed after the one dollar is called the **Decimal Point**. Figures to the left of the decimal point denote whole units. Figures to the right of the decimal point denote parts of a unit, and are called **Decimal Fractions.** The expression $1.10 is read "one dollar and ten cents"; and the expression $ 1.12 is read "one dollar and twelve cents."

Ex. 1.

Write in figures:

1. Two hundred thirty-six, one hundred forty, five hundred two, seven hundred three.
2. Five hundred fourteen, three hundred seventy-six, four hundred thirty, eight hundred two, nine hundred twenty-seven.
3. One hundred ninety, four hundred six, eight hundred ten, two hundred seven.
4. Three hundred ten, two hundred thirteen, six hundred twenty-three, two hundred nineteen.
5. Five hundred fifty, four hundred four, four hundred twenty-five, eight hundred sixty.
6. Eight hundred sixteen, seven hundred eight, nine hundred, seven hundred three.
7. Nine hundred ninety-five, eight hundred eighty, seven hundred, eight hundred seven.
8. Two hundred seventeen, four hundred twelve, four hundred eight, one hundred two.

9. Four hundred seventeen, six hundred nineteen, three hundred six, one hundred eighteen.

Ex. 2.

Read (or write in words):

1.	500,	700,	300,	200,	900,	100.
2.	830,	709,	506,	350,	819,	703.
3.	607,	312,	918,	810,	103,	560.
4.	752,	698,	405,	536,	121,	514.
5.	973,	356,	703,	409,	211,	713.
6.	225,	64,	970,	49,	83,	674.
7.	106,	170,	380,	759,	921,	538.
8.	481,	360,	593,	32,	296,	551.
9.	182,	802,	555,	705,	649,	630.
10.	314,	97,	613,	384,	992,	516.

Ex. 3.

Write in figures:

1. Eight thousand seven hundred three, four thousand forty-five, six thousand three hundred eight, forty-eight hundred.
2. Five thousand forty-eight, nineteen hundred ninety, seven thousand eighty-two, eight thousand fifty.
3. Seven thousand two hundred forty, nine thousand nine hundred nineteen, six thousand seven, eight thousand seven hundred seventy-six.
4. Seven thousand one hundred seven, six thousand eight hundred four, nine thousand one hundred ten, five thousand five hundred fifty.
5. Six thousand eighty-six, four thousand forty, one thousand ten, nine thousand ninety-nine.
6. Eight thousand eighty, seventeen hundred fifty-seven, eleven hundred one, seven thousand seven, forty-five hundred forty-five.

7. Two thousand four hundred ninety-six, eighteen hundred eighty-three, three thousand ninety-five, one thousand eleven.
8. One thousand thirteen, one thousand one, fourteen hundred, thirty-three thousand fourteen.
9. Seventeen hundred thirty-six, three thousand forty-nine, eight thousand eighteen, nine thousand seventy.
10. Four thousand seven hundred nine, fifteen hundred ten, one thousand sixty-nine, sixteen thousand sixteen.

Ex. 4.

Read (or write in words):

1.	8,000,	5,000,	2,000,	6,000,	1,000,	9,000.
2.	9,210,	6,907,	7,402,	9,998,	4,060,	7,210.
3.	5,068,	4,020,	1,400,	7,031,	1,290,	1,010.
4.	8,808,	6,006,	8,482,	3,096,	4,720,	11,973.
5.	12,002,	11,101,	5,812,	1,739,	6,760,	6,903.
6.	4,085,	1,169,	2,615,	5,007,	1,110,	1,460.
7.	4,760,	4,190,	2,607,	5,180,	1,200,	3,746.
8.	9,008,	8,300,	6,804,	2,977,	6,202,	9,620.
9.	6,322,	7,450,	8,673,	2,603,	2,518,	1,508.
10.	7,080,	1,009,	8,070,	5,068,	1,397,	5,782.

Ex. 5.

Write in figures:

1. Twelve and twelve hundredths, twenty-two and eight tenths, three hundred twenty-five and six tenths, one hundred one and one hundred one thousandths.
2. Seventy-five and seventy-five hundredths, eighty-three and twenty-six thousandths, ninety-six and seven hundred four thousandths, one thousand ten and two tenths.
3. Five hundred seventy-three and five hundred seventy-three thousandths, eleven thousand four and sixteen hundredths, three hundred sixty-five and eight tenths, seventy-two and ninety-six hundredths.

4. Three and nineteen thousandths, six hundred fifty-eight and two hundredths, eight hundred and eight hundredths, thirty-seven and five thousandths.
5. Seventy-one and seven tenths, seven and seventeen hundredths, seven hundred and seventeen thousandths, eight hundred ten and one tenth.
6. Eighty-one and one hundredth, eight and one hundred one thousandths, nine hundred sixty-three and two tenths, ninety-six and thirty-two hundredths, nine and six hundred thirty-two thousandths.
7. Six hundred and five tenths, sixty and five hundredths, six and five thousandths.
8. Nine hundred eighty-three and three tenths, ninety-eight and thirty-three hundredths, nine and eight hundred thirty-three thousandths.
9. One hundred twelve and one tenth, eleven and twenty-one hundredths, one and one hundred twenty-one thousandths.
10. Eleven thousand and sixty-three thousandths, twenty-three and eighty-six hundredths, one hundred ten and eleven hundredths.

Ex. 6.

Read (or write in words):

1.	3010.3,	477.12,	60.206,	698.97,	778.15,	84.510.
2.	903.9,	413.9,	17.918,	113.94,	14.613,	204.12.
3.	234.5,	3010.3,	59.106,	43.136,	380.21,	361.73.
4.	6187.8,	785.33,	90.849,	92.294,	27.989,	28.012.
5.	291.59,	29.645,	30.081,	299.07,	30.190,	35.257.
6.	360.4,	3605.9,	361.16,	39.041,	468.64,	463.59.
7.	47.828,	59.184,	600.65,	601.19,	60.108,	52.466.
8.	510.14,	51.028,	580.35,	5804.7,	641.97,	6409.8.
9.	65.002,	69.949,	602.17,	6020.6,	64.058,	76.343.
10.	770.85,	6994.9,	712.06.	719.66,	833.87,	83.493.

Ex. 7.

Write in figures:

1. Fifty thousand three dollars, eighty thousand nine hundred ninety dollars.
2. Twenty-eight million seven hundred forty-four thousand one hundred sixty-nine dollars.
3. Five hundred sixteen dollars and ten cents, twenty-five hundred fifty dollars and sixty-nine cents.
4. Sixteen hundred million thirty thousand three hundred eight dollars and fifty cents.
5. Twenty-seven hundred million one thousand one dollars and eighty-seven cents.
6. Five hundred thousand two hundred one dollars and seventy-five cents.
7. Eight million fourteen thousand three hundred twenty-five dollars and twenty-five cents.
8. Ninety-seven million two hundred thousand one hundred two dollars and five cents.
9. Ten million ten thousand ten dollars and ten cents.
10. Eleven hundred ten thousand dollars and eleven cents.

Ex. 8.

Read (or write in words):

1.	$259,132.10,	$27,186.25.
2.	$1,213,062.50,	$2,763,001.75.
3.	$3,675,321.12,	$3,500,005.15.
4.	$17,360,502.20,	$27,132,857.33.
5.	$55,333,263.36,	$58,785,587.09.
6.	$116,001,556.40,	$275,363,750.11.
7.	$660,878,640.69,	$594,340,000.94.
8.	$600,241,560.02,	$124,271,000.01.
9.	$768,301,520.20,	$802,631,516.73.
10.	$505,631,880.04,	$1,555,676,410.62.

CHAPTER II.

ADDITION.

32. If you put 2 cents with 3 cents, how many cents have you? Answer, 5 cents.

How can you express this operation on your slate?

You can write the figure 2; then the figure 3 beneath it; draw a line underneath, and below the line write the figure 5. The work is shown in the margin.

$$\begin{array}{r} 2 \\ \underline{3} \\ 5 \end{array}$$

Or, you can express it thus: $2 + 3 = 5$.

The sign $+$ is called **plus**, and means that the numbers between which it is placed are to be counted together; and the sign $=$ means **equals**. So that $2 + 3 = 5$ is read 2 plus 3 equals 5.

33. The operation of finding a number equal to two or more numbers taken together is called **addition**; and the result is called their **sum**. The numbers to be added are called **addends.**

Name the sums of the following numbers, and practice naming them until you can name each sum the instant your eye rests upon the numbers to be added.

Ex. 9. (*Oral.*)

$1+1=$	$3+1=$	$1+0=$	$1+7=$	$1+5=$
$2+1=$	$8+1=$	$1+4=$	$6+1=$	$9+1=$
$2+2=$	$2+0=$	$1+2=$	$8+2=$	$2+7=$

$2+5=$	$6+2=$	$3+2=$	$2+4=$	$2+9=$
$3+4=$	$2+3=$	$5+3=$	$1+3=$	$3+7=$
$8+3=$	$3+0=$	$3+6=$	$3+3=$	$9+3=$
$0+4=$	$5+4=$	$4+7=$	$9+4=$	$4+1=$
$8+4=$	$4+3=$	$2+4=$	$4+4=$	$4+6=$
$5+5=$	$5+7=$	$3+5=$	$0+5=$	$5+9=$
$2+5=$	$5+1=$	$4+5=$	$5+6=$	$8+5=$
$6+3=$	$1+6=$	$5+6=$	$6+0=$	$2+6=$
$6+6=$	$4+6=$	$6+9=$	$7+6=$	$6+8=$
$5+7=$	$5+6=$	$7+3=$	$7+1=$	$0+7=$
$8+7=$	$7+2=$	$4+7=$	$7+7=$	$7+9=$
$8+1=$	$5+8=$	$2+8=$	$8+0=$	$7+8=$
$8+3=$	$8+8=$	$4+8=$	$8+6=$	$9+8=$
$9+0=$	$9+9=$	$9+2=$	$1+9=$	$3+9=$
$4+9=$	$9+6=$	$8+9=$	$9+5=$	$7+9=$

$$\begin{array}{cccccccccc}
5 & 6 & 8 & 7 & 4 & 3 & 6 & 6 & 6 & 7 \\
\underline{3} & \underline{7} & \underline{3} & \underline{5} & \underline{5} & \underline{6} & \underline{4} & \underline{8} & \underline{5} & \underline{5}
\end{array}$$

$$\begin{array}{cccccccccc}
7 & 7 & 7 & 7 & 7 & 8 & 5 & 8 & 8 & 8 \\
\underline{7} & \underline{2} & \underline{9} & \underline{4} & \underline{8} & \underline{2} & \underline{8} & \underline{7} & \underline{9} & \underline{8}
\end{array}$$

$$\begin{array}{cccccccccc}
9 & 9 & 9 & 9 & 6 & 2 & 3 & 8 & 3 & 6 \\
\underline{1} & \underline{4} & \underline{7} & \underline{8} & \underline{8} & \underline{5} & \underline{4} & \underline{2} & \underline{6} & \underline{5}
\end{array}$$

$$\begin{array}{cccccccccc}
8 & 7 & 5 & 3 & 7 & 8 & 9 & 5 & 4 & 3 \\
\underline{7} & \underline{2} & \underline{4} & \underline{8} & \underline{3} & \underline{3} & \underline{8} & \underline{3} & \underline{8} & \underline{3}
\end{array}$$

35. Practise the following additions until you can name the results as rapidly as you can count 1, 2, 3, 4, 5, etc.

Ex. 12. (*Oral.*)

Add by twos to 50, beginning 0, 2, 4, 6, 8. Add by twos to 51, beginning 1, 3, 5, 7, 9.

Add by threes to 102, beginning 0, 3, 6. Add by threes to 100, beginning 1, 4, 7. Add by threes to 101, beginning 2, 5, 8.

Add by fours to 100, beginning 0, 4, 8. Add by fours to 101, beginning 1, 5, 9. Add by fours to 102, beginning 2, 6, 10. Add by fours to 103, beginning 3, 7, 11.

Add by fives to 100, beginning 0, 5, 10. Add by fives to 101, beginning 1, 6, 11. Add by fives to 102, beginning 2, 7, 12. Add by fives to 103, beginning 3, 8, 13. Add by fives to 104, beginning 4, 9, 14.

Add by sixes to 102, beginning 0, 6, 12. Add by sixes to 103, beginning 1, 7, 13. Add by sixes to 104, beginning 2, 8, 14. Add by sixes to 105, beginning 3, 9, 15. Add by sixes to 100, beginning 4, 10, 16. Add by sixes to 101, beginning 5, 11, 17.

Add by sevens to 105, beginning 0, 7, 14. Add by sevens to 106, beginning 1, 8, 15. Add by sevens to 100, beginning 2, 9, 16. Add by sevens to 101, beginning 3, 10, 17. Add by sevens to 102, beginning 4, 11, 18. Add by sevens to 103, beginning 5, 12, 19. Add by sevens to 104, beginning 6, 13, 20.

Add by eights to 104, beginning 0, 8, 16. Add by eights to 105, beginning 1, 9, 17. Add by eights to 106, beginning 2, 10, 18. Add by eights to 107, beginning 3, 11, 19. Add by eights to 100, beginning 4, 12, 20. Add by eights to 101, beginning 5, 13, 21. Add by eights to 102, beginning 6, 14, 22. Add by eights to 103, beginning 7, 15, 23.

Add by nines to 108, beginning 0, 9, 18. Add by nines to 100, beginning 1, 10, 19. Add by nines to 101, beginning 2, 11, 20. Add by nines to 102, beginning 3, 12, 21. Add by nines to 103, beginning 4, 13, 22. Add by nines to 104, beginning 5, 14, 23. Add by nines to 105, beginning 6, 15, 24. Add by nines to 106, beginning 7, 16, 25. Add by nines to 107, beginning 8, 17, 26.

36. Practise adding columns of three digits until you can name the sum of any three digits the instant you see them.

Ex. 13. (*Oral.*)

Find the sums of:

1.	3	2	4	5	4	5	6	7	4	3
	5	1	2	3	3	2	3	2	3	2
	4	3	3	4	2	1	4	3	2	7
2.	3	4	5	6	5	7	8	3	4	5
	4	2	4	2	4	2	1	2	3	2
	1	3	2	3	2	3	0	4	2	3
3.	4	5	3	5	4	3	8	6	7	8
	2	2	4	3	1	5	1	2	9	6
	3	1	3	2	3	2	3	5	4	9
4.	1	8	7	6	7	9	5	7	9	6
	3	6	5	7	2	8	6	3	8	7
	5	1	7	9	9	9	4	9	1	8
5.	7	6	5	9	7	6	5	3	9	5
	2	4	5	0	6	6	9	8	9	8
	3	3	4	8	4	6	8	7	9	7

6	3	5	7	6	8	9	7	6	9	8
	9	6	2	3	5	6	5	3	8	3
	7	8	5	4	4	4	7	5	8	3
7	6	6	7	7	4	4	3	5	3	6
	2	5	8	7	9	4	8	5	7	8
	5	3	9	7	4	4	7	5	7	7
8	8	3	2	3	5	5	9	4	2	2
	4	3	2	4	4	6	3	7	9	2
	4	3	9	8	8	6	6	5	8	8
9	9	5	7	8	9	6	7	4	5	8
	9	6	7	4	5	7	4	3	4	2
	7	6	5	7	5	6	8	9	7	9
10	3	2	2	3	6	8	7	8	9	6
	8	9	2	3	7	5	2	2	8	9
	7	8	9	7	4	4	3	2	7	7

87. The quickest way to add columns of four or more digits is to train the eye to see at a glance sums of 20, and simply add these sums. If you add the column given in the margin by single digits, you say to yourself, *ten, thirteen, seventeen, twenty-two, twenty-eight, thirty seven, forty five*; if you add by taking two digits at a time, you say *ten, seventeen, twenty-eight, forty five*; if you add by taking three digits at a time, you say *thirteen, twenty-eight, forty-five*; if you add by 20's, you say *twenty* (separating 5 into 3 and 2), *forty-five*.

8
9
6
5
4
3
2
8

Ex. 14.

Find the sums of:

1.	5	9	6	4	5	7	1	7	8	2
	4	5	4	6	8	3	4	3	4	6
	8	7	9	5	7	8	5	6	3	9
	6	3	7	2	3	6	3	8	2	7
	4	8	5	8	4	9	8	5	7	4
	7	4	8	6	8	2	6	4	8	5
	3	9	2	9	5	7	7	3	2	3
	5	7	6	4	9	3	9	6	8	5

2.	8	5	4	4	5	2	3	5	3	6
	2	3	2	2	3	6	8	3	8	4
	4	6	5	5	2	4	6	8	4	7
	9	8	9	7	8	5	7	9	9	3
	3	2	7	6	4	3	9	5	7	5
	7	8	6	9	8	5	5	7	3	8
	6	5	9	4	6	3	4	6	8	6
	5	3	4	3	2	7	8	9	6	7
	2	4	1	8	1	9	2	4	2	9

3.	7	5	4	2	4	8	4	6	9	7
	3	6	4	6	8	7	9	8	3	9
	5	7	9	8	6	3	6	0	8	6
	8	3	6	4	7	5	7	4	2	3
	2	9	7	7	4	5	4	5	8	4
	8	4	9	9	5	4	3	3	7	5
	6	2	3	8	4	7	7	6	3	8
	3	8	7	2	6	9	6	9	4	6
	4	1	7	3	2	9	2	7	6	8

4.

6	5	9	8	5	9	8	7	9	6
8	8	5	6	9	6	4	9	3	8
5	3	3	7	4	7	5	6	8	3
3	7	4	9	3	5	9	3	9	5
7	9	9	5	7	9	7	4	8	3
4	4	6	8	2	8	6	5	7	6
6	8	8	4	9	3	8	8	3	9
8	3	2	3	6	9	3	6	4	5
2	6	1	2	8	4	2	1	0	2

5.

8	5	3	8	6	3	5	1	3	4
5	9	9	6	5	4	5	3	3	3
5	6	1	4	7	7	6	5	3	2
1	1	1	3	4	3	1	8	2	6
1	1	8	4	9	1	5	5	9	1
9	7	1	0	3	4	1	7	6	6
7	7	9	4	9	3	1	4	2	7
1	1	3	3	1	1	1	0	9	8
6	4	7	9	2	9	8	9	8	9

Ex. 15.

Find the sums of:

1.	2.	3.	4.	5.	6.
50	40	60	30	10	80
20	80	50	10	70	90
70	20	80	90	10	30
60	30	20	40	90	80
30	70	50	20	40	60
90	80	30	50	70	30
80	60	40	70	30	40
10	50	70	80	20	50

7.	8.	9.	10.	11.	12.
40	80	52	30	42	60
21	70	60	23	40	40
90	31	70	52	50	32
50	42	30	91	80	54
83	60	81	70	70	90
70	51	42	33	34	82
62	90	50	80	90	91

13.	14.	15.	16.	17.	18.
51	56	48	36	25	17
46	63	31	42	52	82
30	72	45	50	49	25
25	81	82	81	38	13
32	17	19	14	41	80
47	26	21	35	57	45

19.	20.	21.	22.	23.	24.
18	57	15	44	19	91
24	31	8	21	27	42
91	28	23	36	48	36
33	63	70	8	39	82
64	90	61	14	7	71
75	9	55	27	9	54
37	81	83	59	87	65

25.	26.	27.	28.	29.	30.
48	52	8	16	33	54
9	61	43	48	52	46
17	26	52	85	27	8
29	28	67	7	38	19
83	83	9	26	41	92
75	94	17	35	9	57
21	77	84	54	94	83

31.	32.	33.	34.	35.	36.
55	68	9	13	48	35
67	5	23	99	6	42
78	43	25	7	51	57
9	67	68	85	9	64
4	25	79	64	23	49
18	14	7	39	88	87

Ex. 16.

Add:

1.	2.	3.	4.	5.
123	516	321	225	871
205	341	75	716	215
310	236	184	348	64
79	110	769	519	371
118	196	815	96	296

6.	7.	8.	9.	10.
123	205	310	79	118
516	341	236	110	196
321	75	184	769	815
225	716	348	519	96
871	215	64	371	296

11.	12.	13.	14.	15.
213	421	85	231	526
327	87	222	624	448
98	116	376	785	379
716	615	584	923	87
825	399	972	84	999

16.	17	18.	19.	20.
213	327	98	716	825
421	87	116	615	379
85	222	376	584	972
231	624	785	923	84
526	448	379	87	999

Ex. 17.

Add:

1.	2.	3.	4.
1234	4321	2345	345
368	6450	3456	2783
5721	378	4567	1497
1050	4291	5678	5840
4862	5782	689	9010
9215	6431	7890	2709

5.	6.	7.	8.
5207	3426	2358	9210
3584	783	7291	1029
2671	5279	5946	291
987	1085	7368	3587
3512	9270	5492	2785
6705	876	876	8899

Ex. 18.

Add:

1.	2.	3.	4.
12345	23456	5	92583
3275	72564	23	4620
4721	3785	936	973
371	23584	6543	25
51028	987	92840	9
61234	96	72104	17

5.	6.	7.	8.
23504	358	56789	123456
4368	9246	3587	258071
25	14376	296	589347
9	845	89	258923
36	29	7	720145
378	7	12345	396012

9.	10.	11.	12.	13.
[illegible]	[illegible]	5	345	584321
42384	[illegible]	24	6197	92047
527913	3792	356	52713	3681
80235	589	1497	6204	927
726045	75	36725	571	1078
4386	[illegible]	157345	89	92569

Ex. 19.

Add:

1.	2.	3.
5203461	2587409	1357924
9350472	3582764	6804281
1456849	1357906	5975325
2604030	4670253	7101584
5876543	8492056	9276432
1234567	4759841	6789009

4.	5.
8274108	5791350
3509270	246801
4680259	1384650
3584672	2794589
9876543	6532108
5279614	7999888

38. It is obvious that numbers can be added only when they refer to the same things. Five oranges and three books when "put together" are still 5 oranges and 3 books, and not 8 oranges or 8 books.

It is also obvious that digits can be added only when they refer to the same order of units. Nine hundreds and eight tens when put together are still 9 hundreds and 8 tens, and not 17 hundreds or 17 tens.

Care must be taken, therefore, in writing numbers to be added, that *all the units' digits shall fall in one column, all the tens' digits in the next column (to the left), and all the hundreds' digits in the next column, and so on.*

39. To add columns of digits with absolute accuracy and great rapidity is a real accomplishment, and the operation of addition should be continued until both these results are secured. The beginner, however, will need some **test** of the accuracy of his work. One test is to begin at the bottom of the right-hand column in adding, and write on a piece of waste-paper the entire sum of each column; then to begin at the top of the left-hand column and write also the entire sum of each column; finally, to add the sums obtained in the first addition, and the sums obtained in the second addition, and compare the results.

The study of an example will make the process understood.

Beginning at the top of the left-hand column in adding, and writing the entire sum of each column, we have:		Beginning at the bottom of the right-hand column in adding, and writing the entire sum of each column, we have:
	871254	
	123456	
	789098	
	357912	
	993286	
	3135006	

```
Left-hand first:      Right-hand first:
28                          26
 31                        28
  23                      17
   17                    23
    28                  31
     26                28
-------              -------
3135006              3135006
```

By comparing the results we find each sum to be ***3,135,006, and so infer*** **that the operation is correct.**

Ex. 20.

Find the sums of:

1. 427, 342, 856, 728.
2. 483, 1000, 8000, 648, 3750, 9840.
3. 15, 603, 1145, 6342.
4. 41, 725, 60, 425, 7000, 4900, 398.
5. 39, 876, 5742, 3000, 478, 9873.
6. 327, 4960, 5000, 749, 3000, 7849.
7. 4284, 32, 679, 43, 5006, 7897.
8. 325, 6007, 983, 4050, 678, 9874.
9. 856, 9193, 8765, 4287, 6696, 9185, 979.
10. 7964, 5000, 303, 9784, 5673, 9004.
11. 9007, 34, 6876, 400, 9344, 7879.
12. 45,678, 96, 375, 4784, 9673, 11,980.
13. 7865, 3586, 4321, 8576.
14. 900,542 + 308,970 + 555,674 + 498,785.
15. 456,789 + 304,590 + 600,792 + 480,893 + 514,763.
16. 357,963 + 478,497 + 323,484 + 596,372 + 300,409.
17. 706,963 + 78,405 + 907,342 + 503,476.
18. A man bought a sleigh for \$142, a carriage for \$325, and a pair of horses for \$476. What was the cost of all?
19. A man collected on Monday, \$1290; on Tuesday, \$340; on Wednesday, \$1008. How much was collected in all?
20. A lady paid \$912 for a piano, \$342 for furniture, \$187 for linen, \$46 for silver. What did she pay for all?
21. A farmer had in one flock of sheep, 407; in another, 96; and in a third, 2584. How many had he in all?

22. A man owns four houses; the first is worth $47,050; the second, $9106; the third, $1492; the fourth, $512. What is the value of them all?

23. Five loads of flour weighed as follows: 3500 pounds, 4967 pounds, 3974 pounds, 7982 pounds, 7963 pounds. What was the weight of the whole?

24. A house was bought for $7895; repairs amounted to $1500; new fences, $97; repairs on stable, $463; furniture, $1285. What was the cost of the whole?

25. The population of six towns is: 1674, 9008, 3769, 4000, 7096, 3784. Find the whole population.

26. A house-lot cost $675; for building the house and furnishing materials the carpenters were paid $2245, the masons $540, the painters $320. What was expended on house and lot?

27. A merchant bought carpets to the amount of $4670; curtains, $300; paper-hangings, $1275; matting, $9765. What was the cost of the whole?

28. Find the sum of three hundred thousand six hundred fifty, seven thousand eight hundred thirty-two, eleven thousand five hundred sixty-seven, ten thousand fifty-six, four hundred seventy-two.

29. Find the sum of one hundred sixty-seven thousand, three hundred sixty-seven thousand, nine hundred six thousand, two hundred forty-seven thousand, ten thousand, seven hundred thousand, nine hundred seventy-six thousand, one hundred ninety-five thousand, ninety-seven thousand.

30. Find the sum of two hundred seven, three hundred sixty-two, nine hundred forty-five, two thousand three hundred forty-three, fifteen thousand six hundred twenty-two, forty-five thousand eight.

31 Add 3 thousand 4 hundred 92, one thousand four, 6 thousand 5 hundred seventy, 42 hundred eleven.

32. Add 386 million 591, 546 million 311 thousand 122, 796 thousand 351, 84 hundred 1, 9 thousand, 86 thousand 521, 3 hundred fifty-eight thousand 6 hundred, 8 million 888 thousand eight hundred eighty-eight, 1 hundred million.

33. Find the sum of six million sixty thousand six, seven million nine hundred fifty thousand ninety-nine, ten million nine thousand eight hundred seven, three hundred sixty-seven thousand forty-five.

34. Find the sum of 200 million 302 thousand, 200 thousand two hundred, 50 million 50 thousand 50, 25 million 860 thousand, 47 million 467 thousand, 202 million 6367.

35. What is the sum of eighteen thousand three hundred twenty, seventy-four thousand five hundred six, ten hundred seventeen thousand nine hundred twenty-one, fifty-three thousand seven hundred eleven, five hundred seventy-six thousand three hundred four, six hundred fifty thousand forty-four?

36. A man drew five loads of bricks; in the first load there were 4068; in the second, 1342; in the third, 3927; in the fourth, 1694; in the fifth, 2009. How many in all the loads?

37. What is the united population of the following cities: Utica, 28,804; Lowell, 40,928; Lynn, 28,236; Salem, 24,100; Erie, 19,500; Auburn, 17,225?

38. A fruit-grower sent to market the produce of six peach orchards; from the first, 7000 baskets; from the second, 6973; from the third, 1004; from the fourth, 3276; from the fifth, 1594; from the sixth, 3976. How many baskets in all?

39. The distance from Boston to Springfield is 98 miles, from Springfield to New Haven 62 miles, from New Haven to New York 76 miles. How many miles is it from Boston to New York?

40. An army officer paid at one time $7038 for horses, at another time $7776, at another time $9948. How many dollars did he pay in all?

41. A farmer sold his wheat for $8742, his corn for $13,569, and his oats for $9528. How much did he receive for the whole?

42. A bank has $40,317 in specie, $91,256 in bills, $18,317 in cash items. Find the whole amount.

43. The army of Napoleon at Waterloo consisted of 48,950 infantry, 15,765 cavalry, 7732 artillery. What was the whole number?

44. The Duke of Wellington's army at Waterloo consisted of 20,661 infantry, 8735 cavalry, 6877 artillery. There were also 33,413 allies. What was the whole number of his army?

45. The area of England is 50,535 square miles, of Scotland 29,167 square miles, and of Wales 8125 square miles. How many square miles in England, Scotland, and Wales together?

46. New Hampshire furnished 12,497 soldiers for the Revolution, Massachusetts 67,907, Rhode Island 5908, Connecticut 31,939. How many did these four states furnish?

47. A country merchant has in his store flour worth $656, sugar worth $480, molasses worth $325, cotton cloth worth $125, tea worth $56, canned goods worth $78. What is the whole value of his goods?

48. A farmer sold four loads of hay. The first weighed 2007 pounds, the second 1963 pounds, the third 2585 pounds, the fourth 2614 pounds. How many pounds did the whole weigh?

49. If Abraham was born at the beginning of the year B.C. 1996, how many years from the date of his birth to the end of the year 1889?

50. An orchard contains 112 apple trees, and an equal number of pear trees; 56 peach trees, and an equal number of plum trees; and 19 cherry trees. How many trees are there in the orchard?

51. How many times does a clock strike from half past twelve o'clock at night to half past twelve o'clock at noon?

52. The area of Maine in square miles is 29,895, of New Hampshire 9005, of Vermont 9135, of Massachusetts 8040, of Rhode Island 1085, of Connecticut 4845. What is the area of New England in square miles?

53. The area of New York in square miles is 47,620, of Pennsylvania 44,985, of Virginia 40,125, of North Carolina 48,580, of Ohio 40,760. What is the area of these five states in square miles?

54. The area of Illinois in square miles is 56,000, of Michigan 57,430, of Wisconsin 54,450, of Iowa 55,475, of Missouri 68,735. What is the area of these five states in square miles?

55. The area of Texas in square miles is 262,290, of California 155,980, of Dakota 147,700, of Montana 145,310, of New Mexico 122,460, of Arizona 112,920. Find their total area in square miles.

CHAPTER III.

SUBTRACTION.

40. What number must be added to four to make seven? What, then, will be left if 4 is taken from 7?

What number must be added to seven to make ten? What, then, will be left if 7 is taken from 10?

Copy the following set of numbers, and find what number must be added to each one in the upper row to make the number below the line. Write the required numbers in the empty places above the lines:

7	6	12	4	2	9	0	5	6
—	—	—	—	—	—	—	—	—
17	14	20	7	12	10	5	7	12

13	19	24	7	13	8	9	4	10
—	—	—	—	—	—	—	—	—
15	25	28	12	20	10	11	16	21

When you have done this, you will see that, since 7 and 10 make 17, 7 taken from 17 leaves 10; since 6 and 8 make 14, 6 taken from 14 leaves 8; so with each set of numbers.

41. In the following set, under each number in the lower row, write the number that must be added to it to make the upper number:

9	12	7	12	15	10	6	9	7
3	4	2	8	6	5	0	5	4
—	—	—	—	—	—	—	—	—

11	18	17	5	10	9	16	8	3
5	2	10	1	3	2	5	2	0
—	—	—	—	—	—	—	—	—

To 3 we have to add 6 to make 9, so we write 6 under the 3. To 4 we must add 8 to make 12, so we write 8 under the 4.

Now in finding what number must be added to 3 to make 9, we have really found what number will be left if 3 is taken from 9. In finding what number must be added to 4 to make 12, we have really found what number will remain if 4 is taken from 12.

42. The operation of finding the number that remains, when a smaller number is taken from a larger, is called **subtraction.** The result is called the **remainder** or **difference.**

43. The number which is to be subtracted is called the **subtrahend**; and the number which is to be diminished (that is, the number from which the subtraction is made), is called the **minuend.**

44. A dash — is the sign of subtraction, and when placed between two numbers means that the first number is to be diminished by the second. It is called the **minus** sign.

The expression $4 - 1 = 3$ is read *four minus one equals three.*

45. Three dots $\therefore$ are often used for the word *therefore.* The expression $6+2=8$, $\therefore 8-6=2$, is read *six plus wo equals eight, therefore eight minus six equals two.*

Ex. 21. (*Oral.*)

1. What number with 5 makes 10?
What number with 3 makes 10?
What number with 2 makes 10?
What number with 4 makes 10?

2. What number taken from 10 leaves 2?
What number taken from 10 leaves 4?
What number taken from 10 leaves 3?
What number taken from 10 leaves 5?

3. 5 is one part of 12, what is the other?
8 is one part of 12, what is the other?
3 is one part of 12, what is the other?
7 is one part of 12, what is the other?
9 is one part of 12, what is the other?
6 is one part of 12, what is the other?
10 is one part of 12, what is the other?

4. What number taken from 12 leaves 11?
What number taken from 12 leaves 9?
What number taken from 12 leaves 5?
What number taken from 12 leaves 8?
What number taken from 12 leaves 2?
What number taken from 12 leaves 6?
What number taken from 12 leaves 7?
What number taken from 12 leaves 1?

5. $9+2=$ $\quad \therefore 11-2=$ $\quad$ and $11-9=$
$8+3=$ $\quad \therefore 11-3=$ $\quad$ and $11-8=$
$6+5=$ $\quad \therefore 11-5=$ $\quad$ and $11-6=$
$10+1=$ $\quad \therefore 11-1=$ $\quad$ and $11-10=$

6.	$8+5=$	$\therefore 13-5=$	**and** $13-8=$
	$6+7=$	$\therefore 13-7=$	**and** $13-6=$
	$9+4=$	$\therefore 13-4=$	**and** $13-9=$
7.	$6+8=$	$\therefore 14-6=$	**and** $14-8=$
	$5+9=$	$\therefore 14-9=$	**and** $14-5=$
	$7+7=$	$\therefore 14-7=$	
8.	$7+8=$	$\therefore 15-7=$	**and** $15-8=$
	$9+6=$	$\therefore 15-6=$	**and** $15-9=$
	$9+3=$	$\therefore 12-9=$	**and** $12-3=$
9.	$8+8=$	$\therefore 16-8=$	
	$7+9=$	$\therefore 16-7=$	**and** $16-9=$
	$9+8=$	$\therefore 17-9=$	**and** $17-8=$

10. Subtract by threes, from 100 to 1; from 102 to 0; by fours, from 101 to 1; from 102 to 2; from 103 to 3.

11. Subtract by fives, from 102 to 2; from 103 to 3; from 104 to 4; from 100 to 5.

12. Subtract by sixes, from 103 to 1; from 104 to 2; from 105 to 3; from 100 to 4; from 102 to 6.

13. Subtract by sevens, from 106 to 1; from 100 to 2; from 101 to 3; from 102 to 4; from 103 to 5; from 104 to 6; from 105 to 7.

14. Subtract by eights, from 105 to 1; from 106 to 2; from 107 to 3; from 100 to 4; from 101 to 5; from 102 to 6; from 103 to 7; from 104 to 8.

15. Subtract by nines, from 100 to 1; from 101 to 2; from 102 to 3; from 103 to 4; from 104 to 5; from 105 to 6; from 106 to 7.

Ex. 22. (*Oral.*)

$5+4=$	$\therefore 9-5=$	$9-4=$
$9+3=$	$\therefore 12-9=$	$12-3=$
$6+5=$	$\therefore 11-6=$	$11-5=$
$7+6=$	$\therefore 13-7=$	$13-6=$
$9+6=$	$\therefore 15-6=$	$15-9=$
$7+9=$	$\therefore 16-9=$	$16-7=$

$14-8=$	$16-9=$	$18-6=$	$17-8=$	$25-9=$
$11-3=$	$33-8=$	$45-6=$	$76-8=$	$32-9=$
$16-7=$	$24-9=$	$37-8=$	$48-6=$	$53-9=$
$17-8=$	$35-8=$	$43-7=$	$50-4=$	$63-6=$
$12-4=$	$44-7=$	$24-8=$	$31-3=$	$26-9=$
$15-7=$	$68-9=$	$56-7=$	$43-5=$	$29-7=$
$13-6=$	$27-8=$	$34-9=$	$40-9=$	$50-7=$
$11-8=$	$13-8=$	$15-8=$	$13-9=$	$31-3=$
$27-9=$	$86-8=$	$85-9=$	$87-6=$	$84-5=$
$32-8=$	$73-5=$	$62-7=$	$26-9=$	$23-7=$
$25-4=$	$75-9=$	$73-7=$	$72-6=$	$83-8=$
$17-9=$	$31-8=$	$42-9=$	$50-3=$	$39-8=$
$42-3=$	$30-6=$	$38-9=$	$40-4=$	$93-7=$
$37-9=$	$58-9=$	$52-6=$	$63-8=$	$41-3=$
$24-7=$	$70-8=$	$21-9=$	$22-7=$	$38-9=$
$45-8=$	$42-3=$	$54-7=$	$71-8=$	$65-7=$
$19-8=$	$60-3=$	$65-9=$	$64-6=$	$17-9=$
$34-6=$	$95-6=$	$82-8=$	$79-9=$	$76-8=$
$28-9=$	$72-7=$	$90-9=$	$65-6=$	$81-7=$
$54-5=$	$77-8=$	$85-7=$	$69-9=$	$71-4=$

From 876 take 631.

Write units under units, tens under tens, and so on. Then 1 unit from 6 units leaves 5 units, and we write 5 under the units' column; 3 tens from 7 tens leave 4 tens, and we write 4 under the tens' column; 6 hundreds from 8 hundreds leave 2 hundreds, and we write 2 under the hundreds' column. The remainder, therefore, is 2 hundreds 4 tens 5 units; that is, 245.

Operation.

Minuend,	876
Subtrahend,	631
Remainder,	245

46. The minuend is the sum of the subtrahend and the remainder. Hence, to test the accuracy of the work, add the subtrahend and remainder together, and if the work is correct, their sum will be equal to the minuend.

47. It is obvious that one number can be subtracted from another only when both numbers refer to the same things. Thus, we can subtract 3 oranges from 5 oranges, but we cannot subtract 3 apples from 5 oranges.

Ex. 23.

Find the results of:

1. 59 — 23.	**13.** 89 — 41.	**25.** 786 — 45.
2. 54 — 23.	**14.** 67 — 23.	**26.** 674 — 52.
3. 67 — 14.	**15.** 58 — 17.	**27.** 569 — 38.
4. 65 — 32.	**16.** 75 — 34.	**28.** 857 — 43.
5. 78 — 25.	**17.** 96 — 53.	**29.** 294 — 82.
6. 75 — 41.	**18.** 87 — 42.	**30.** 348 — 37.
7. 85 — 33.	**19.** 69 — 37.	**31.** 489 — 76.
8. 78 — 25.	**20.** 78 — 26.	**32.** 768 — 47.
9. 96 — 42.	**21.** 64 — 43.	**33.** 976 — 53.
10. 97 — 54.	**22.** 98 — 35.	**34.** 897 — 75.
11. 87 — 54.	**23.** 89 — 53.	**35.** 588 — 64.
12. 86 — 31.	**24.** 77 — 46.	**36.** 467 — 45.

37. 874 — 632.	**42.** 6982 — 5431.	**47.** 725,419 — 613,208.
38. 792 — 261.	**43.** 7629 — 4518.	**48.** 965,420 — 342,100.
39. 798 — 627.	**44.** 7824 — 6821.	**49.** 854,267 — 723,150.
40. 764 — 532.	**45.** 8542 — 6131.	**50.** 549,830 — 438,820.
41. 862 — 741.	**46.** 8792 — 6281.	**51.** 628,300 — 517,200.

48. If the number of units of any order in the minuend is less than the number of units of the corresponding order in the subtrahend, one of the next higher order of units in the minuend must be added to the units of the order we are considering. The process will be understood by an example.

From 783 take 469.

Operation.

Minuend,	783
Subtrahend,	469
Remainder,	314

Since we cannot take 9 units from 3 units, we add 1 of the 8 *tens* to the 3 *units*, making 13 *units*; then 9 units from 13 units leave 4 units. Now as we have added 1 of the 8 *tens* to the 3 *units* of the minuend, we have only 7 tens remaining, and 6 tens from 7 tens leave 1 ten; 4 hundreds from 7 hundreds leave 3 hundreds. The remainder, therefore, is 3 hundreds 1 ten 4 units; that is, 314.

From 359 take 186.

Operation.

Minuend,	359
Subtrahend,	186
Remainder,	173

Here 6 units from 9 units leave 3 units. Since we cannot take 8 tens from 5 tens we add 1 of the 3 *hundreds* to the 5 *tens*, making 15 *tens*; then 8 tens from 15 tens leave 7 tens. Now as we have added 1 of the 3 *hundreds* to the 5 *tens* of the minuend, we have only 2 hundreds remaining; and 1 hundred from 2 hundreds leaves 1 hundred. The remainder, therefore, is 1 hundred 7 tens 3 units; that is, 173.

Ex. 24.

1. 867 — 325.
2. 985 — 312.
3. 746 — 213.
4. 384 — 132.
5. 479 — 235.
6. 679 — 215.
7. 857 — 324.
8. 956 — 532.
9. 795 — 362.
10. 687 — 321.
11. 978 — 333.
12. 835 — 214.
13. 90 — 35.
14. 40 — 13.
15. 70 — 26.
16. 50 — 24.
17. 80 — 32.
18. 60 — 33.
19. 60 — 47.
20. 70 — 45.
21. 70 — 52.
22. 80 — 36.
23. 90 — 28.
24. 90 — 27.
25. 70 — 28.
26. 50 — 13.
27. 80 — 37.
28. 60 — 48.
29. 90 — 25.
30. 50 — 27.
31. 80 — 43.
32. 70 — 36.
33. 90 — 32.
34. 60 — 27.
35. 80 — 49.
36. 90 — 36.

Ex. 25.

1. 52 — 26.
2. 73 — 38.
3. 81 — 49.
4. 94 — 57.
5. 72 — 48.
6. 91 — 64.
7. 75 — 48.
8. 92 — 48.
9. 83 — 26.
10. 95 — 47.
11. 86 — 57.
12. 95 — 66.
13. 63 — 29.
14. 74 — 37.
15. 92 — 68.
16. 81 — 56.
17. 75 — 38.
18. 96 — 48.
19. 85 — 57.
20. 93 — 75.
21. 54 — 18.
22. 81 — 27.
23. 75 — 29.
24. 94 — 58.
25. 680 — 247.
26. 570 — 236.
27. 860 — 218.
28. 690 — 254.
29. 750 — 419.
30. 830 — 214.
31. 690 — 275.
32. 750 — 326.
33. 860 — 247.
34. 970 — 358.
35. 580 — 149.
36. 870 — 146.

Ex. 26.

1. 407 — 84.
2. 308 — 75.
3. 609 — 58.
4. 205 — 81.
5. 506 — 63.
6. 807 — 42.
7. 462 — 38.
8. 374 — 57.
9. 281 — 65.
10. 592 — 83.
11. 476 — 68.
12. 852 — 39.
13. 608 — 247.
14. 706 — 253.
15. 805 — 364.
16. 904 — 472.
17. 809 — 581.
18. 705 — 694.

19. 508 — 294.
20. 609 — 385.
21. 707 — 246.
22. 806 — 324.
23. 405 — 132.
24. 709 — 328.
25. 781 — 246.
26. 892 — 387.
27. 643 — 418.
28. 954 — 216.
29. 763 — 419.
30. 655 — 247.
31. 461 — 239.
32. 572 — 238.
33. 693 — 447.
34. 754 — 536.
35. 835 — 226.
36. 973 — 237.

Ex. 27.

1. 612 — 78.
2. 523 — 64.
3. 845 — 87.
4. 417 — 58.
5. 731 — 94.
6. 324 — 65.
7. 942 — 74.
8. 635 — 89.
9. 522 — 56.
10. 417 — 68.
11. 325 — 86.
12. 712 — 94.
13. 732 — 458.
14. 816 — 237.
15. 624 — 158.
16. 936 — 489.
17. 567 — 298.
18. 715 — 348.
19. 623 — 417.
20. 861 — 375.
21. 453 — 286.
22. 817 — 329.
23. 643 — 457.
24. 415 — 186.
25. 531 — 352.
26. 642 — 263.
27. 763 — 174.
28. 824 — 296.
29. 915 — 468.
30. 812 — 357.
31. 514 — 136.
32. 972 — 489.
33. 624 — 248.
34. 512 — 136.
35. 713 — 364.
36. 817 — 259.

Ex. 28.

1. 500 — 78.
2. 600 — 83.
3. 700 — 92.
4. 800 — 64.
5. 600 — 57.
6. 400 — 76.
7. 802 — 68.
8. 304 — 95.
9. 506 — 87.
10. 403 — 75.
11. 902 — 94.
12. 504 — 69.
13. 600 — 235.
14. 800 — 217.
15. 900 — 386.
16. 700 — 427.
17. 400 — 128.
18. 800 — 372.
19. 600 — 345.
20. 700 — 562.
21. 800 — 427.
22. 900 — 368.
23. 500 — 321.
24. 600 — 487.
25. 902 — 146.
26. 805 — 347.
27. 704 — 215.
28. 607 — 238.
29. 503 — 267.
30. 906 — 387.
31. 904 — 328.
32. 802 — 467.
33. 705 — 258.
34. 603 — 318.
35. 701 — 427.
36. 705 — 348.

Ex. 29.

1. 7689 — 2345.
2. 6837 — 4216.
3. 9876 — 1234.
4. 8697 — 3274.
5. 7586 — 2145.
6. 6789 — 4321.
7. 8470 — 2138.
8. 6790 — 3245.
9. 9580 — 5136.
10. 7480 — 2367.
11. 9560 — 1423.
12. 8670 — 4324.
13. 8700 — 3218.
14. 9600 — 2745.
15. 9600 — 4347.
16. 7200 — 3647.
17. 8300 — 2746.
18. 7400 — 2843.
19. 8020 — 3647.
20. 7050 — 6873.
21. 6040 — 2895.
22. 8030 — 2746.
23. 7050 — 4873.
24. 6020 — 2748.

Ex. 30.

1. 6005 — 2347.
2. 8002 — 2636.
3. 8003 — 2746.
4. 6005 — 2748.
5. 9004 — 2615.
6. 6003 — 2846.
7. 7035 — 2648.
8. 7023 — 2896.
9. 8021 — 3472.
10. 8064 — 2397.
11. 9012 — 3684.
12. 7054 — 2768.
13. 7000 — 2546.
14. 7000 — 3748.
15. 8000 — 5318.
16. 8000 — 3526.
17. 9000 — 3725.
18. 9000 — 2745.
19. 6324 — 2538.
20. 6245 — 3789.
21. 4517 — 1638.
22. 7253 — 4867.
23. 9215 — 4757.
24. 7214 — 4869.

Ex. 31.

1. 56,739 — 24,316.
2. 68,507 — 47,623.
3. 47,865 — 12,341.
4. 72,006 — 48,315.
5. 65,043 — 17,872.
6. 81,000 — 25,143.
7. 90,000 — 30,906.
8. 90,503 — 47,628.
9. 41,009 — 31,214.
10. 43,020 — 36,748.
11. 26,735 — 9,856.
12. 75,986 — 43,264.
13. 59,001 — 16,739.
14. 89,076 — 569.
15. 60,020 — 24,156.
16. 57,490 — 598.
17. 70,000 — 25,487.
18. 70,000 — 4,139.
19. 60,800 — 36,428.
20. 70,302 — 5,648.
21. 80,040 — 23,619.
22. 63,008 — 47,236.
23. 50,004 — 47,825.
24. 80,047 — 26,578.

Ex. 32.

1. 431,250 – 153,697.
2. 920,503 – 476,829.
3. 523,146 – 286,759.
4. 647,352 – 268,574.
5. 502,304 – 186,475.
6. 625,030 – 274,384.
7. 720,301 – 368,596.
8. 842,003 – 459,687.
9. 715,324 – 369,857.
10. 900,500 – 465,783.
11. 512,435 – 126,867.
12. 600,000 – 285,436.
13. 723,514 – 536,945.
14. 801,050 – 469,872.

Ex. 33.

1. What number must be added to 7428 to make 8047?
2. What number must be taken from 3015 to leave 2405?
3. If the minuend is 78,206, and the subtrahend 35,264, what is the remainder?
4. A man owed $4689. He paid at one time $3894. How much did he still owe?
5. A flour merchant had on hand 2038 barrels of flour. He sold 1299 barrels. How many barrels had he left?
6. Mr. Brown's yearly income is $5067. His expenses are $4093. How much does he save?
7. The population of New England in 1870 was 3,487,924, in 1880, 4,010,529. Find the increase.
8. A house cost $9468. If payments to the amount of $5889 have been made to the builder, how much still remains due?
9. The sum of two numbers is 890,375, and one of them is 309,007. What is the other?

10. A is worth $98,760; B is worth $4586 less than A. How much is B worth?

11. In 1880 the population of Boston was 369,832, and the population of Baltimore was 332,313. How much greater was the population of Boston than that of Baltimore?

12. A tank holding 370 gallons of water was filled by pouring 77 gallons into it. How many gallons were there already in the tank?

13. What number increased by 15,639 will be 28,984?

14. What number subtracted from nine hundred eighty-seven thousand three hundred fifty-nine will leave three hundred thousand two hundred eight?

15. A cotton planter raised 9675 pounds of cotton. He sold 7876 pounds. How many pounds had he left?

16. There were 322 apples on a tree, of which 198 were gathered, and 87 were blown off by the wind. How many were left on the tree?

17. There are 60 minutes in an hour; how many minutes between 4 minutes after 10 o'clock and 3 minutes before 11 o'clock? Between 9 minutes after 1 o'clock and 3 minutes before 2 o'clock?

18. A man purchases a farm for $24,669, and pays down $13,708. How much remains unpaid?

19. Eight hundred seventy-six thousand four hundred twenty-five added to a certain number makes eleven million seven hundred nine thousand three hundred four. What is the number?

20. Two men, A and B, start together from the same place and travel in the same direction. A walks the first day 29 miles, B rides the first day 67 miles. How many miles apart are they at the end of the first day? How many miles would they have been apart if they had travelled in *opposite* directions?

21. A merchant deposited in a bank $10,040; and afterwards drew a check for $3780. How much had he in the bank after the check was paid?

22. What is the difference between 106,074 and 28,999?

23. A man went to market with $10.25. He paid for steak $2; for sugar, $1; for coffee, $1; for fruit, $2; for flour, $2. How much money had he left?

24. A horse cost $397 and was sold for $563. How much was gained?

25. A horse and carriage were bought for $458, and were sold for $539. What was the gain?

26. A cow was sold for $171.25. The cow cost $152. What was the gain?

27. A man bought a house lot for $1290 and sold it for $1196. How much did he lose?

28. A horse, harness, and saddle were bought for $378, and were sold for $423.50. How much was gained?

29. A man owing $7862.50 has paid $5678. How much is still due?

30. From a $50 bank-note a bill of $38.50 was paid. What change was given back?

31. In the siege of Gibraltar (1779–1783) the English fired 57,163 round shot, and the French, 175,741. How many more did the French fire than the English?

32. The length of the Missouri River from its source to the Mississippi is three thousand ninety-six miles, and from its source to the Gulf of Mexico four thousand five hundred six miles. How many miles is it from the junction of the two rivers to the Gulf of Mexico?

33. A lady bought articles in a store amounting to nine dollars and seventy-five cents. She gave in payment a ten-dollar bill. How much change should she receive?

34. A gentleman received from his father $65,784. He paid for a house $28,598. How much had he left?

35. If the area of the Mississippi Valley is 1,237,111 square miles, and the area of the Atlantic slope is 967,576 square miles, find the excess of the Mississippi Valley over the Atlantic slope in square miles.

36. Lake Erie covers 9600 square miles, and Massachusetts contains 8040 square miles. How many more square miles in Lake Erie than in Massachusetts?

37. The foreign immigration into the United States was, in 1883, 603,322, and in 1885, 395,346. How much greater was the number in 1883 than in 1885?

38. The consumption of imported sugar in the United States was, in 1882, 866,517 tons, and in 1880, 730,519 tons. How many more tons were consumed in 1882 than in 1880?

39. A lady bought goods amounting to two dollars and thirty-four cents. She gave a five-dollar bill in payment. What change should she receive?

40. The polar diameter of the earth is 41,707,620 feet, and the equatorial diameter is 41,847,426 feet. Find the difference in feet.

41. The Secretary of the Treasury of the United States estimated the revenue for 1885 to be $330,000,000. The actual revenue was $323,690,706. How much did the actual fall short of the estimated revenue?

42. The population of Chicago in 1860 was 109,260, in 1880, 503,185. Find the increase.

43. The gross earnings of the Eastern Railroad for 1883 were $3,584,506, and the expenses were $2,310,830. Find the net earnings for the year.

44. The population of New York City in 1880 was 1,206,299, in 1860 it was 805,651. Find the increase for twenty years.

45. In 1880 Kentucky raised 149,017,855 pounds of tobacco, and Virginia raised 78,421,860 pounds. How many more pounds did Kentucky raise than Virginia?

46. The value of the tobacco raised in Kentucky in 1880 was $10,431,250, the value of that raised in Virginia was $6,273,749. Find the difference.

47. The population of Massachusetts in 1880 was 1,783,085, and of Virginia 1,512,565. Find the difference.

48. The population of New York in 1880 was 5,082,871, and of Ohio 3,198,062. Find the difference.

49. The population of the United States in 1880 was 50,155,783, in 1870, 38,558,371. Find the increase.

50. In 1885 the railroads of the United States earned from freight $519,690,992, and from passengers $200,883,911. How much more was earned from freight than from passengers?

51. The imports of raw cotton into England in 1871 were 1,778,139,776 pounds, the exports were 362,075,616 pounds. How many more pounds were imported than exported?

[illegible]

MULTIPLICATION.

[illegible] required. [illegible]

[illegible]

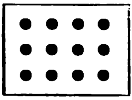
[illegible] written be- [illegible] 7 must be [illegible] and this [illegible] 42. Thus:

$$\begin{array}{r} 7 \\ 6 \\ \hline 42 \end{array}$$

[illegible] this operation 7 is called the multiplicand, 6 the multiplier, and 42 the product. The multiplier 6 is the [illegible] and the product 42 is the sum of six 7s. [illegible] that:

Multiplication is an operation by which, when two numbers are given, called multiplicand and multiplier, a third number is found, called product, which is formed from the multiplicand as the multiplier is formed from unity.

52. The multiplicand is the number to be multiplied. The multiplier is the number by which we multiply. The product is the result obtained. The multiplicand and multiplier are called **factors** of the product. The product of two or more factors is the same in whatever order they are taken. Thus, $3 \times 4 = 4 \times 3$. The dots in the margin, read horizontally, make 3 fours; read vertically, make 4 threes.



53. The sign of multiplication is $\times$. When the multiplier precedes the multiplicand, the sign $\times$ is read *times*. Thus, $6 \times \$7 = \$42$ is read 6 times \$7 equal \$42.

54. When the multiplier follows the multiplicand the sign $\times$ is read *multiplied by*. Thus, $\$7 \times 6 = \$42$ is read \$7 multiplied by 6 equal \$42; and means \$7 *taken* 6 *times* equal \$42. In all cases the product refers to the *same kind of units* as the multiplicand.

55. Products of two factors, which are each less than ten, must be learned by heart.

They can all be readily found by addition. Thus, if the product of 4 times 6 is required. we see that the multiplier 4 is the sum of four 1's, and the multiplicand is 6; hence, the product is the sum of four 6's, and we write

$$\begin{array}{r} 6 \\ 6 \\ 6 \\ \underline{6} \\ 24 \end{array} \qquad \text{Thus, } 4 \times 6 = 24.$$

In the same way every product is found when each of its two factors is less than ten; and the results are all written in the following multiplication table:

MULTIPLICATION TABLE.

2 TIMES	3 TIMES	4 TIMES	5 TIMES
1 ARE 2	1 ARE 3	1 ARE 4	1 ARE 5
2 ARE 4	2 ARE 6	2 ARE 8	2 ARE 10
3 ARE 6	3 ARE 9	3 ARE 12	3 ARE 15
4 ARE 8	4 ARE 12	4 ARE 16	4 ARE 20
5 ARE 10	5 ARE 15	5 ARE 20	5 ARE 25
6 ARE 12	6 ARE 18	6 ARE 24	6 ARE 30
7 ARE 14	7 ARE 21	7 ARE 28	7 ARE 35
8 ARE 16	8 ARE 24	8 ARE 32	8 ARE 40
9 ARE 18	9 ARE 27	9 ARE 36	9 ARE 45
6 TIMES	**7 TIMES**	**8 TIMES**	**9 TIMES**
1 ARE 6	1 ARE 7	1 ARE 8	1 ARE 9
2 ARE 12	2 ARE 14	2 ARE 16	2 ARE 18
3 ARE 18	3 ARE 21	3 ARE 24	3 ARE 27
4 ARE 24	4 ARE 28	4 ARE 32	4 ARE 36
5 ARE 30	5 ARE 35	5 ARE 40	5 ARE 45
6 ARE 36	6 ARE 42	6 ARE 48	6 ARE 54
7 ARE 42	7 ARE 49	7 ARE 56	7 ARE 63
8 ARE 48	8 ARE 56	8 ARE 64	8 ARE 72
9 ARE 54	9 ARE 63	9 ARE 72	9 ARE 81

Ex. 34. (*Oral.*)

1. Of what number are 2 and 4 the factors? 3 and 3? 5 and 3? 2 and 5? 3 and 6?
2. What are the factors of 14? of 9? of 8? of 18? of 6? of 21? of 10?
3. 4 is one factor of 8; what is the other?
   3 is one factor of 12; what is the other?
   9 is one factor of 18; what is the other?
4. 3 times what number make 15?
   6 times what number make 18?
   5 times what number make 25?
   4 times what number make 28?
   3 times what number make 21?
   7 times what number make 14?
5. 8 times what number make 24?
   6 times what number make 24?
   6 times what number make 12? 42? 30? 18?
6. 7 times what number make 21? 63? 35? 49? 56? 14?
7. 8 times what number make 32? 64? 16? 40? 24? 56?
8. 9 times what number make 27? 72? 45? 63? 36? 18?

9.	10.	11.	12.
$6\times2=$	$7\times3=$	$8\times4=$	$0\times5=$
$8\times2=$	$9\times3=$	$1\times4=$	$3\times5=$
$3\times2=$	$0\times3=$	$0\times4=$	$2\times5=$
$9\times2=$	$4\times3=$	$3\times4=$	$7\times5=$
$7\times2=$	$3\times8=$	$9\times4=$	$1\times5=$
$1\times2=$	$3\times3=$	$7\times4=$	$8\times5=$
$0\times2=$	$3\times0=$	$4\times4=$	$6\times5=$
$5\times2=$	$3\times5=$	$6\times4=$	$4\times5=$
$4\times2=$	$3\times1=$	$4\times4=$	$9\times5=$

13.	$1 \times 6 =$	**14.**	$3 \times 7 =$	**15.**	$4 \times 8 =$	**16.**	$1 \times 9 =$
	$9 \times 6 =$		$0 \times 7 =$		$0 \times 8 =$		$7 \times 9 =$
	$8 \times 6 =$		$1 \times 7 =$		$3 \times 8 =$		$0 \times 9 =$
	$3 \times 6 =$		$2 \times 7 =$		$9 \times 8 =$		$3 \times 9 =$
	$5 \times 6 =$		$4 \times 7 =$		$7 \times 8 =$		$8 \times 9 =$
	$0 \times 6 =$		$8 \times 7 =$		$1 \times 8 =$		$6 \times 9 =$
	$7 \times 6 =$		$5 \times 7 =$		$5 \times 8 =$		$9 \times 9 =$
	$2 \times 6 =$		$9 \times 7 =$		$8 \times 8 =$		$5 \times 9 =$
	$4 \times 6 =$		$6 \times 7 =$		$8 \times 6 =$		$9 \times 9 =$
17.	$4 \times 6 =$	**18.**	$3 \times 2 =$	**19.**	$5 \times 2 =$	**20.**	$7 \times 4 =$
	$7 \times 3 =$		$7 \times 9 =$		$8 \times 2 =$		$8 \times 8 =$
	$9 \times 2 =$		$8 \times 3 =$		$6 \times 4 =$		$0 \times 2 =$
	$5 \times 3 =$		$4 \times 5 =$		$7 \times 3 =$		$1 \times 9 =$
	$7 \times 4 =$		$9 \times 6 =$		$0 \times 9 =$		$6 \times 5 =$
	$8 \times 2 =$		$7 \times 4 =$		$7 \times 6 =$		$7 \times 7 =$
	$5 \times 7 =$		$8 \times 9 =$		$8 \times 5 =$		$9 \times 9 =$
	$9 \times 3 =$		$6 \times 5 =$		$9 \times 5 =$		$4 \times 8 =$
	$4 \times 9 =$		$7 \times 8 =$		$3 \times 6 =$		$7 \times 2 =$

56. When the multiplicand consists of two or more digits, and the multiplier is a single digit, it is necessary to multiply each digit of the multiplicand by the multiplier. Thus, the product of 6×4587 is the sum of six numbers, each the same as the multiplicand.

$$\begin{array}{r} 4587 \\ 4587 \\ 4587 \\ 4587 \\ 4587 \\ \underline{4587} \end{array} \quad \text{or} \quad \begin{array}{r} 4587 \\ \underline{6} \\ 27522 \end{array}$$

The sum of the six 7's is 6 times 7 = 42, and we write the 2 units in the column of units, and reserve the 4 tens to be added to the product of the tens; then 6 times 8 tens = 48 tens, which, with the 4 tens, make 52 tens, or 5 hundreds and 2 tens, and we write the 2 tens in the column of tens; then 6 times 5 hundreds = 30 hundreds, which, with the 5 hundreds, make 35 hundreds, or 3 thousands and 5 hundreds, and we write the 5 hundreds in the column of hundreds: then 6 times 4 thousands = 24 thousands, which, with the 3 thousands, make 27 thousands, and we write 27 to the left of the 5 hundreds.

57. When the multiplier is 10, 100, 1000, etc., the product is obtained by simply annexing as many zeros to the multiplicand as are found in the multiplier. Thus:

$$10 \times 4587 = 45,870.$$

Likewise, when the multiplier is any one of the nine significant* digits followed by zeros, the product is obtained by multiplying the multiplicand by the significant digit and annexing to the result as many zeros as are found in the multiplier. Thus, if the multiplicand is 4587, and the multiplier is 600, we multiply by 6 and obtain 27,522, and annex to this result 2 zeros, and have for the required product 2,752,200:

$$\begin{array}{r} 4587 \\ 600 \\ \hline 2,752,200 \end{array}$$

Ex. 35.

Find the products of:

1. 4×80.	**13.** 6×32.	**25.** 3×97.	**37.** 9×96.
2. 8×40.	**14.** 2×62.	**26.** 8×57.	**38.** 6×59.
3. 9×70.	**15.** 7×47.	**27.** 8×75.	**39.** 4×83.
4. 7×60.	**16.** 3×53.	**28.** 9×74.	**40.** 7×84.
5. 5×60.	**17.** 8×54.	**29.** 9×28.	**41.** 5×94.
6. 9×80.	**18.** 4×87.	**30.** 2×86.	**42.** 8×96.
7. 6×90.	**19.** 9×63.	**31.** 2×67.	**43.** 8×86.
8. 9×40.	**20.** 5×96.	**32.** 3×95.	**44.** 9×78.
9. 7×40.	**21.** 5×78.	**33.** 7×85.	**45.** 7×53.
10. 5×90.	**22.** 6×58.	**34.** 4×79.	**46.** 8×83.
11. 8×50.	**23.** 4×86.	**35.** 8×74.	**47.** 9×68.
12. 5×70.	**24.** 7×89.	**36.** 5×68.	**48.** 7×94.

* **The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant* digits.**

Ex. 36.

Find the products of:

1. 7 × 800.	**13.** 6 × 703.	**25.** 5 × 974.	**37.** 8 × 948.
2. 4 × 200.	**14.** 9 × 507.	**26.** 4 × 789.	**38.** 9 × 827.
3. 9 × 700.	**15.** 5 × 809.	**27.** 4 × 947.	**39.** 7 × 825.
4. 5 × 300.	**16.** 7 × 604.	**28.** 5 × 987.	**40.** 8 × 493.
5. 8 × 600.	**17.** 4 × 906.	**29.** 6 × 896.	**41.** 9 × 672.
6. 7 × 400.	**18.** 6 × 803.	**30.** 6 × 456.	**42.** 7 × 756.
7. 6 × 750.	**19.** 2 × 986.	**31.** 7 × 627.	**43.** 8 × 359.
8. 4 × 340.	**20.** 2 × 593.	**32.** 7 × 645.	**44.** 6 × 387.
9. 7 × 960.	**21.** 3 × 593.	**33.** 5 × 865.	**45.** 9 × 865.
10. 6 × 580.	**22.** 3 × 486.	**34.** 8 × 329.	**46.** 5 × 739.
11. 8 × 680.	**23.** 4 × 867.	**35.** 6 × 496.	**47.** 9 × 648.
12. 8 × 630.	**24.** 3 × 837.	**36.** 9 × 584.	**48.** 4 × 867.

Ex. 37.

Find the products of:

1. 9 × 6000.	**13.** 8 × 6070.	**25.** 2 × 6007.
2. 4 × 8000.	**14.** 4 × 9080.	**26.** 9 × 7008.
3. 7 × 8000.	**15.** 6 × 5080.	**27.** 3 × 8005.
4. 7 × 9000.	**16.** 7 × 4070.	**28.** 8 × 4007.
5. 8 × 6000.	**17.** 3 × 7040.	**29.** 4 × 6009.
6. 6 × 7000.	**18.** 9 × 3050.	**30.** 7 × 5006.
7. 6 × 7300.	**19.** 9 × 6320.	**31.** 7 × 8026.
8. 6 × 7400.	**20.** 7 × 3980.	**32.** 6 × 7054.
9. 7 × 8500.	**21.** 6 × 8570.	**33.** 5 × 9045.
10. 6 × 8600.	**22.** 5 × 7390.	**34.** 4 × 6072.
11. 5 × 3900.	**23.** 6 × 8570.	**35.** 9 × 6038.
12. *7 × 7500.*	**24.** 8 × 6780.	**36.** 5 × 5076.

Ex. 38.

Find the products of:

1. 7×7204.
2. 3×6305.
3. 8×9308.
4. 6×4706.
5. 4×6407.
6. 9×3809.
7. 7×3628.
8. 8×6984.
9. 8×5746.
10. 4×4968.
11. 9×9786.
12. 7×3715.
13. 2×4716.
14. 3×3825.
15. 4×6918.
16. 5×5724.
17. 6×6375.
18. 7×8413.
19. 8×5823.
20. 9×3285.
21. 2×7619.
22. 3×9167.
23. 4×4682.
24. 5×2864.
25. 6×3725.
26. 7×5273.
27. 8×6531.
28. 9×1365.
29. 2×8417.
30. 3×7148.
31. 4×6528.
32. 5×8256.
33. 6×3748.
34. 7×4873.
35. 8×5329.
36. 9×9235.

Ex. 39.

Multiply by 2; by 3; and so on to 9:

1. 2739.
2. 4519.
3. 8526.
4. 7658.
5. 5396.
6. 5783.
7. 7463.
8. 8367.
9. 8562.
10. 6483.
11. 3526.
12. 5417.

Multiply by 20; by 30; and so on to 90:

13. 5732.
14. 6749.
15. 8345.
16. 7952.

Multiply by 200; by 300; and so on to 900:

17. 6738.
18. 3579.
19. 5742.
20. 5793.

Multiply by 2000; by 3000; and so on to 9000:

21. 4827.
22. 9357.
23. 6519.
24. 7953.

58. Suppose the product of 649×4587 is required. The multiplier 649 is $600+40+9$, and the product is found by multiplying by 9, then by 40, and then by 600, and adding the partial products. Thus,

	4587	
	649	
9 times the multiplicand =	41283	Partial products.
40 times the multiplicand =	183480	
600 times the multiplicand =	2752200	
649 times the multiplicand =	2976963	

59. The zeros at the right of the partial products do not affect the result of the addition, and may be omitted if care is taken to put the right-hand digit of each partial product directly under the multiplier used. Thus,

```
   4587
    649
  41283
 18348
27522
2976963
```

60. If the multiplier contains zeros, the products that correspond to them will be zero, and need not be written.

Find the product of 2007×4587.

```
   4587                     2007
   2007                     4587
  32109                    14049
9174            Proof:    16056
9206109                  10035
                        8028
                        9206109
```

61. To test the accuracy of the work in multiplication, interchange the multiplicand and the multiplier. If the numerical result is the same in both cases, as in the last *example, the work* may be assumed to be correct.

Ex. 40.

Find the products of:

1.	27×8436.	13.	83×8495.
2.	26×7358.	14.	86×5283.
3.	36×3579.	15.	91×5246.
4.	37×5684.	16.	93×6475.
5.	45×5823.	17.	26×8167.
6.	43×4263.	18.	29×7384.
7.	53×4271.	19.	38×7496.
8.	54×7538.	20.	34×4976.
9.	64×9057.	21.	47×4982.
10.	65×8154.	22.	46×8217.
11.	78×6381.	23.	56×6284.
12.	74×9472.	24.	57×9582.

Ex. 41.

Find the products of:

1.	364×6492.	13.	843×6527.
2.	327×4756.	14.	935×5729.
3.	283×5718.	15.	297×7186.
4.	465×3862.	16.	487×8526.
5.	592×4718.	17.	752×3849.
6.	583×5926.	18.	594×6392.
7.	647×8529.	19.	265×6973.
8.	637×6548.	20.	378×7495.
9.	741×9438.	21.	374×8247.
10.	758×4857.	22.	648×9238.
11.	824×3741.	23.	864×9753.
12.	826×3297.	24.	798×5937.

Ex. 42.

1. What will 29 acres of land cost at $475 an acre?

2. What will 39 passenger cars cost at $3785 a car?

3. A square mile contains 640 acres. How many acres in a county containing 936 square miles?

4. If a cotton factory makes 9360 yards of cloth daily, how many yards will the factory make in a year (313 days)?

5. The cost of building a certain road was, on the average, $1789 a mile. What was the cost of 327 miles of this road?

6. If a field contains 2340 hills of potatoes, and the average number of potatoes in a hill is 12, how many potatoes are there in the field?

7. If a saw mill turns out 5708 feet of boards in a day, how many feet will it turn out in 294 days?

8. A pound of platinum is worth $85. If 4730 pounds are obtained yearly from South America and the Ural Mountains, what is the value of the whole amount?

9. Two cities 294 miles apart are to be connected by a railroad, at a cost of $24,645 per mile. What will be the cost of the road?

10. If 125 tons of steel rails are required for one mile of railroad, how many tons will be necessary for 389 miles?

11. A mile contains 5280 feet. How many feet in 542 miles?

12. The garrison of a fort consumes 785 pounds of bread a day. How many pounds will be consumed in 3 years of 365 days?

13. If a railway train runs 38 miles in an hour, how many miles will it run in 84 trips of 3 hours each?

14. A square mile contains 640 acres. How many acres are there in 3481 square miles?

15. At the rate of 1275 words in an hour, how many words can be sent over a telegraph line in 108 hours?

16. A clock strikes 156 times a day. How many times does it strike in a leap year (366 days)?

17. If a swallow destroys daily 500 insects, how many will it destroy in 92 days?

18. At 27 bushels an acre, how many bushels of wheat will be harvested from 640 acres?

19. A good cow yields 168 pounds of butter a year. If it takes 215,000 cows to supply London with butter, how many pounds of butter are consumed in that city annually?

20. If sound travels at the rate of 1120 feet in a second, how many feet distant is a cloud where the thunder clap follows the flash of lightning in 9 seconds?

21. Find the weight in pounds of 5792 iron bars, each weighing 24 pounds.

22. From what number can 847 be subtracted 307 times, and leave a remainder of 49?

23. If 19 men can do a piece of work in 31 days, how many days will it take one man to do it?

24. If an army consists of 24 regiments averaging 913 men each, how many men are there in the whole army?

25. If one acre produces 211 pounds of cotton, how many pounds will 933,000 acres produce?

26. If one acre produces 154 pounds of tobacco, how many pounds will 10,070 acres produce?

27. If one acre produces 28 bushels of oats, how many bushels will 911,200 acres produce?

28. If one acre produces 42 bushels of corn, how many bushels will 201,106 acres produce?

29. If one acre produces 17 bushels of wheat, how many bushels will 613,263 acres produce?

30. If one acre produces 227 bushels of potatoes, how many bushels will 19,121 acres produce?

31. If one acre produces 23 bushels of barley, how many bushels will 237,769 acres produce?

32. If one acre produces 19 bushels of winter rye, how many bushels will 27,119 acres produce?

33. British India has a population of 150 to the square mile, and contains 1,004,616 square miles. Find its population.

CHAPTER V.

DIVISION.

62. To divide \$42 by 6 is to find *the number of dollars* that must be taken 6 times to make \$42. Again, to divide \$42 by \$6 is to find *the number of times* that it is necessary to take \$6 to make \$42. In either case, the *product* and *one factor* are given and *the other factor* is required. Hence,

63. Division is an operation by which when **the product** and **one factor** are given **the other factor** is found.

64. The number to be divided is called the **dividend,** the number by which the dividend is to be divided is called the **divisor,** and the result is called the **quotient.**

65. Division is indicated by *the sign of division* $\div$, or by writing the dividend over the divisor with a line between them. Thus, each of the expressions $42 \div 6 = 7$, and $\frac{42}{6} = 7$, means and is read "forty-two **divided by** six equals seven."

Ex. 43. (*Oral.*)

$2 \times 8 =$	$\therefore 16 \div 2 =$	$2 \times 6 =$	$\therefore 12 \div 6 =$
	$16 \div 8 =$		$12 \div 2 =$
$2 \times 2 =$	$\therefore 4 \div 2 =$	$2 \times 3 =$	$\therefore 6 \div 3 =$
			$6 \div 2 =$
$2 \times 5 =$	$\therefore 10 \div 2 =$	$2 \times 7 =$	$\therefore 14 \div 7 =$
	$10 \div 5 =$		$14 \div 2 =$

$2 \times 9 =$	$\therefore 18 \div 2 =$	$5 \times 9 =$	$\therefore 45 \div 5 =$
	$18 \div 9 =$		$45 \div 9 =$
$3 \times 4 =$	$\therefore 12 \div 4 =$	$5 \times 8 =$	$\therefore 40 \div 8 =$
	$12 \div 3 =$		$40 \div 5 =$
$3 \times 3 =$	$\therefore 9 \div 3 =$	$5 \times 3 =$	$\therefore 15 \div 3 =$
			$15 \div 5 =$
$3 \times 6 =$	$\therefore 18 \div 6 =$	$5 \times 6 =$	$\therefore 30 \div 6 =$
	$18 \div 3 =$		$30 \div 5 =$
$3 \times 9 =$	$\therefore 27 \div 9 =$	$5 \times 4 =$	$\therefore 20 \div 4 =$
	$27 \div 3 =$		$20 \div 5 =$
$3 \times 7 =$	$\therefore 21 \div 3 =$	$5 \times 7 =$	$\therefore 35 \div 7 =$
	$21 \div 7 =$		$35 \div 5 =$
$3 \times 8 =$	$\therefore 24 \div 8 =$	$6 \times 9 =$	$\therefore 54 \div 6 =$
	$24 \div 3 =$		$54 \div 9 =$
$3 \times 5 =$	$\therefore 15 \div 5 =$	$6 \times 3 =$	$\therefore 18 \div 6 =$
	$15 \div 3 =$		$18 \div 3 =$
$4 \times 5 =$	$\therefore 20 \div 4 =$	$6 \times 6 =$	$\therefore 36 \div 6 =$
	$20 \div 5 =$		
$4 \times 3 =$	$\therefore 12 \div 4 =$	$6 \times 7 =$	$\therefore 42 \div 6 =$
	$12 \div 3 =$		$42 \div 7 =$
$4 \times 6 =$	$\therefore 24 \div 6 =$	$6 \times 8 =$	$\therefore 48 \div 8 =$
	$24 \div 4 =$		$48 \div 6 =$
$4 \times 9 =$	$\therefore 36 \div 9 =$	$6 \times 5 =$	$\therefore 30 \div 6 =$
	$36 \div 4 =$		$30 \div 5 =$
$4 \times 7 =$	$\therefore 28 \div 7 =$	$6 \times 4 =$	$\therefore 24 \div 6 =$
	$28 \div 4 =$		$24 \div 4 =$
$4 \times 8 =$	$\therefore 32 \div 4 =$	$7 \times 3 =$	$\therefore 21 \div 3 =$
	$32 \div 8 =$		$21 \div 7 =$
$4 \times 4 =$	$\therefore 16 \div 4 =$	$7 \times 9 =$	$\therefore 63 \div 9 =$
			$63 \div 7 =$
$5 \times 5 =$	$\therefore 25 \div 5 =$	$7 \times 7 =$	$\therefore 49 \div 7 =$

$7 \times 4 =$	$\therefore 28 \div 4 =$ $28 \div 7 =$	$8 \times 9 =$	$\therefore 72 \div 9 =$ $72 \div 8 =$
$7 \times 8 =$	$\therefore 56 \div 7 =$ $56 \div 8 =$	$8 \times 4 =$	$\therefore 32 \div 4 =$ $32 \div 8 =$
$7 \times 5 =$	$\therefore 35 \div 7 =$ $35 \div 5 =$	$9 \times 3 =$	$\therefore 27 \div 9 =$ $27 \div 3 =$
$7 \times 6 =$	$\therefore 42 \div 6 =$ $42 \div 7 =$	$9 \times 5 =$	$\therefore 45 \div 9 =$ $45 \div 5 =$
$8 \times 8 =$	$\therefore 64 \div 8 =$	$9 \times 9 =$	$\therefore 81 \div 9 =$
$8 \times 3 =$	$\therefore 24 \div 8 =$ $24 \div 3 =$	$9 \times 6 =$	$\therefore 54 \div 9 =$ $54 \div 6 =$
$8 \times 7 =$	$\therefore 56 \div 7 =$ $56 \div 8 =$	$9 \times 8 =$	$\therefore 72 \div 8 =$ $72 \div 9 =$
$8 \times 5 =$	$\therefore 40 \div 5 =$ $40 \div 8 =$	$9 \times 7 =$	$\therefore 63 \div 9 =$ $63 \div 7 =$

66. In the following exercises, the divisor for each line of dividends is written at the left. The quotients should be named without a moment's hesitation.

Ex. 44. (*Oral.*)

1.	6)	54	18	42	12	6	24	36	60	30	48
2.	4)	28	16	20	32	40	36	4	24	12	8
3.	8)	32	48	64	16	40	24	56	8	72	0
4.	9)	0	27	72	81	18	63	54	9	45	36
5.	7)	21	0	14	7	28	42	35	56	63	49
6.	5)	30	45	20	5	40	50	25	15	35	10
7.	3)	9	12	3	30	27	18	15	21	6	24

Ex. 45. (*Oral.*)

Give the quotients and remainders in the following examples:

1.	2) 11	7	9	15	13	14	18	19	16	17
2.	3) 7	11	10	13	16	14	17	15	5	19
3.	4) 19	7	13	17	25	15	22	31	33	29
4.	9) 71	83	15	25	34	17	19	62	26	44
5.	7) 15	19	27	38	40	54	48	60	17	39
6.	8) 23	14	31	17	9	25	37	68	71	28
7.	6) 20	15	19	27	32	10	13	45	57	40
8.	5) 14	9	13	21	43	12	49	32	29	38
9.	7) 11	18	26	37	34	53	47	59	16	33
10.	5) 13	11	22	16	42	32	48	31	49	37
11.	6) 21	16	31	19	11	26	39	46	56	41
12.	8) 33	34	39	18	27	41	69	70	75	63
13.	9) 73	16	84	29	35	43	51	64	80	70
14.	5) 37	41	34	27	36	23	28	33	44	48
15.	6) 37	44	17	10	51	58	25	34	59	50
16.	8) 26	39	30	42	53	20	36	43	51	57
17.	9) 21	37	23	41	47	11	55	50	60	65
18.	7) 13	23	61	46	55	69	25	58	62	18
19.	8) 55	67	73	44	[illegible]	[illegible]	74	65	52	77

Short Division.

67. When the divisor is so small that the work can be performed mentally, the process is called **Short Division,** and will be understood from the following examples:

(1) Divide 697,425 by 3.

The divisor is written at the left of the dividend, as in the margin.

$$3\overline{)697425}$$
$$232475$$

Wording. 3 in 6, **2**; in 9, **3**; in 7, **2**; in 14, **4**; in 22, **7**; in 15, **5**.

Here the divisor is contained in 6 twice, in 9 three times, and in 7 twice with remainder 1; this 1 is equal to 10 of the next lower order, and with the 4, the next order of the dividend, makes 14. Then 14 is divided by 3; the quotient is 4 with remainder 2; this 2 is equal to 20 of the next lower order, and with the 2 makes 22. Then 22 is divided by 3; the quotient is 7 with remainder 1. Then 15 is divided by 3, and the quotient is **5.**

(2) Divide 4,236,158 by 7.

$$7\overline{)4236158}$$
$$605165 \text{ \textit{with remainder} } 3.$$

In this example, 7 is not contained in 3, so 0 is the second figure of the quotient: then the next figure 6 of the dividend is joined to the 3, making 36, and the division is continued. When the division is finished, there is a remainder 3.

(3) Divide 54,123 by 9.

$$9\overline{)54123}$$
$$6013 \text{ \textit{with remainder} } 6.$$

Each quotient figure is of the same order of units as the right-hand figure of that part of the dividend used in obtaining it. Thus, 54 in this example are 54 thousands, and the first figure of the quotient is 6 thousands.

(4) Divide $23,087 by 5.

$$5\overline{)\$23087}$$
$$\$4617 \text{ \textit{with} } \$2 \text{ \textit{remaining.}}$$

In this example, we are required to divide 23087 dollars into five equal parts, and find the *number of dollars* in each part. The answer is 4617 *dollars*, with 2 dollars over. The complete quotient may be written $\$4617\frac{2}{5}$.

(5) Divide \$23,087 by \$5.

$$\$5)\underline{\$23087}$$
$$4617 \textit{ with } \$2 \textit{ remaining.}$$

In this example, we are required to find the *number of times* we can take away \$5 from \$23,087, and the answer is 4617 *times*, with \$2 left over. The complete quotient may be written $4617\frac{2}{5}$; and the meaning is, that we can take \$5 away 4617 times from \$23,087, and the next time have \$2 to take away.

68. The last two examples illustrate the different meanings of division. When the divisor corresponds to the multiplier in multiplication the quotient corresponds to the multiplicand, and denotes *the same kind of units as the dividend;* when the divisor corresponds to the multiplicand the quotient corresponds to the multiplier, and denotes the *number of times* the divisor must be taken to obtain a quantity equal to the dividend.

69. A number, when divided by 10, will have a quotient consisting of the same series of figures, the last one being cut off for the remainder. Thus, $35764 \div 10 = 3576$ *with remainder* 4. In this case, the value of each figure in the result is diminished ten-fold, the *tens* becoming *units*, the hundreds becoming *tens*, and so on. A number, when divided by 100, 1000, etc., will have the same series of figures in the quotient, the last *two*, *three*, etc., figures being cut off for the remainder. Hence,

When a divisor ends in one or more zeros, cut off the zeros and an equal number of figures from the right of the dividend, perform the division with the numbers left,

and for the total remainder annex the figures cut off from the dividend to the remainder from the division.

Divide 5,786,342 by 200.

$$2\cancel{0}\cancel{0})\underline{57863\cancel{4}\cancel{2}}$$
$$28931 \textit{ with remainder } 142.$$

In this example, we cut off the two zeros at the right of the divisor and two figures at the right of the dividend; then we divide, putting the first figure of the quotient under the figure 8, which is the right-hand figure of the first partial dividend when the entire divisor 200 is used.

70. The product of the divisor and quotient increased by the remainder is equal to the dividend. Hence,

To test the accuracy of the work of division, find the product of the divisor and quotient, and to this product add the remainder; this result will be equal to the dividend if the work is correct.

Thus, in the last example,

$$200 \times 28{,}931 = 5{,}786{,}200,$$

and $$5{,}786{,}200 + 142 = 5{,}786{,}342 \text{ (the dividend).}$$

Ex. 46.

Find the quotients of:

1. $48 \div 2$.	**10.** $75 \div 5$.	**19.** $91 \div 8$.	**28.** $815 \div 5$.
2. $72 \div 3$.	**11.** $98 \div 7$.	**20.** $94 \div 9$.	**29.** $714 \div 6$.
3. $56 \div 4$.	**12.** $92 \div 4$.	**21.** $94 \div 5$.	**30.** $826 \div 7$.
4. $85 \div 5$.	**13.** $57 \div 2$.	**22.** $87 \div 4$.	**31.** $952 \div 8$.
5. $96 \div 6$.	**14.** $83 \div 3$.	**23.** $95 \div 6$.	**32.** $972 \div 9$.
6. $84 \div 7$.	**15.** $75 \div 4$.	**24.** $77 \div 3$.	**33.** $912 \div 8$.
7. $96 \div 8$.	**16.** $48 \div 5$.	**25.** $734 \div 2$.	**34.** $492 \div 4$.
8. $99 \div 9$.	**17.** $77 \div 6$.	**26.** $768 \div 3$.	**35.** $675 \div 5$.
9. $90 \div 6$.	**18.** $82 \div 7$.	**27.** $956 \div 4$.	**36.** $918 \div 6$.

37. 513 ÷ 2.	**53.** 9354 ÷ 6.	**69.** 4017 ÷ 7.
38. 719 ÷ 3.	**54.** 8176 ÷ 7.	**70.** 7139 ÷ 8.
39. 623 ÷ 4.	**55.** 9456 ÷ 8.	**71.** 9415 ÷ 6.
40. 749 ÷ 5.	**56.** 8568 ÷ 9.	**72.** 8793 ÷ 5.
41. 875 ÷ 6.	**57.** 3712 ÷ 8.	**73.** 3794 ÷ 2.
42. 643 ÷ 7.	**58.** 2226 ÷ 7.	**74.** 7929 ÷ 3.
43. 927 ÷ 8.	**59.** 2550 ÷ 6.	**75.** 6728 ÷ 4.
44. 705 ÷ 9.	**60.** 2895 ÷ 5.	**76.** 6380 ÷ 5.
45. 591 ÷ 8.	**61.** 5391 ÷ 2.	**77.** 8322 ÷ 6.
46. 853 ÷ 7.	**62.** 7418 ÷ 3.	**78.** 9219 ÷ 7.
47. 735 ÷ 6.	**63.** 5327 ÷ 4.	**79.** 7395 ÷ 2.
48. 923 ÷ 5.	**64.** 8236 ÷ 5.	**80.** 7684 ÷ 3.
49. 7594 ÷ 2.	**65.** 7129 ÷ 6.	**81.** 7315 ÷ 4.
50. 7458 ÷ 3.	**66.** 8513 ÷ 7.	**82.** 8369 ÷ 5.
51. 9656 ÷ 4.	**67.** 9237 ÷ 8.	**83.** 5869 ÷ 6.
52. 7985 ÷ 5.	**68.** 5682 ÷ 9.	**84.** 4239 ÷ 7.

Divide by 2; by 3; and so on to 9:

85. 5794. **86.** 4572. **87.** 9785. **88.** 7163.

Divide by 20; by 30; and so on to 90.

89. 8239. **90.** 5197. **91.** 3274. **92.** 5834.

Divide by 200; by 300; and so on to 900:

93. 4571. **94.** 5768. **95.** 9563. **96.** 9876.

Ex. 47.

1. There were 72 children in a Sunday-school, and they walked two and two to church. How many rows would they make? How many rows would there have been if they had walked three and three?

2. A boy had 97 filberts. He kept 34 for himself, and divided the rest equally among his 9 class-mates. How many did he give to each?

3. How many times must we take the number 7 to make 819? How many times the number 9?

4. Divide a paper of 264 pins equally into 8 papers.

5. 2691 poles were used in a certain hop-yard, and 3 were required for each plant. How many plants were there?

6. A blacksmith uses 7 nails in putting on one shoe, and in one day he used 336 nails. How many hoofs did he shoe?

7. A forest of 1995 trees is to be thinned by cutting down 1 tree in 7. How many will be taken out?

8. A regiment consists of 1200 men and 60 officers. How many men are there to each officer?

9. When beef is $7 per hundred-weight, how many hundred-weight can be bought for $9,700,327?

10. How many tons of coal, at $9, can be bought for $3,596,301?

11. A wagon travels 58,068 feet. How many times will a wheel 12 feet in circumference turn in going that distance?

12. A square yard contains 9 square feet. How many square yards in 3,917,502 square feet?

13. Aaron Reed left $325,645 for his wife and four children. How much had each, if the property was divided equally among them?

14. A grocer sells brown sugar at $9 per hundred-weight. If he receives $976,482, how many hundred-weight does he sell?

15. John Brown paid $375,008 for a tract of wild land, at $8 per acre. How many acres did he buy?

16. How many tons of coal, at $7 per ton, can be purchased for $3,785,908?

17. A merchant received $397,640 in selling a quantity of flour, at $8 per barrel. How many barrels did he sell?

18. What must be paid for 12 yards of cloth, if 5 yards cost $25?

SOLUTION. If 5 yards cost $25, to find the cost of 1 yard $25 must be divided by 5; $25 ÷ 5 = $5, cost of 1 yard. 12 yards will cost 12 × $5 = $60. *Ans.*

19. A drover paid $20 for 5 sheep. What will be the cost of 125 sheep?

20. Three cows cost $156. What must be paid for 27 cows?

21. If 7 tons of hay cost $105, what will be the cost of 63 tons?

22. If 9 barrels of flour are worth $63, how many barrels of apples, at $3 a barrel, will pay for 72 barrels of flour?

23. If 7 cords of birch wood are worth $28, how many cords of birch wood will pay for 6 barrels of sugar worth $16 a barrel?

24. If 12 men do a piece of work in 12 hours, how many *hours* would it take 8 men to do the same work?

LONG DIVISION.

71. The process of Long Division is the same as that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend.

Divide 41,668 by 78.

The beginner will find it convenient to form a table of products of the divisor by the numbers 1, 2, 3, ..., as follows:

$1 \times 78 = 78$	$4 \times 78 = 312$	$7 \times 78 = 546$
$2 \times 78 = 156$	$5 \times 78 = 390$	$8 \times 78 = 624$
$3 \times 78 = 234$	$6 \times 78 = 468$	$9 \times 78 = 702$

The third product is found by adding the first and second products, the fourth by adding the first and third, and so on.

As 78 is more than 41, it is necessary to take *three* figures of the dividend for the first partial dividend. Of the products in the table that do not exceed 419 the greatest is 390, that is, 5×78. Hence the first quotient figure is 5, and is written over the 9 in the dividend; then 390 is subtracted from 419. To the remainder 29, the next figure 9 of the dividend is annexed. Of the products that do not exceed 299, the greatest is 234, that is, 3×78. Hence 3 is the next figure of the quotient, and the next remainder is 65, to which the 8 of the dividend is annexed. Of the products that do not exceed 658, the greatest is 624, that is, 8×78. Hence the next figure of the quotient is 8, and the remainder 34.

OPERATION.

$$\begin{array}{r@{\,}l}
 & 538 \\
78) & 41998 \\
 & 390 \\ \hline
 & \ \ 299 \\
 & \ \ 234 \\ \hline
 & \ \ \ \ 658 \\
 & \ \ \ \ 624 \\ \hline
 & \ \ \ \ \ \ 34 \text{ remainder.}
\end{array}$$

After a little practice the operation of division can be performed without the aid of a table of products. Each quotient figure is estimated by taking for a trial divisor the *left-hand figure of the* divisor (or the left-hand figure in-

creased by 1, when the next figure is greater than 5), and by taking for a trial dividend one or two figures only of each partial dividend. When the trial divisor is increased by 1, the trial dividend should be increased by 1.

Divide 2,791,163 by 394.

The first partial dividend is 2791. As 9, the second figure of the divisor, is greater than 5, we take 4 for a trial divisor. As we have increased the trial divisor, we increase the trial dividend by 1, making it 28. 4 is contained 7 times in 28. We write the 7 over the 1, and multiply the divisor 394 by 7. We subtract the product 2758 from 2791 and have for a remainder 33, to which we annex the 1 of the dividend. As 331 is less than 394, the next quotient figure is 0. To 331 we annex the next figure 6 of the dividend. 4 is contained 8 times in 34. We therefore write 8 for the next quotient figure, and find the product of 8×394 to be 3152. The remainder obtained by subtracting 3152 is 164, to which the 3 of the dividend is annexed. 4 is contained 4 times in 17. The product of 4×394 is 1576, and this subtracted from 1643 leaves 67 for the final remainder.

OPERATION.

$$\begin{array}{r}
7084 \\
394\overline{)2791163} \\
\underline{2758} \\
3316 \\
\underline{3152} \\
1643 \\
\underline{1576} \\
67 \text{ remainder.}
\end{array}$$

NOTE. If the product of the divisor by the quotient figure is greater than the partial dividend, the quotient figure is too large, and must be diminished; and, if the difference between the partial dividend and the product of the divisor by the quotient figure is greater than the divisor, the quotient figure is too small and must be increased.

Ex. 48.

Find the quotients of:

1. $4386 \div 21$.
2. $5271 \div 31$.
3. $8056 \div 41$.
4. $7158 \div 51$.
5. $9357 \div 61$.
6. $5263 \div 71$.
7. $3046 \div 82$.
8. $7219 \div 92$.
9. $6985 \div 22$.
10. $9876 \div 32$.
11. $2378 \div 42$.
12. $4068 \div 52$.

13. 8359 ÷ 63.
14. 4573 ÷ 73.
15. 7358 ÷ 84.
16. 3985 ÷ 94.
17. 6973 ÷ 25.
18. 7413 ÷ 36.
19. 8765 ÷ 47.
20. 7654 ÷ 57.
21. 6,543 ÷ 68.
22. 8,319 ÷ 78.
23. 5,432 ÷ 89.
24. 9,753 ÷ 99.
25. 41,268 ÷ 21.
26. 74,306 ÷ 31.
27. 89,415 ÷ 42.
28. 67,834 ÷ 52.
29. 79,853 ÷ 63.
30. 82,569 ÷ 73.
31. 94,365 ÷ 84.
32. 98,765 ÷ 94.
33. 82,639 ÷ 25.
34. 64,372 ÷ 35.
35. 59,036 ÷ 46.
36. 42,837 ÷ 56.

Ex. 49.

Find the quotients of:

1. 84,317 ÷ 67.
2. 72,659 ÷ 77.
3. 64,980 ÷ 88.
4. 52,196 ÷ 98.
5. 47,028 ÷ 29.
6. 74,369 ÷ 39.
7. 54,371 ÷ 14.
8. 68,594 ÷ 15.
9. 73,109 ÷ 16.
10. 82,563 ÷ 17.
11. 94,069 ÷ 18.
12. 47,938 ÷ 19.
13. 437,650 ÷ 23.
14. 657,320 ÷ 35.
15. 327,045 ÷ 47.
16. 632,008 ÷ 59.
17. 437,650 ÷ 33.
18. 657,320 ÷ 45.
19. 327,045 ÷ 57.
20. 632,008 ÷ 69.
21. 437,650 ÷ 43.
22. 657,320 ÷ 55.
23. 327,045 ÷ 67.
24. 632,008 ÷ 79.
25. 437,650 ÷ 53.
26. 657,320 ÷ 65.
27. 327,045 ÷ 77.
28. 632,008 ÷ 89.
29. 437,650 ÷ 63.
30. 657,320 ÷ 75.
31. 327,045 ÷ 87.
32. 632,008 ÷ 99.
33. 437,650 ÷ 73.
34. 657,320 ÷ 85.
35. 327,045 ÷ 97.
36. 632,008 ÷ 29.

Ex. 50.

Find the quotients of:

1. 50,576 ÷ 101.
2. 50,576 ÷ 102.
3. 50,576 ÷ 203.
4. 50,576 ÷ 205.
5. 50,576 ÷ 302.
6. 50,576 ÷ 106.
7. 76,593 ÷ 415.
8. 76,593 ÷ 516.
9. 76,593 ÷ 621.
10. 76,593 ÷ 732.
11. 76,593 ÷ 843.
12. 76,593 ÷ 954.
13. 96,432 ÷ 781.
14. 96,432 ÷ 592.
15. 96,432 ÷ 864.
16. 96,432 ÷ 972.
17. 96,432 ÷ 492.
18. 96,432 ÷ 993.

Ex. 51.

Find the quotients of:

1. 861,345 ÷ 4001.
2. 861,345 ÷ 2048.
3. 861,345 ÷ 3507.
4. 861,345 ÷ 6409.
5. 861,345 ÷ 8157.
6. 861,345 ÷ 3965.
7. 730,604 ÷ 8403.
8. 972,817 ÷ 7184.
9. 854,235 ÷ 8794.
10. 730,604 ÷ 5748.
11. 972,817 ÷ 4981.
12. 730,604 ÷ 1984.

Ex. 52.

1. How many stoves can be bought for $1120, if one stove costs $35?

2. If a carriage is valued at $144, how many carriages can be bought at the same rate for $54,000?

3. A horse dealer bought a horse for $125. How many horses could he buy for $60,625, at the same rate?

4. How many barrels of sugar can be bought for $8352 when $36 is paid for one barrel?

5. A merchant sold 297 barrels of flour for $2673. How much did he get a barrel?

6. George Clifford paid $10,250 for oxen, paying on the average $82 an ox. How many did he buy?

7. How many days will it take a man to dig a ditch 864 feet long, if he can dig 48 feet a day?

8. One share of a certain bank stock is worth $98. How many shares can be bought for $22,050?

9. A farmer sold 19 sheep for $152. For how much a head did he sell them?

10. John Jones paid $1752 for lambs at an average price of $4. How many did he buy?

11. A fruit grower received $1755 for 195 barrels of cranberries. What was the price per barrel?

12. In one square foot there are 144 square inches. How many square feet in 1,375,920 square inches?

13. A public library has a yearly circulation of 56,966 books. How many books are taken daily, if the library is open 313 days in a year?

14. One mile contains 320 rods. How many miles in 348,160 rods?

15. If the dividend is 514,478, the divisor 327, and the remainder 107, what is the quotient?

16. A railroad 478 miles in length cost $3,500,872. What was the average cost per mile?

17. How many house lots, at $321 for each, can be bought for $772,326?

18. A company of 547 men took equal shares in a mine valued at $705,083. How much money did each man invest?

19. If 325 workmen are paid $583,700, what sum does each receive?

20. At $89 per acre, how many acres of land can be purchased for $713,513?

21. Divide one million three hundred seventy-five thousand eight hundred nine by two hundred eighty-seven.

22. A ship averaging 215 miles per day has to sail 3678 miles. How many days will be required for the trip?

23. A New Orleans merchant sends to New York 376,705 gallons of molasses. How many casks will there be if each cask contains 235 gallons?

24. The capital and surplus of a bank amounting to $518,077 belonged to 679 stockholders. What is the average amount belonging to each stockholder?

25. If 34,823 tons of coal are required for 97 steamships, what is the average number of tons for each?

26. A carpet factory running 45 looms makes 17,820 yards of carpet in a fortnight. What number of yards is woven by each loom on the average?

27. A cotton planter raises 428,243 pounds of cotton. If the cotton is put into bales, weighing on the average 401 pounds, what will be the whole number of bales?

28. In one cubic foot there are 1728 cubic inches. How many cubic feet are there in a pile of wood containing 3,507,840 cubic inches?

29. In how many hours will a cistern holding 3330 gallons be filled by a pipe that discharges into it 185 gallons an hour?

30. An army officer paid $107 for a horse. At that rate how many horses can he buy for $317,897?

31. A man having an income of $3874 a year (52 weeks) spent $2314 and saved the rest. How much did he save per week on the average?

32. What number subtracted 88 times from 80,005 will leave 13 as a remainder?

33. How many rolls of carpet at $75 a roll can be bought for $1275?

34. A has 425 horses valued at $58,650; B has 382 acres of land worth $48,514. What is the difference in value between one of A's horses and an acre of B's land?

35. If the dividend is 325,682, the divisor 284, and the remainder 218, what is the quotient?

36. What is the nearest number to 7196 that will contain 372 without a remainder?

37. New York contains 47,620 square miles, Texas 262,290. How many states as large as New York can be made out of Texas, and how many square miles will be left over?

38. Dakota contains 147,700 square miles, Massachusetts 8040. How many states as large as Massachusetts can be made out of Dakota, and how many square miles will be left over?

39. In 1880 Texas produced 550,872,000 pounds of cotton. Allowing 400 pounds to a bale, how many bales of cotton did Texas raise that year?

40. If one pound of sugar is obtained from 18 sugar canes, how many pounds will be obtained from 1,233,216 canes?

CHAPTER VI.

DECIMALS.

72. Numbers which denote **whole units** are called **Integral** numbers; but it is often necessary to express **parts** of a unit.

If a unit is divided into **two equal parts,** each part is called **one-half,** and is expressed by $\frac{1}{2}$. If a unit is divided into **three equal parts,** each part is called **one-third,** and is expressed by $\frac{1}{3}$; two of the parts are called **two-thirds,** and are expressed by $\frac{2}{3}$. Again, if a unit is divided into **four equal parts,** each part is called **one-fourth,** and is expressed by $\frac{1}{4}$; into **five equal parts,** each part is called **one-fifth,** and is expressed by $\frac{1}{5}$; into **six equal parts,** each part is called **one-sixth,** and is expressed by $\frac{1}{6}$; into **seven equal parts,** each part is called **one-seventh,** and is expressed by $\frac{1}{7}$; into **eight equal parts,** each part is called **one-eighth,** and is expressed by $\frac{1}{8}$; into **nine equal parts,** each part is called **one-ninth,** and is expressed by $\frac{1}{9}$; into **ten equal parts,** each part is called **one-tenth,** and is expressed by $\frac{1}{10}$.

If AB (see page opposite) represent a unit of length, each division of the line next below AB represents one-half of a unit; and each division of the second line below AB represents one-third of a unit; and so on.

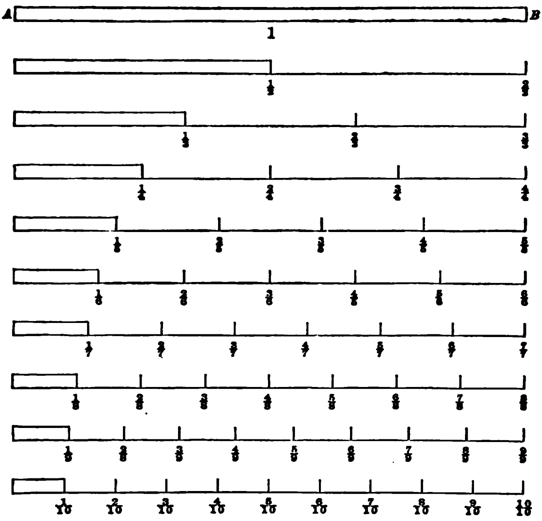
How many halves of a unit make a whole unit?

How many fourths make a half? how many make a whole unit?

How many sixths make a third? how many make a half? how many make a whole unit?

How many eighths make a half? a fourth? a whole unit?

How many tenths make a fifth? a half? a whole unit?

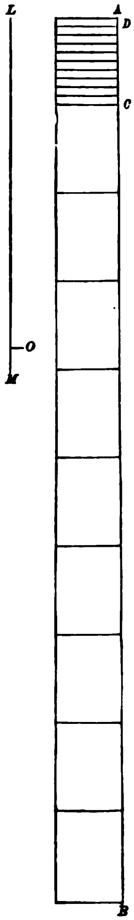


73. When a unit is divided into **ten equal parts,** and we wish to express in figures one or more of these parts, we do not usually write them $\frac{1}{10}$, $\frac{2}{10}$, etc., but we write 1, 2, 3, etc., and separate the number which denotes *parts* of a unit from the number which denotes *whole* units by a decimal point. Thus, two units and three-tenths of a unit are written, 2.3.

If each *tenth* of a unit is divided into ten equal parts, that is, the entire unit into a **hundred equal parts,** each part is called a **hundredth** of the unit; and if each hundredth is divided into ten equal parts, that is, the entire unit into a **thousand equal parts,** each part is called a **thousandth** of the unit; and so on.

These tenth-parts are called **Decimal** parts, from the Latin word *decem,* which means *ten;* and these parts are commonly called **Decimal Fractions.**

Let AB, for example, represent the unit of length by which a certain distance is to be measured. Suppose the given distance to contain AB 137 times, and a remainder LM to be left, which is less than AB. Take AC, a tenth of AB, and suppose AC is contained in LM 4 times, with a remainder OM less than AC. Again, suppose AD, a tenth of AC (that is, a hundredth of AB), to be contained in OM 3 times, with a remainder less than AD. And again, suppose a tenth of AD (that is, a thousandth of AB), to be contained in this last remainder 9 times. Then the whole distance expressed in lengths of AB will be 137.439.



The series of figures 137.439 means 1 hundred + 3 tens + 7 units + 4 tenths + 3 hundredths + 9 thousandths; as 1 hundred = 10 tens = 100 units; and 3 tens = 30 units, the integral value is 137 units; so, 4 tenths = 40 hundredths = 400 thousandths, and 3 hundredths = 30 thousandths; the decimal value therefore is 439 thousandths.

If the unit is the yard-stick, the whole is read "one hundred thirty-seven and four hundred thirty-nine thousandths yards"; if the unit is the meter-stick, the whole is read "137 and 439 thousandths meters."

NOTE. The pupil will get the clearest notions of decimals by taking a meter-stick (which is divided in tenths, hundredths, and thousandths) and measuring given lengths; such as, the length of the side of the room, of the platform, of the window-sill, etc., etc., and writing down the result in each case. Whenever the length measured is less than a meter, he should write down 0, and after it the decimal point, then the actual measure. Thus, if the length is found to be 8 tenths 2 hundredths and 7 thousandths, it is expressed by 0.827, and read "eight hundred twenty-*seven thousandths* of a meter."

74. It will be seen that 1 tenth = 10 hundredths, 1 hundredth = 10 thousandths; and, conversely, 10 thousandths = 1 hundredth, 10 hundredths = 1 tenth, 10 tenths = 1 unit; so that in *decimal* numbers, as in *integral* numbers, 10 in any place is equal to 1 in the next place to the left, and 1 in any place is equal to 10 in the next place to the right.

Hence figures in the *first* decimal place denote *tenths*, in the *second* place *hundredths*, in the *third* place *thousandths*, in the *fourth* place *ten-thousandths*, in the *fifth* place *hundred-thousandths*, in the *sixth* place *millionths*, and so on.

75. In reading decimals, read precisely as if the decimal were an integral number, and add the name of the lowest decimal place. It is best to pronounce the word "and" at the decimal point, and omit it in all other places. Thus, 100.023 is read one hundred *and* twenty-three thousandths. Ambiguity in reading, from having zeros at the end of a decimal, is avoided by a pause; thus, 0.300 is read three hundred . . . thousandths, while 0.00003 is read three . . . hundred-thousandths.

76. Read the following numbers:

0.3; 0.7; 0.65; 0.99; 37.5; 26.9; 425.312; 617.624; 94.57; 83.28; 0.9; 0.96; 57.09; 3.207; 2.03; 3.045; 40.7; 0.055; 0.074; 0.0215; 7.3945; 0.14875; 0.00005; 2.000375; 100.015625; 3.7525; 2.1136257.

77. Express in the decimal notation:

Seven tenths; nine tenths; eleven hundredths; eight hundredths; one hundred thirty-four thousandths; twenty-five thousandths; two hundred and thirty-four thousandths; nineteen *and* forty-one hundred-thousandths; *twenty-five* *and* sixteen ten-thousandths;

thirteen *and* two hundred one hundred-thousandths; six hundred fifty-eight thousand three hundred forty-two millionths; eighty-six *and* eight hundred three thousand three hundred four millionths; three *and* twenty-nine hundredths; fifteen *and* six hundred seventy-one thousandths; fifty-three ten-thousandths; twenty-two *and* sixty-seven hundredths; fourteen *and* two thousand three hundred fifty-one ten-thousandths; two *and* two hundred nineteen thousandths; three *and* one hundred fifty-seven thousandths.

78. Zeros occurring at the end of a decimal do not affect its value. Thus, 3.50700 means 3 units + 5 tenths + 0 hundredths + 7 thousandths + 0 ten-thousandths + 0 hundred-thousandths, and is, therefore, 3 and 507 thousandths, the same as 3.507.

79. The arrangement and method of working employed in decimals is precisely like that employed in integral numbers, the decimal point being the only new consideration.

Addition of Decimals.

Add 17.5163, 236.3, 1.7162, 0.00132.

Operation.

$$\begin{array}{r}
17.5163 \\
236.3 \\
1.7162 \\
\underline{0.00132} \\
255.53382
\end{array}$$

Write the numbers in columns, units under units, tens under tens, tenths under tenths, and so on, so that the decimal points will fall in a vertical line, and add as in *integral numbers.*

Ex. 53.

Find the value of:

1. $2.514 + 3.7 + 9.6304 + 0.24876.$

2. $1.916 + 6.3 + 0.4782 + 9.35634.$

3. $0.415 + 8.0 + 6.3746 + 8.29426.$

4. $7.516 + 9.6 + 1.9238 + 7.21442.$

5. $7.03 + 7.2456 + 0.483 + 9.23579 + 8.3.$

6. $2.576 + 3.4203 + 1.5 + 6.27948 + 0.362357.$

7. $3.29 + 15.671 + 0.0053 + 22.67.$

8. $14.2351 + 651 + 2.219 + 3.157.$

9. $213.7 + 2.913 + 14.769 + 0.007871.$

10. $1.4178 + 0.2 + 2.356709 + 1.14 + 2.0.$

11. $4.96 + 3.2728 + 0.7 + 3.54219 + 4.7.$

12. $1.198 + 3.5 + 7.635487 + 4.23 + 1.5724.$

13. $4.372 + 9.5 + 7.369248 + 1.72 + 3.2948.$

14. $0.4293 + 0.7 + 6.954326 + 3.14 + 7.005.$

15. $3.87 + 2.6493 + 0.8 + 2.63495 + 9.3.$

16. $6.9 + 5.71 + 0.0431 + 329.2 + 4.4.$

17. $3.571 + 0.008 + 12.51 + 649 + 3.051.$

18. $15.753 + 2.069 + 17.6143 + 3.2107.$

19. $1.1 + 20.02 + 13 + 2.845 + 1.0001.$

20. $31.826 + 3.471 + 0.004 + 45 + 0.6.$

21. $82.537 + 2000 + 1.354 + 0.006 + 13.$

22. $64.27 + 1.1 + 23 + 17.12 + 8.8.$

23. $72.5 + 140 + 340.03 + 21.5715 + 4.00087.$

24. $0.96 + 7.3004 + 8010 + 0.00093 + 124650.$

SUBTRACTION OF DECIMALS.

8[illegible]. Subtract 37.286 from 41.1325; and 1.00523 from 9.3.

OPERATION.	OPERATION.
41.1325	9.30000
37.286	1.00523
3.8465	8.29477

Write the subtrahend under the minuend, so that the decimal points may fall in a vertical line. If the number of decimal places in the subtrahend exceed the number in the minuend, zeros may be annexed to the minuend, as such zeros have no effect on its value.

Ex. 54.

1. 0.58 − 0.39.
2. 0.67 − 0.59.
3. 3.927 − 1.836.
4. 4.825 − 1.763.
5. 4.325 − 1.672.
6. 6.283 − 3.576.
7. 9.025 − 6.387.
8. 6.275 − 3.829.
9. 7.57 − 6.385.
10. 9.26 − 2.375.
11. 8.4 − 3.228.
12. 9.5 − 2.732.
13. 14.3846 − 4.8003.
14. 3.4370 − 0.3045.
15. 0.3290 − 0.0089.
16. 136.0200 − 1.5423.
17. 1.9990 − 0.063.
18. 18.5298 − 10.0060.
19. 2.1808 − 0.0009.
20. 1.9870 − 1.0873.
21. 48.9370 − 30.3000.
22. 0.9990 − 0.9009.
23. 15.1409 − 3.8579.
24. 5.9009 − 0.0909.
25. 1.3993 − 0.9090.
26. 10.1010 − 0.0999.
27. 3.5 − 0.075.
28. 517 − 0.0076.
29. 1.325 − 0.4736.
30. 192.3 − 17.294.
31. 175.8 − 1.0024.
32. 186.257 − 13.794.
33. 0.715 − 0.70451.
34. 1111.116 − 22.22222.
35. 71.0047 − 9.0008167.
36. 9161.0098 − 7149.16716.

Multiplication of Decimals.

81. A change in position of the decimal point of a number will affect the local value of each figure of that number. Thus, if in place of 79.213 we write 792.13, we increase the value of each figure ten-fold, the 7 tens become 7 hundreds, the 9 units become 9 tens, the 2 tenths become 2 units, the 1 hundredth becomes 1 tenth, and the 3 thousandths become 3 hundredths, and, as the value of every figure is increased ten-fold, the entire number is increased ten-fold. If the decimal point is moved one place to the left, the local value of each figure is diminished ten-fold, and consequently the value of the entire number is diminished ten-fold. Hence,

To multiply a decimal by 10, 100, 1000, etc., we have only to move the decimal point in the multiplicand as many places to the *right*, annexing zeros if necessary, as there are zeros in the multiplier.

To divide a decimal by 10, 100, 1000, etc., we have only to move the decimal point in the dividend as many places to the *left*, prefixing zeros if necessary, as there are zeros in the divisor.

Thus, $100 \times 36.123 = 3612.3$, and $1000 \times 36.1 = 36100$; $36.123 \div 10 = 3.6123$, and $36.123 \div 1000 = 0.036123$.

82. To multiply a number by 0.1, 0.01, 0.001, etc., we have, by the definition of multiplication, to divide the multiplicand by 10, 100, 1000, etc.; that is, to remove the decimal point one place, two places, etc., to the left.

To divide by 0.1, 0.01, 0.001, etc., we have only to move the decimal point in the dividend one place, two places, etc., to the right.

Thus, $0.1 \times 86.32 = 8.632$, and $0.01 \times 1.236 = 0.01236$; $86.32 \div 0.1 = 863.2$, and $1.236 \div 0.01 = 123.6$.

83. Multiply 123.826 by 3.

Here 3×6 thousandths $= 18$ thousandths, or 1 hundredth and 8 thousandths; the 8 therefore is written in the thousandths' column; then, 3×2 hundredths $= 6$ hundredths, which, with the 1 hundredth, make 7 hundredths, and the 7 is written in the hundredths' column; then, 3×8 tenths $= 24$ tenths, or 2 units and 4 tenths, and the 4 is written in the tenths' column; then, 3×3 units $= 9$ units, which, with the 2 units, make 11 units, and so on.

OPERATION.

$$\begin{array}{r} 123.826 \\ 3 \\ \hline 371.478 \end{array}$$

Multiply 123.826×0.3.

OPERATION.

$$\begin{array}{r} 123.826 \\ 0.3 \\ \hline 37.1478 \end{array}$$

The multiplier $0.3 = 3 \times 0.1$. We therefore multiply first by 3, and the resulting product by 0.1. But multiplying by 0.1 simply moves the decimal point in the product one place to the left. Hence, the product will have three decimal places for the decimal in the multiplicand, and one more place for the decimal in the multiplier.

Multiply 123.826 by 0.32.

OPERATION.

$$\begin{array}{r} 123.826 \\ 0.32 \\ \hline 247652 \\ 371478 \\ \hline 39.62432 \end{array}$$

The multiplier $0.32 = 32 \times 0.01$. We therefore multiply first by 32, and the resulting product by 0.01. But multiplying by 0.01 simply moves the decimal point in the product two places to the left. Hence, the product has three decimal places for the decimal in the multiplicand, and two more places for the decimal in the multiplier.

In the multiplication of decimals, therefore, point off in the product as many decimal places as there are in the multiplicand and multiplier taken together.

Ex. 55.

Find the products of:

1. 5×0.3.
2. 8×0.27.
3. 12×0.375.
4. 15×0.256.
5. 9×0.7.
6. 6×0.75.
7. 16×0.284.
8. 11×0.386.
9. 10×0.65.
10. 100×0.721.
11. 1000×3.736.
12. 1000×0.074.
13. 10×0.99.
14. 100×0.615.
15. 1000×2.409.
16. 1000×0.055.
17. 0.5×37.
18. 0.9×99.
19. 0.25×428.
20. 0.36×7384.
21. 0.9×26.
22. 0.7×67.
23. 0.48×237.
24. 0.18×3692.
25. 0.312×425.
26. 0.716×388.
27. 0.725×96.
28. 0.085×88.
29. 0.624×617.
30. 0.358×776.
31. 0.145×48.
32. 0.017×44.
33. 57×9.4.
34. 26×3.8.
35. 3×972.3.
36. 65×87.2.
37. 2.8×83.
38. 3.2×64.
39. 7.8×369.
40. 3.7×815.
41. 1.44×9.6.
42. 2.88×4.8.
43. 3.21×72.5.
44. 2.16×40.7.
45. 3.26×4.37.
46. 2.03×3.207.
47. 2.472×9.525.
48. 3.264×3.045.
49. 0.7×0.5.
50. 0.9×0.57.
51. 0.45×0.57.
52. 0.72×0.324.
53. 0.6×0.9.
54. 0.8×0.96.
55. 0.72×0.72.
56. 0.36×0.648.
57. 416×0.416.
58. 57×0.015.
59. 693×0.83.
60. 4.625×7.14.
61. 99.9×4.09.
62. 753×0.672.
63. 928×8.302.
64. 56.704×0.413.
65. 2.052×0.0037.
66. 0.00948×29.
67. 372×0.468.
68. 9.43×0.054.
69. 786×3.62.
70. 0.632×85.
71. 2.406×0.008.
72. 6824×3.7.
73. 42.53×0.685.
74. 0.832×59.
75. 763.24×4.078.

Division of Decimals.

84. In Division, if the dividend and divisor are both multiplied or both divided by the same number, the quotient is not changed. Thus, $18 \div 6 = 3$, and (when both dividend and divisor are multiplied by 2) $36 \div 12 = 3$. Again (when both dividend and divisor are divided by 2), $9 \div 3 = 3$.

If, therefore, the divisor contains decimal places, we may remove the decimal point from the divisor, provided we carry the decimal point in the dividend as many places to the right as there are decimal places in the divisor.

Divide 78.528 by 0.8.

Here the decimal point is removed from the divisor, and the decimal point in the dividend is carried one place to the right; that is, both dividend and divisor are multiplied by **10**.

OPERATION.

$$\begin{array}{r} 8\overline{)\,785.28} \\ 98.16 \end{array}$$

When the divisor is a whole number, each quotient figure is of the same **order of units** as the right-hand figure of the partial dividend used in obtaining it. Hence, *the decimal point is put in the quotient as soon as the decimal point in the dividend is reached.*

Divide 28.3696 by 1.49.

OPERATION.

$$\begin{array}{r} 19.04 \\ 149\overline{)2836.96} \\ \underline{149} \\ 1346 \\ \underline{1341} \\ 596 \\ \underline{596} \end{array}$$

Here the decimal point is removed from the divisor, and is moved two places to the right in the dividend; in other words, both dividend and divisor are multiplied by 100.

If the divisor is not contained in the dividend without a remainder, ciphers may be mentally annexed to the dividend, and the division continued.

Divide 0.39842 by 3.7164 to four decimal places.

OPERATION.

```
          0.1072
37164 ) 3984.2
        37164
         267800
         260148
           76520
           74328
            2192
```

If the divisor is a whole number, and ends in zeros, we may cut off the zeros from the divisor, and move the decimal point in the dividend as many places *to the left* as there are zeros cut off.

Divide 42.08 by 8000.

OPERATION.

```
8 ) 0.04208
    0.00526
```

Here the three zeros are cut off from the divisor, and the decimal point in the dividend is moved three places to the left. In other words, both divisor and dividend are divided by 1000.

Ex. 56.

Find the quotients of:

1. $34.24 \div 4.28$.
2. $24.56 \div 6.14$.
3. $52.90 \div 5.75$.
4. $37.576 \div 6.832$.
5. $281.232 \div 7.812$.
6. $97.524 \div 5.16$.
7. $738.0930 \div 0.023$.
8. $5.18466 \div 1.02$.
9. $0.018 \div 9.6$.
10. $34.96818 \div 0.381$.

11. 0.003125 ÷ 25.
12. 859.95 ÷ 136.5.
13. 5.488 ÷ 0.08.
14. 3.4922 ÷ 0.0023.
15. 0.0044408 ÷ 0.0112.
16. 0.20412 ÷ 0.0084.
17. 0.07504 ÷ 23.45.
18. 0.00025 ÷ 2.5.
19. 0.08217 ÷ 1250.
20. 171.99 ÷ 27.3.
21. 0.012305 ÷ 1.07.
22. 15.625 ÷ 2.5.
23. 5.418 ÷ 2.58.
24. 0.59064 ÷ 0.0276.
25. 0.73807 ÷ 0.023.
26. 15.4546 ÷ 0.019.
27. 6.7288 ÷ 64.7.
28. 72.36 ÷ 144.
29. 0.01124 ÷ 11.24.
30. 15.625 ÷ 5.
31. 8.192 ÷ 0.00128.
32. 0.00512 ÷ 2.048.
33. 0.00972 ÷ 0.0004.
34. 0.07504 ÷ 23.45.
35. 15.21 ÷ 11.7.
36. 6 ÷ 0.008.
37. 4.8 ÷ 0.00016.
38. 1562.5 ÷ 0.00025.
39. 64 ÷ 0.016.
40. 5.76 ÷ 0.0048.
41. 3.012 ÷ 0.0006.
42. 91844152.5 ÷ 1.1575.
43. 7 ÷ 0.0035.
44. 0.39237 ÷ 0.319.
45. 0.3230864 ÷ 0.5072.
46. 3.1 ÷ 0.0025.
47. 63.8406 ÷ 0.18345.
48. 181.3 ÷ 0.00037.
49. 12.5 ÷ 2.56.
50. 284.7432 ÷ 0.00004.
51. 130.4 ÷ 0.0004.
52. 113.4 ÷ 0.0108.
53. 68.97516 ÷ 0.9246.
54. 0.022185 ÷ 0.0306.
55. 0.276766 ÷ 0.371.
56. 286 ÷ 0.013.
57. 0.10724 ÷ 0.003125.
58. 0.03 ÷ 0.001.
59. 105 ÷ 43.75.
60. 8.468 ÷ 0.0292.

Ex. 57.

1. To enclose a certain lot 225 yards of fence are needed. What will be the cost of the fence at the rate of $0.50 a yard?

2. A section of land costs $49,878; what must be paid for 0.375 of a section?

3. When 0.7 of a ton of coal is worth $6.30, what will be the cost of 12.5 tons?

4. Coal being worth $7.00 per ton, what part of a ton can be bought for $2.59?

5. If a man can build 0.45 of a rod of wall in one hour, how many rods will 4 men build in 3.8 days, working 7.5 hours per day?

6. Twelve dozen penknives cost $90. If they are sold at $0.75 each, what will be the gain on each?

7. Divide $125.15 by $25.03.

8. Twelve yards of velvet cost $150. At that rate, what must be paid for 18 yards?

9. What will be the cost of 9.75 cords of white oak wood at the rate of $10 a cord?

10. Twenty-five hundredths of a farm cost $5000; what will nine-tenths of it cost?

11. A merchant bought 575 pounds of sugar for $51.75; he sold four-tenths of it at $0.11 per pound, and the remainder at $0.125. What was his gain?

12. A railroad train has 201 miles to run. If it averages 26.8 miles per hour, how many hours will be required?

8. A [illegible] dealer paid [illegible] per M for cedar shingles and sold them for [illegible]. What did he gain on 35 M?

9. A [illegible] contracted for [illegible] at the rate of [illegible] per M. His bill for [illegible] amounted to [illegible]. How many thousand feet did he buy?

To find the cost of goods sold by the ton.

Point off three places to the right in the number denoting the *quantity* [illegible], then multiply the *price* of a ton by [illegible] and divide this result by 2. The reason for this operation follows from the fact that *two* thousand pounds make a ton.

Find the retail price of 7846 pounds of coal at $8.75 a ton.

OPERATION

$$
\begin{array}{r}
\$8.75 \\
7.846 \\
\hline
5250 \\
3500 \\
7000 \\
6125 \\
\hline
2\,\underline{)\,68.65250} \\
\$34.32625
\end{array}
$$

10. What must be paid for 9785 pounds of plaster at $6.75 per ton?

11. If 25,000 pounds of plaster cost $131.25, what is that per ton?

12. A dealer in New York retails coal at $7.75 per ton. If a ton costs $3.75 at the mine and $0.75 for freight, what will he make on 8758 pounds of coal?

13. At $10.50 per ton what is the cost of 25,000 pounds of plaster?

14. What is the retail price of coal per ton when 17,520 pounds are sold for $74.46?

15. How many pounds of plaster at $10.50 per ton can be bought for $131.25?

16. An errand boy receives $2.75 a week. In how many weeks will he earn $44?

17. How many cords of pine wood at $3.375 a cord must be given for 12 yards of broadcloth at $2.25 a yard?

18. The milk from a herd of 75 cows at 6 cents a quart amounted in one summer to $2025. How many quarts were sold?

19. A merchant sold 3 pieces of matting, each containing 45.5 yards, at $0.375 per yard. How much money did he receive?

20. If a man earns $12 a week and spends on the average $10.125, in how many weeks will he save $97.50?

21. A grocer bought 156 boxes of oranges at $5.625 each, and sold the whole for $916.875. How much did he gain?

22. A Western farmer's wheat crop at $1.08 per bushel brings $831.60. How many bushels did he raise?

23. Find the cost of 15,964 feet of boards at $39.25 a thousand.

24. Find the cost of 19,500 laths at 35 cents a hundred.

25. What will be paid for shipping 1500 tons of wheat from Buffalo to New York at the rate of 5 cents a bushel? (A bushel of wheat weighs 60 pounds.)

Ex. 59.

1. Find the price of 30 Parian statuettes at $8.875 each.

2. In February, 1884, the number of days during which rain fell in New England was 22, and the amount which fell was 4.57 inches. Find the daily average for the 22 days.

3. How many acres are in a park containing 0.08 of 115.1875 acres?

4. If 31.75 rods of fence are made for $10.90, what is the cost of a rod?

5. On a certain day in February, 1884, the thermometer at the highest was 51.1°, and at the lowest 29.4°. Find the difference.

6. Of 100 parts of matter in beans, sugar and gum form 61.10, other vegetable matter forms 31.55, and moisture 5 parts. Of how many parts does the remainder, which is mineral matter, consist?

7. If 0.1571 of the weight of superphosphate is organic matter, find the weight in tons of organic matter in 80 tons of superphosphate.

8. In January, 1884, the barometer at the highest was 30.543 inches, and at the lowest 28.843 inches. Find the difference.

9. A cubic inch of pure water weighs 252.458 grains. Find the weight in grains of a cylindrical inch, which is 0.7854 of a cubic inch.

10. Divide $31.40 among 6 men and 11 youths, giving a youth 0.525 of a man's share. What is each man's *share?*

11. Four men together paid \$20,000 for some land. The first puts in \$2350, the second \$5820.35, the third \$7640.75. How much must the fourth man pay?

12. What will be the cost of uniforms for a base-ball nine at \$2.87 for each uniform?

13. At \$15.87 a ton, what will be the value of 637 tons of hay?

14. If peaches are worth \$1.25 a basket, and it takes 3 dozen for a basket, what is the value of 2892 dozen peaches?

15. If 964 baskets of peaches are sold for \$1301.40, what is the price per basket?

16. If 324 men contribute together \$2647.08, what is the contribution of each?

17. A boy picks blueberries in a pasture, giving to the owner of the pasture for the privilege 1 quart out of every 8 quarts. In 2 days he picks 48 quarts, and sells his share of the berries for \$3.78. What did he get a quart?

18. If 150 men work on a railroad at the same price per day, and if, at the end of the week, they all together receive \$1575, what price per day does each man receive?

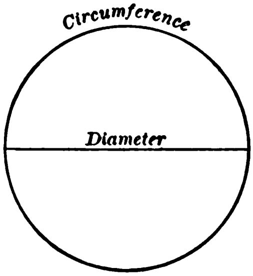
19. If a kite-string is 213.86 feet long, and the kite breaks away and carries off 94.38 feet of the string, how much will be left? How much more must be bought to make up 1000 feet?

20. At \$9.17 a barrel, how many barrels of flour can be bought for \$876.35, and how much money will be left over?

Ex. 60.

If the length of the diameter of a circle is multiplied by 3.1416, the product is the length of the circumference.

1. Find the length in inches of the circumference of a circle if the diameter is 6 inches.



2. Find the length in inches of the circumference of a circle if the diameter is 17 inches.
3. If a carriage wheel is 4 feet in diameter, what is its circumference in feet?
4. If the carriage wheel in Example 3 rolls on the ground without slipping, how many feet will it go in turning 27 times?
5. How many times will the carriage wheel in Example 3 turn in going 1160 feet?

If the length of the circumference of a circle is divided by 3.1416, the quotient is the length of the diameter.

6. Find the diameter in feet of a circle if its circumference is 1000 feet.
7. Find the diameter in feet of a wheel which revolves 19.5 times in going 253.5 feet.
8. If the circumference of a circle is 198 yards, what is its diameter in yards?

9. How deep is a well if the wheel whose diameter is 2 feet makes 10 revolutions in raising the bucket?

10. If a carriage wheel makes 440 revolutions in travelling a mile (5280 feet), what is its diameter in feet?

11. If 1964.52 bushels of corn are to be put into bags holding 2.14 bushels each, how many bags will it take?

12. A boy has 3 pieces of twine: one is 58.74 feet long, another is 97.86 feet, and a third 57.26 feet. How long a kite-string can he make, making no allowance for knots?

13. Boys in playing hare and hound run 3.876 miles. The hares drop a piece of paper every 4.75 feet on the average. How many pieces do they drop? A mile is 5280 feet.

14. If a man earns $23.25 a day, how many days will it take him to earn $1964.87?

15. A grain merchant bought corn at 60 cents and rye at 75 a bushel. He bought the same number of bushels of both kinds of grain and paid for both together $607.50. How many bushels of each kind did he buy?

16. When potatoes are worth $0.77 per bushel, and corn $1.10, how many bushels of corn should a farmer receive in exchange for 50 bushels of potatoes?

17. How many gallons of 231 cubic inches are contained in a cubic foot (1728 cubic inches)?

18. How many cubic feet are contained in a bushel, a bushel containing 2150.42 cubic inches?

19. For $7624.13 how many tons of hay can be bought at $18.75 a ton?

20. The large wheel of a bicycle is 14.37 feet around. How many times will it turn in going a mile (5280 feet)?

CHAPTER VII.

MULTIPLES AND MEASURES.

85. When the multiplier is an **integral number**, the product is called a **multiple** of the **multiplicand**; and, in division, when the quotient is an **integral number**, the divisor is called a **measure** of the dividend. Thus, $8 \times 7 = 56$; the number 56 is a multiple of 7. Again, $56 \div 7 = 8$; the number 7 is a measure of 56.

86. A number which cannot be divided by any other number except unity without remainder is called a **prime number.**

Thus, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc., are prime numbers.

87. Other numbers are each the product of a **fixed set** of prime numbers, and are called **composite numbers.**

88. Numbers which can be divided by 2 without remainder are called **even numbers**; and all other numbers are called **odd numbers.** Even numbers end in 2, 4, 6, 8, or 0; odd numbers end in 1, 3, 5, 7, or 9.

89. By way of distinction, when a number is used without reference to any designated unit, it is called an **abstract number**; and, when used with reference to a specified unit, it is called a **concrete number.**

Thus, 5, 7, 8 are abstract numbers, and 5 horses, 7 chairs, 8 dollars are called, by way of distinction, concrete numbers.

90. To factor a composite number is to separate the number into its factors.

Find the prime factors of 144.

$$\begin{array}{r|r}
2 & 144 \\ \hline
2 & 72 \\ \hline
2 & 36 \\ \hline
2 & 18 \\ \hline
3 & 9 \\ \hline
 & 3
\end{array}$$

That is, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

91. To avoid the necessity of writing long rows of equal factors, a small figure called the **exponent** is written at the right of a number to show how many times the number is taken as a factor.

Thus, $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is written $2^4 \times 3^2$.

The expression 2^4 is called the fourth **power** of 2, and 3^2 is called the second power of 3.

92. It is evident from § 90 that the method of separating a composite number into its prime factors is,

Divide the given number by any prime number that is contained in it without remainder; then the quotient by any prime number that is contained in it without remainder; and so on until the quotient is itself a prime number. The several divisors and the last quotient are the prime factors.

If no prime factor is found before the quotient becomes equal to or less than the divisor, the number is prime.

93. The following tests are useful for determining without actual division if a number contains certain factors:

1. A number is divisible by 2 if its *last*, or *right hand, digit is even.*
2. A number is divisible by 4 (2^2) if the number denoted by the *last two digits* is divisible by 4.

3. A number is divisible by 8 (2^3) if the number denoted by the *last three digits* is divisible by 8.

4. A number is divisible by 3 if the *sum of its digits* is divisible by 3.

5. A number is divisible by 9 (3^2) if the *sum of its digits* is divisible by 9.

6. A number is divisible by 5 if its *last digit* is either 5 or 0.

7. A number is divisible by 25 (5^2) if the number denoted by the *last two digits* is divisible by 25.

8. A number is divisible by 125 (5^3) if the number denoted by the *last three digits* is divisible by 125.

9. A number is divisible by 6 if its *last digit is even*, and the *sum of its digits* is divisible by 3.

10. A number is divisible by 11 if the *difference between the sum of the digits in the even places and the sum of the digits in the odd places* is either 0 or a multiple of 11.

Ex. 61.

Find the prime factors of:

1. 32; 48; 56; 60; 75; 63; 92; 44; 88; 72; 84; 85.
2. 51; 69; 68; 87; 54; 98; 74; 90; 86; 70; 42; 62.
3. 112; 140; 132; 216; 162; 176; 252; 240; 360; 384.
4. 484; 476; 512; 525; 560; 572; 632; 648; 696; 720.
5. 748; 775; 824; 876; 888; 948; 960; 925; 117; 119.

94. The number 1.56 may be put in the form of 156×0.01, and separated into $2^2 \times 3 \times 13 \times 0.01$.

Ex. 62.

Find the prime factors of:

1. 1.05; 12.5; 14.3; 1.65; 19.2; 2.42; 62.4; 27.5.
2. 34.3; 5.39; 62.1; 118.8; 1.331; 1.452; 1.584; 92.4.

GREATEST COMMON MEASURE.

95. The measures of 12 are 1, 2, 3, 4, 6, 12, and the measures of 18 are 1, 2, 3, 6, 9, 18. These two numbers have the measures 1, 2, 3, 6 in common, and of these measures 6 is the greatest.

The measures that two or more numbers have in common are called their *common measures*, and the greatest of these is called their **Greatest Common Measure,** which, for the sake of brevity, is denoted by the letters G.C.M.

If two or more numbers have no common measure they are said to be *prime to each other.* Thus, 27 and 125 are prime to each other.

96. The prime factors of 12 are 2^2, 3.

The prime factors of 18 are 2, 3^2.

The prime factors common to 12 and 18 are 2, 3. The G.C.M. of 12 and 18, namely 6, is 2×3.

That is, the G.C.M. of two or more numbers is,

The product of the prime factors common to the numbers, each prime factor having the least exponent that it has in any one of the numbers.

Hence, to find the G.C.M. of two or more numbers,

Separate the numbers into their prime factors.

Select the lowest power of each factor that is common to the given numbers, and find the product of these powers.

Find the G.C.M. of 84, 105, 63.

$$\begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$84 = 2^2 \times 3 \times 7.$ $\quad 105 = 3 \times 5 \times 7.$ $\quad 63 = 3^2 \times 7.$

Hence, *G.C.M.* $= 3 \times 7$ or 21.

97. Common factors of two or more numbers may be taken out of the numbers simultaneously, as follows:

$$\begin{array}{r|rrr} 3 & 84 & 105 & 63 \\ \hline 7 & 28 & 35 & 21 \\ \hline & 4 & 5 & 3 \end{array}$$

The number 3 is seen to be a factor of all the numbers, and 7 of the resulting quotients 28, 35, 21. The quotients 4, 5, and 3 have no common factor. Therefore, 3 and 7 are the only common factors, and the G.C.M. is 3×7, or 21.

Ex. 63.

Find the G.C.M. of:

1. 48, 128.
2. 36, 90.
3. 64, 256.
4. 24, 105.
5. 125, 600.
6. 56, 138.
7. 63, 108.
8. 40, 600.
9. 65, 91.
10. 39, 273.
11. 56, 126.
12. 232, 493.
13. 365, 511.
14. 148, 592.
15. 216, 360.
16. 279, 403.
17. 294, 378.
18. 210, 294.
19. 182, 196.
20. 225, 375.
21. 195, 299.
22. 288, 360.
23. 133, 152.
24. 23, 111.
25. 352, 384.
26. 123, 579.
27. 960, 1536.
28. 336, 884.
29. 352, 364.
30. 1344, 1536.
31. 21, 35, 56.
32. 42, 133, 56.
33. 32, 48, 128.
34. 27, 36, 108.
35. 96, 48, 60, 108.
36. 33, 297, 198.
37. 56, 63, 315.
38. 75, 225, 500.
39. 232, 290, 493.
40. 365, 511, 803.

98. When it is required to find the G.C.M. of two or more numbers that cannot readily be separated into factors, the method to be employed is as follows:

Find the G.C.M. of 63 and 217.

OPERATION.

```
63)217(3
   189
   ---
    28)63(2
       56
       --
        7)28(4
          28
          --
```

Therefore, the G.C.M. is 7.

Hence, by this method,

Divide the greater number by the less, and then the divisor by the remainder, and so on till there is no remainder. The last divisor will be the G. C. M. required.

To find the G.C.M. of several numbers, find the G.C.M. of two of the numbers, then of that result and a third number, and so on. The last G.C.M. is the one required.

Ex. 64.

Find the G.C.M. of:

1. 342, 665.	**6.** 1131, 2639.	**11.** 3927, 5049.
2. 841, 899.	**7.** 9889, 986.	**12.** 1287, 1551.
3. 961, 1178.	**8.** 1792, 1832.	**13.** 1537, 1802.
4. 1243, 1469.	**9.** 1847, 1792.	**14.** 3056, 3629.
5. 1001, 1287.	**10.** 1850, 1517.	**15.** 2108, 3813.

16. 4844, 5536.	**22.** 216, 105, 405.
17. 696, 1305.	**23.** 112, 192, 128.
18. 232, 3219.	**24.** 168, 132, 352.
19. 949, 1387.	**25.** 198, 495, 209, 660.
20. 1081, 1311.	**26.** 146, 730, 365, 219.
21. 4067, 2573.	**27.** 924, 378, 612, 246.

Least Common Multiple.

99. The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, etc.

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, etc.

The multiples common to 3 and 5 are 15, 30, etc., and of these 15 is the least.

100. The multiples that two or more numbers have in common are called their *common multiples*, and the least of these is called their **Least Common Multiple**, which is denoted by the letters L. C. M.

Find the L. C. M. of 7, 8, 9, 21.

The L. C. M. of 7, 8, 9, 21, must contain the factor 7, or it would not be a multiple of 7. It must also contain 2^3 to be a multiple of 8, and 3^2 to be a multiple of 9. It must contain the factors 3 and 7 to be a multiple of 21. That is, the L. C. M. of 7, 8, 9, 21, is the product of the factors 7, 2^3, 3^2; therefore, it is $7 \times 8 \times 9 = 504$. Hence,

To find the L. C. M. of two or more numbers,

Separate each number into its prime factors.

Select from these the **highest power** *of each factor, and find the product of these powers.*

Find the L. C. M. of 16, 21, 24, 30, 32.

$$16 = 2^4,$$
$$21 = 3 \times 7,$$
$$24 = 2^3 \times 3,$$
$$30 = 2 \times 3 \times 5,$$
$$32 = 2^5.$$

Hence, the L. C. M. $= 2^5 \times 3 \times 5 \times 7 = 3360.$

The L.C.M. of 16, 21, 24, 30, 32, may be found as follows:

$$\begin{array}{r|rrrrr}
2 & \cancel{16} & 21 & 24 & 30 & 32 \\ \hline
2 & & 21 & 12 & 15 & 16 \\ \hline
2 & & 21 & 6 & 15 & 8 \\ \hline
3 & & 21 & \cancel{3} & 15 & 4 \\ \hline
 & & 7 & & 5 & 4
\end{array}$$

Hence, the L.C.M. $= 2^3 \times 3 \times 7 \times 5 \times 4 = 3360$.

Since 16 is a measure of 32, it is elided, for any multiple of 32 is also a multiple of 16. The even numbers are divided by 2; the quotients and the odd numbers are written below the horizontal line. This operation is repeated so long as 2 is a measure of more than one number. In the fourth line 3, a measure of 15, is elided. The division by 3 leaves in the fifth line the numbers 7, 5, 4, which are prime to each other.

Therefore, the factors contained in the numbers are 2, 2, 2, 3, and 7, 5, 4.

Hence, the L. C. M. $= 2 \times 2 \times 2 \times 3 \times 7 \times 5 \times 4 = 3360$.

When two or more numbers are *prime* to each other, their L.C.M. is their product. Thus, the L.C.M. of 3, 5, 7, is $3 \times 5 \times 7$.

Ex. 65.

Find the L.C.M. of:

1. 3, 9, 27, 54.
2. 6, 9, 24, 40.
3. 144, 12, 18, 96.
4. 3, 8, 12, 22.
5. 16, 30, 48, 15.
6. 12, 24, 63, 84.
7. 9, 27, 33, 54.
8. *12, 20, 36, 54.*
9. 22, 44, 88, 108.
10. 15, 30, 45, 60.
11. 8, 16, 24, 32.
12. 13, 15, 26, 39.
13. 7, 17, 51, 119.
14. 8, 6, 28, 32.
15. 4, 21, 42, 63.
16. 3, 6, 18, 22.

17. 5, 15, 24, 30.

18. 7, 2, 3, 5.

19. 13, 5, 2, 26.

20. 5, 10, 20, 100.

21. 19, 38, 2, 76.

22. 3, 9, 27, 81.

23. 6, 18, 22, 99.

24. 18, 26, 117, 312.

25. 13, 26, 39, 65.

26. 9, 36, 3, 45.

27. 16, 24, 13, 7.

28. 5, 9, 14, 96, 128.

29. 32, 36, 49, 56, 42.

30. 20, 24, 25, 27, 45.

31. 28, 30, 32, 36, 42.

32. 35, 40, 42, 49, 28.

33. 14, 18, 21, 32, 28.

34. 24, 27, 32, 36, 56.

35. 21, 24, 27, 28, 35.

36. 28, 32, 56, 72, 96.

101. If the given numbers are large, and contain no prime factors that can readily be detected, the common factors may be obtained by the process of finding the G.C.M. under like circumstances.

Find the L.C.M. of 1189 and 2117.

```
1189)2117(1
     1189
     ----
      928)1189(1
           928
           ---
           261)928(3
               783
               ---
               145)261(1
                   145
                   ---
                   116)145(1
                       116
                       ---
                        29)116(4
                           116
                           ---
```

Hence, the G.C.M. = 29.

Therefore, 1189 = 29 × 41; 2117 = 29 × 73.

Therefore, the **L.C.M. = 29 × 41 × 73 = 1189 × 73.**

From this process it will be seen that:

The L.C.M. of two numbers may be found by dividing one of the numbers by their G.C.M., and multiplying the quotient by the other number.

Ex. 66.

Find the L.C.M. of:

1. 510 and 595.
2. 217 and 643.
3. 506 and 308.
4. 296 and 407.
5. 645 and 275.
6. 468 and 923.
7. 187 and 255.
8. 1261 and 663.
9. 255 and 357.
10. 432 and 840.
11. 949 and 2920.
12. 1247 and 1769.

Ex. 67.

1. A farmer owns 132 acres of wood-land, and 99 acres of pasture; he wishes to divide them into equal lots of the largest possible size. How many lots will there be, and what number of acres in each one?
2. A merchant has 75 yards of one kind of silk, 225 of a second, and 200 of a third; if he cut them into dress patterns of equal size, what is the largest number of yards which each pattern can contain?
3. Simeon Jones has 260 bushels of rye, 384 of oats, and 416 of wheat. He sends his grain to market in bags of equal size. What is the greatest number of bushels which each bag can hold, provided there is no mixture of the different kinds of grain?
4. What width of carpet will fit three rooms, the first 15 feet wide, the second 21 feet, and the third 33 feet?

Ex. 69.

1. How many yards of cloth, at $3 a yard, can be bought for 12 tons of hay, at $15 per ton?
2. How many pairs of boots, at $4 a pair, can be bought for 40 pounds of butter, at 40 cents per pound?
3. How many jars of lard of 36 pounds each, at 8 cents per pound, must be given for 16 pieces of cloth containing 24 yards each, at 48 cents a yard?
4. How many coats, at $4 each, can be bought for 32 yards of broadcloth, at $2.50 a yard?
5. A milkman having 30 cows which daily give 8 quarts each, sells the milk at 5 cents per quart. How many pieces of cloth containing 40 yards each, at 12 cents per yard, ought he to receive for the milk of 6 days?
6. A market gardener sold 16 lots of celery, 120 bunches in each, at 28 cents per bunch; how many 240-pound barrels of sugar, at 8 cents a pound, will the celery pay for?
7. John Peters sold 9 firkins of butter weighing 78 pounds each, at 25 cents per pound; how many pieces of matting having 45 yards in a piece, at 30 cents per yard, should he receive?
8. A workman has received for 15 days' work of 7 hours each, 21 dollars. How much would he receive for 19 days' work of 5 hours each?
9. Thirty workmen have made in 9 days 215 yards of wall. At the same rate, how much would 36 workmen make in 15 days?
10. A telegraph operator transmits 50 words, averaging 4 letters each, in the space of 5 minutes. At the same rate, how many minutes will be required to send a *dispatch of* 120 words, averaging 5 letters each?

CHAPTER VIII.

COMMON FRACTIONS.

103. What is the name of one of the parts when a unit is divided into:

1. Two equal parts?
2. Three equal parts?
3. Four equal parts?
4. Five equal parts?
5. Six equal parts?
6. Eight equal parts?
7. Ten equal parts?
8. Twelve equal parts?
9. Sixteen equal parts?
10. Twenty equal parts?

A unit contains how many:

1. Halves?
2. Thirds?
3. Fourths?
4. Fifths?
5. Sixths?
6. Eighths?
7. Tenths?
8. Twelfths?
9. Sevenths?
10. Ninths?
11. Elevenths?
12. Thirteenths?
13. Twentieths?
14. Twenty-fourths?
15. Thirtieths?
16. Thirty-seconds?

When a unit is divided into twelve equal parts, what is the name of:

1. One part?
2. Three parts?
3. Five parts?
4. Two parts?
5. Four parts?
6. Six parts?
7. Eight parts?
8. Nine parts?
9. Twelve parts?

Express in figures:

1. Three-sevenths.
2. Five-ninths.
3. Seven-eighths.
4. Five-twelfths.
5. Seven-sixteenths.
6. Five-eighteenths.
7. Four-elevenths.
8. Nine-twentieths.

Read: $\frac{7}{10}$, $\frac{6}{13}$, $\frac{5}{12}$, $\frac{4}{21}$, $\frac{11}{16}$, $\frac{78}{100}$, $\frac{28}{24}$, $\frac{11}{32}$, $\frac{18}{55}$, $\frac{7}{28}$, $\frac{9}{28}$.

104. The expression $\frac{7}{9}$ means:

I. Seven of the parts when a unit has been divided into nine equal parts.

II. One-ninth of seven units; for, if *seven* units be divided into nine equal parts, one of these parts will be *seven* times as great as one of the parts obtained by dividing one unit into nine equal parts.

III. The quotient of seven divided by nine.

105. In the fraction $\frac{7}{9}$, the lower figure shows the number of equal parts into which the whole has been divided, and is therefore a **divisor**; but, since it shows the number of parts into which the whole has been divided, it shows the **name** of each part, and is therefore called the **denominator.**

The upper figure shows the **number** of these parts taken, and is therefore called the **numerator.**

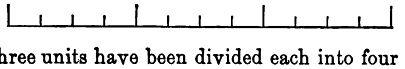
The figure, then, above the line denotes **number,** the figure below the line **name.**

106. The numerator and denominator are called the **terms** of a fraction.

107. A **proper fraction** is one of which the numerator is *less than* the denominator; as $\frac{7}{9}$.

108. An **improper fraction** is one of which the numerator equals or exceeds the denominator; as $\frac{9}{9}$, $\frac{17}{4}$.

When the numerator is greater than the denominator, more than one unit must be regarded as divided into equal parts; thus, $\frac{9}{4}$



means that three units have been divided each into four equal parts, and that all the parts of two units and one part of the third unit are taken.

109. A **mixed number** is an expression consisting of a whole number and a fraction; as $4\frac{3}{7}$, 5.35. These expressions are read four *and* three-sevenths, five *and* thirty-five hundredths.

Every mixed number means that some entire units are taken, and the fraction of another unit.

Select the proper fractions, the improper fractions, and mixed numbers from the following expressions:

$\frac{8}{9}$, $\frac{21}{2}$, $4\frac{1}{5}$, $\frac{3}{8}$, $9\frac{4}{7}$, $\frac{125}{39}$, $\frac{7}{8}$, $\frac{16}{17}$, $\frac{17}{7}$, $6\frac{3}{7}$, $\frac{15}{22}$, $\frac{487}{385}$, $\frac{4}{5}$, $\frac{9}{8}$, $5\frac{1}{4}$, $\frac{9}{10}$, $8\frac{4}{9}$, $19\frac{4}{12}$, $\frac{17}{9}$, $6\frac{3}{4}$, $\frac{25}{7}$, $18\frac{2}{3}$, $\frac{5}{4}$.

110. An improper fraction represents a quantity which can also be represented by a whole number or else by a mixed number. Thus, $\frac{19}{7} = 2\frac{5}{7}$.

For, if we suppose several units to be divided each into seven equal parts, and we take 19 of these parts, 14 (that is, 2×7) will make 2 units, and the five remaining parts will be five-sevenths of another unit.

111. To reduce an improper fraction to a whole or mixed number,

Divide the numerator by the denominator.

The quotient will be the whole number, and the remainder, if any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.

Ex. 70. (*Oral.*)

Reduce to whole or mixed numbers:

1. $\frac{15}{8}$.	7. $\frac{19}{7}$.	13. $\frac{9}{5}$.	19. $\frac{13}{4}$.
2. $\frac{17}{5}$.	8. $\frac{29}{6}$.	14. $\frac{35}{5}$.	20. $\frac{37}{10}$.
3. $\frac{19}{8}$.	9. $\frac{56}{9}$.	15. $\frac{14}{12}$.	21. $\frac{40}{8}$.
4. $\frac{24}{12}$.	10. $\frac{13}{4}$.	16. $\frac{28}{7}$.	22. $\frac{45}{11}$.
5. $\frac{28}{5}$.	11. $\frac{14}{8}$.	17. $\frac{30}{3}$.	23. $\frac{23}{6}$.
6. $\frac{48}{12}$.	12. $\frac{12}{5}$.	18. $\frac{35}{6}$.	24. $\frac{70}{11}$.

Ex. 71.

Reduce to whole or mixed numbers:

1. $\frac{27}{16}$.	9. $\frac{397}{78}$.	17. $\frac{575}{79}$.	25. $\frac{13505}{72}$.
2. $\frac{219}{24}$.	10. $\frac{563}{45}$.	18. $\frac{587}{38}$.	26. $\frac{39679}{88}$.
3. $\frac{416}{17}$.	11. $\frac{584}{92}$.	19. $\frac{1334}{91}$.	27. $\frac{14271}{256}$.
4. $\frac{683}{25}$.	12. $\frac{796}{97}$.	20. $\frac{3597}{429}$.	28. $\frac{3615}{411}$.
5. $\frac{497}{39}$.	13. $\frac{847}{109}$.	21. $\frac{13322}{111}$.	29. $\frac{1384}{37}$.
6. $\frac{498}{47}$.	14. $\frac{693}{420}$.	22. $\frac{5142}{117}$.	30. $\frac{3145}{48}$.
7. $\frac{576}{58}$.	15. $\frac{318}{95}$.	23. $\frac{360}{13}$.	31. $\frac{3782}{78}$.
8. $\frac{800}{87}$.	16. $\frac{417}{89}$.	24. $\frac{9172}{63}$.	32. $\frac{580}{87}$.

112. A whole number or a mixed number represents a quantity which can also be represented by an improper fraction. Thus, $\$3\frac{3}{4} = \$\frac{15}{4}$.

For each dollar contains 4 *fourths;* therefore 3 dollars contain 3×4 *fourths* or 12 *fourths;* which, together with the 3 *fourths,* make 15 *fourths.* Hence,

113. To reduce a mixed number to an improper fraction,

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

114. A whole number may be expressed as a fraction with any given denominator. Thus, $9 = \frac{63}{7}$.

For, as 1 unit contains 7 *sevenths*, 9 units contain 9×7 *sevenths*, or 63 *sevenths*.

A whole number may be written in the form of a fraction with 1 for a denominator. Thus, $9 = \frac{9}{1}$.

Ex. 72. (*Oral.*)

Reduce to improper fractions:

1. $4\frac{1}{5}$.	**7.** $12\frac{1}{3}$.	**13.** $11\frac{3}{7}$.	**19.** $25\frac{1}{2}$.
2. $7\frac{2}{3}$.	**8.** $10\frac{5}{8}$.	**14.** $13\frac{1}{10}$.	**20.** $30\frac{2}{5}$.
3. $8\frac{1}{2}$.	**9.** $7\frac{4}{5}$.	**15.** $7\frac{3}{13}$.	**21.** $17\frac{3}{10}$.
4. $9\frac{3}{7}$.	**10.** $3\frac{9}{10}$.	**16.** $9\frac{6}{12}$.	**22.** $40\frac{1}{4}$.
5. $5\frac{4}{9}$.	**11.** $8\frac{4}{11}$.	**17.** $20\frac{2}{12}$.	**23.** $50\frac{7}{8}$.
6. $6\frac{3}{7}$.	**12.** $3\frac{5}{12}$.	**18.** $15\frac{1}{3}$.	**24.** $80\frac{2}{3}$.

25. Change 12 to thirds; 8 to fourths; 7 to fifths; 9 to halves; 12 to ninths; 13 to sixths; 11 to sevenths; 14 to eighths.

Ex. 73.

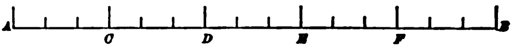
Change to improper fractions:

1. $15\frac{1}{23}$.	**6.** $45\frac{5}{12}$.	**11.** $5\frac{27}{160}$.	**16.** $155\frac{29}{37}$.
2. $36\frac{11}{12}$.	**7.** $56\frac{7}{15}$.	**12.** $46\frac{5}{126}$.	**17.** $17\frac{29}{84}$.
3. $5\frac{61}{79}$.	**8.** $77\frac{3}{17}$.	**13.** $2\frac{7}{30}$.	**18.** $167\frac{37}{56}$.
4. $3\frac{13}{55}$.	**9.** $183\frac{5}{21}$.	**14.** $9\frac{32}{79}$.	**19.** $29\frac{18}{19}$.
5. $12\frac{15}{61}$.	**10.** $72\frac{31}{43}$.	**15.** $10\frac{8}{17}$.	**20.** $29\frac{39}{68}$.

21. Change 25 to 94ths; 218 to 23ds; 375 to 87ths.

Reduction of Fractions to Lower Terms.

115. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction is not altered.



Thus, if the line AB be divided into 5 equal parts at the points C, D, E, and F, then AF is $\frac{4}{5}$ of AB.

Now, if each of the parts be sub-divided into 3 equal parts, AB will contain 15 of these sub-divisions, and AF 12 of these sub-divisions. Therefore AF is $\frac{12}{15}$ of AB.

Since AF is $\frac{4}{5}$ of AB and also $\frac{12}{15}$ of AB, it follows that $\frac{12}{15} = \frac{4}{5}$. But $\frac{4}{5}$ is obtained from $\frac{12}{15}$ by dividing both numerator and denominator by 3. Therefore,

To reduce a fraction to lower terms,

Divide the numerator and denominator by any common factor.

A fraction is expressed in its lowest terms when both the numerator and denominator are divided by the greatest common divisor.

Reduce $\frac{336}{784}$ to its lowest terms.

$$\frac{336}{784} = \frac{84}{196} = \frac{21}{49} = \frac{3}{7}.$$

The common factors cancelled are 4, 4, and 7.

Reduce $\frac{259}{333}$ to its lowest terms.

Since no common factor can readily be detected, we find the G.C.M.

$$\begin{array}{l}
259)333(1 \\
\quad\ \underline{259} \\
\quad\ \ 74)259(3 \\
\qquad\quad \underline{222} \\
\qquad\quad\ 37)74(2 \\
\qquad\qquad\ \underline{74}
\end{array}$$

Divide 259 and 333 each by 37, their G.C.M. Then $\frac{259}{333} = \frac{7}{9}$.

Ex. 74. (*Oral.*)

Reduce to lowest terms by inspection:

1. $\frac{9}{12}$.	8. $\frac{11}{22}$.	15. $\frac{25}{30}$.	22. $\frac{14}{30}$.
2. $\frac{15}{20}$.	9. $\frac{10}{20}$.	16. $\frac{21}{28}$.	23. $\frac{25}{50}$.
3. $\frac{10}{12}$.	10. $\frac{15}{18}$.	17. $\frac{18}{24}$.	24. $\frac{30}{42}$.
4. $\frac{14}{16}$.	11. $\frac{7}{14}$.	18. $\frac{14}{21}$.	25. $\frac{27}{45}$.
5. $\frac{12}{15}$.	12. $\frac{9}{15}$.	19. $\frac{9}{21}$.	26. $\frac{24}{56}$.
6. $\frac{16}{24}$.	13. $\frac{18}{27}$.	20. $\frac{21}{27}$.	27. $\frac{32}{48}$.
7. $\frac{12}{18}$.	14. $\frac{16}{20}$.	21. $\frac{13}{26}$.	28. $\frac{36}{63}$.

Ex. 75.

Reduce to lowest terms by the method of inspection or by the method of finding the G.C.M.

1. $\frac{125}{500}$.	13. $\frac{78}{416}$.	25. $\frac{330}{660}$.	37. $\frac{492}{744}$.
2. $\frac{144}{192}$.	14. $\frac{308}{506}$.	26. $\frac{128}{256}$.	38. $\frac{55}{121}$.
3. $\frac{96}{264}$.	15. $\frac{135}{255}$.	27. $\frac{256}{1280}$.	39. $\frac{744}{906}$.
4. $\frac{120}{180}$.	16. $\frac{147}{189}$.	28. $\frac{252}{288}$.	40. $\frac{505}{707}$.
5. $\frac{36}{252}$.	17. $\frac{136}{168}$.	29. $\frac{365}{511}$.	41. $\frac{420}{864}$.
6. $\frac{154}{220}$.	18. $\frac{180}{234}$.	30. $\frac{775}{1800}$.	42. $\frac{882}{1512}$.
7. $\frac{48}{128}$.	19. $\frac{336}{576}$.	31. $\frac{144}{960}$.	43. $\frac{630}{882}$.
8. $\frac{180}{234}$.	20. $\frac{47}{83}$.	32. $\frac{198}{297}$.	44. $\frac{344}{2268}$.
9. $\frac{28}{154}$.	21. $\frac{63}{217}$.	33. $\frac{315}{745}$.	45. $\frac{168}{224}$.
10. $\frac{28}{348}$.	22. $\frac{196}{324}$.	34. $\frac{225}{450}$.	46. $\frac{216}{648}$.
11. $\frac{48}{224}$.	23. $\frac{384}{416}$.	35. $\frac{154}{220}$.	47. $\frac{1242}{2328}$.
12. $\frac{108}{252}$.	24. $\frac{209}{253}$.	36. $\frac{119}{121}$.	48. $\frac{2268}{3444}$.

NOTE. In the answers to *all* examples, fractions should be left in their lowest *terms.*

MULTIPLICATION OF FRACTIONS.

116. 7×3 *horses* $= 21$ *horses.*

7×3 *fifths* $= 21$ *fifths.*

If three like quantities are taken 7 times, the result will be 7 times 3 of the *same quantities.*

$\frac{3}{5} \times 15$ means $\frac{3}{5}$ of 15, which equals 9. Hence,

117. To find the product of a whole number and a fraction,

Find the product of the numerator and whole number, and divide the result by the denominator.

A factor common to the whole number and the denominator of the fraction may be cancelled. For, cancelling a factor common to the whole number and the denominator of the fraction *before* the multiplication, is evidently equivalent to dividing the numerator and denominator of the resulting fraction by that factor *after* the multiplication. Which may be done by § 115.

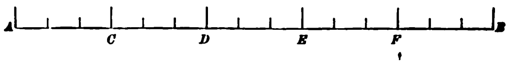
Ex. 76. (*Oral.*)

Find the products of:

1. $18 \times \frac{1}{3}$.	**10.** $14 \times \frac{1}{2}$.	**19.** $8 \times \frac{11}{20}$.	**28.** $18 \times \frac{3}{20}$.
2. $25 \times \frac{1}{5}$.	**11.** $20 \times \frac{3}{5}$.	**20.** $16 \times \frac{9}{20}$.	**29.** $25 \times \frac{3}{10}$.
3. $27 \times \frac{5}{9}$.	**12.** $16 \times \frac{1}{4}$.	**21.** $36 \times \frac{5}{12}$.	**30.** $\frac{4}{13} \times 26$.
4. $10 \times \frac{1}{5}$.	**13.** $7 \times \frac{11}{21}$.	**22.** $24 \times \frac{5}{8}$.	**31.** $\frac{1}{6} \times 9$.
5. $24 \times \frac{2}{3}$.	**14.** $16 \times \frac{3}{4}$.	**23.** $32 \times \frac{3}{16}$.	**32.** $\frac{7}{15} \times 12$.
6. $12 \times \frac{1}{4}$.	**15.** $\frac{17}{20} \times 10$.	**24.** $12 \times \frac{5}{16}$.	**33.** $22 \times \frac{5}{33}$.
7. $21 \times \frac{1}{7}$.	**16.** $\frac{11}{20} \times 12$.	**25.** $27 \times \frac{2}{9}$.	**34.** $\frac{5}{21} \times 28$.
8. $30 \times \frac{5}{6}$.	**17.** $27 \times \frac{7}{9}$.	**26.** $18 \times \frac{5}{9}$.	**35.** $\frac{3}{14} \times 21$.
9. $16 \times \frac{1}{8}$.	**18.** $\frac{13}{20} \times 15$.	**27.** $12 \times \frac{7}{9}$.	**36.** $\frac{4}{15} \times 35$.

118. To multiply a fraction by a fraction.

Multiply $\frac{4}{5}$ by $\frac{2}{3}$.



$\frac{4}{5}$ multiplied by $\frac{2}{3}$ means $\frac{2}{3}$ of $\frac{4}{5}$.

If the line AB be divided into 5 equal parts at the points C, D, E, and F, AF will be $\frac{4}{5}$ of AB. Now, if each part be sub-divided into three equal parts, there will evidently be 15 such parts in the whole line, and each part will be $\frac{1}{15}$ of the line.

That is, $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ of the whole.

$\frac{1}{3}$ of $\frac{4}{5}$ will be $\frac{1}{15}+\frac{1}{15}+\frac{1}{15}+\frac{1}{15}$, or $\frac{4}{15}$ of the whole. And $\frac{2}{3}$ of $\frac{4}{5}$ will be twice $\frac{4}{15}$; that is, $\frac{8}{15}$ of the whole. Therefore,

To multiply a fraction by a fraction,

Find the product of the numerators for the required numerator, and of the denominators for the required denominator.

Mixed numbers must first be reduced to improper fractions.

Any factor common to a numerator and denominator should be cancelled before the multiplication.

(1) $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

(2) $\frac{13}{30} \times 2\frac{8}{13} \times \frac{10}{17}$.

Reducing the mixed number to an improper fraction, we have,

$$\frac{13}{30} \times \frac{34}{13} \times \frac{10}{17}.$$

By cancellation,

$$\frac{\cancel{13}}{\underset{3}{\cancel{30}}} \times \frac{\overset{2}{\cancel{34}}}{\cancel{13}} \times \frac{\overset{1}{\cancel{10}}}{\cancel{17}} = \frac{2}{3}.$$

Ex. 77. (*Oral.*)

Find the products of:

1. $\frac{1}{3} \times \frac{3}{5}$.
2. $\frac{1}{2} \times \frac{8}{9}$.
3. $\frac{1}{3} \times \frac{9}{8}$.
4. $\frac{1}{2} \times \frac{3}{8}$.
5. $\frac{1}{3} \times \frac{3}{7}$.
6. $\frac{5}{12} \times \frac{7}{9}$.
7. $\frac{6}{7} \times \frac{7}{4}$.
8. $\frac{4}{25} \times \frac{5}{6}$.
9. $\frac{7}{30} \times 10$.
10. $\frac{7}{30} \times \frac{10}{3}$.
11. $\frac{1}{5} \times \frac{10}{3}$.
12. $\frac{17}{20} \times \frac{5}{4}$.

Ex. 78.

Find the products of:

1. $\frac{1}{5} \times \frac{15}{19}$.
2. $\frac{2}{3} \times 4\frac{1}{2}$.
3. $2\frac{1}{5} \times \frac{3}{11} \times \frac{2}{7}$.
4. $4\frac{1}{8} \times 3\frac{7}{11} \times \frac{2}{5}$.
5. $\frac{5}{9}$ of $\frac{12}{25}$ of $1\frac{1}{5}$.
6. $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$.
7. $\frac{68}{95} \times \frac{2}{5} \times \frac{4}{5}$.
8. $3\frac{2}{3} \times 5\frac{1}{9} \times \frac{27}{46}$.
9. $\frac{12}{23} \times 9\frac{1}{5} \times \frac{25}{6}$.
10. $2\frac{1}{9} \times \frac{3}{7} \times \frac{11}{38}$.
11. $2\frac{2}{9}$ of $\frac{9}{2}$ of $\frac{3}{25}$.
12. $2\frac{1}{2}$ of $\frac{7}{4}$ of $\frac{13}{21}$.
13. $\frac{3}{4} \times 7\frac{1}{2} \times \frac{3}{25}$.
14. $\frac{4}{3} \times 7\frac{1}{2} \times \frac{3}{25}$.
15. $2\frac{1}{3}$ of $3\frac{1}{9}$ of $\frac{30}{7}$.
16. $\frac{7}{18}$ of $\frac{3}{14}$ of $7\frac{1}{5}$.
17. *$3\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{10}{11}$.*
18. $3\frac{1}{3}$ of $\frac{9}{40}$ of $\frac{9}{10}$.
19. $\frac{6}{19}$ of $2\frac{1}{9}$ of 21.
20. $\frac{7}{4}$ of $1\frac{1}{14}$ of $3\frac{1}{5}$.
21. $8\frac{2}{3} \times \frac{2}{3}$ of $\frac{7}{8}$.
22. $16 \times \frac{4}{7}$ of $\frac{3}{13}$.
23. $3 \times 7\frac{1}{2} \times \frac{11}{15} \times 3\frac{8}{11}$.
24. $\frac{3}{10}$ of $\frac{6}{7} \times \frac{8}{9} \times \frac{10}{11} \times 77$.
25. $\frac{16}{25} \times \frac{4}{5} \times \frac{75}{94} \times \frac{1}{4}$.
26. $\frac{3}{13} \times \frac{20}{81} \times \frac{13}{11} \times \frac{11}{12}$.
27. $3\frac{2}{3} \times 4\frac{7}{8} \times 15$.
28. $\frac{9}{10}$ of $7 \times \frac{11}{15}$ of $87\frac{3}{11}$.
29. $11\frac{3}{7} \times 16\frac{4}{11}$ of $\frac{7}{80}$ of $\frac{1}{90}$.
30. $\frac{1}{3}$ of $\frac{9}{8}$ of $2\frac{2}{3} \times 15\frac{2}{7}$.
31. $3\frac{1}{3}$ of $\frac{7}{15} \times 2\frac{1}{7}$ of $\frac{5}{9}$.
32. $3\frac{2}{3}$ of $\frac{51}{70}$ of $6\frac{2}{3}$ of $\frac{21}{25}$.
33. $\frac{1}{5} \times 2\frac{1}{9} \times 4\frac{1}{2} \times \frac{5}{19}$.
34. $\frac{2}{5} \times 2\frac{1}{7} \times 7\frac{1}{2} \times 4\frac{1}{6} \times \frac{3}{5}$.

35. $2\frac{1}{2} \times \frac{7}{9} \times \frac{4}{5} \times 3\frac{3}{8} \times 5\frac{1}{8}$.

36. $\frac{1}{7} \times \frac{2}{9} \times 7\frac{1}{5} \times 5\frac{1}{4} \times 6\frac{1}{8}$.

37. $5\frac{1}{9} \times \frac{20}{23} \times 7\frac{1}{5} \times \frac{15}{29} \times 7\frac{1}{4}$.

38. $\frac{3}{13}$ of $\frac{60}{81}$ of $\frac{78}{55}$ of $\frac{11}{12}$.

39. $3\frac{2}{5}$ of $\frac{2}{13}$ of $\frac{1}{5}$ of 10.

40. $\frac{5}{18} \times 2\frac{1}{3} \times 3\frac{3}{5} \times 2\frac{1}{4} \times \frac{8}{45}$.

119. When the product of a mixed number and a whole number is required, it is generally best to find the product of the whole number and the fractional part of the mixed number, then the product of the whole number and the integral part of the mixed number, and combine the results. Thus,

The product of 9 times $7\frac{1}{8}$ is found as follows:

OPERATION.

$$\begin{array}{r} 7\frac{1}{8} \\ 9 \\ \hline 64\frac{1}{8} \end{array}$$

Here 9 times $\frac{1}{8}$ equals $1\frac{1}{8}$, the $\frac{1}{8}$ is written, and the 1 is carried to the product of 9×7, making 64.

Ex. 79.

Find the products of:

1. $3 \times 4\frac{5}{7}$.	11. $5\frac{5}{8} \times 9$.	21. $20 \times 5\frac{1}{5}$.
2. $5 \times 7\frac{2}{9}$.	12. $2\frac{3}{10} \times 15$.	22. $4\frac{3}{4} \times 17$.
3. $21 \times 18\frac{3}{7}$.	13. $2\frac{11}{12} \times 20$.	23. $5\frac{2}{3} \times 18$.
4. $22 \times 29\frac{3}{13}$.	14. $5\frac{3}{5} \times 12$.	24. $6\frac{7}{8} \times 15$.
5. $25 \times 12\frac{3}{4}$.	15. $7\frac{1}{3} \times 8$.	25. $9\frac{3}{4} \times 21$.
6. $6 \times 2\frac{2}{3}$.	16. $6\frac{3}{4} \times 9$.	26. $10\frac{1}{2} \times 41$.
7. $7 \times 2\frac{3}{4}$.	17. $9 \times 3\frac{3}{5}$.	27. $11\frac{2}{3} \times 32$.
8. $8 \times 2\frac{3}{5}$.	18. $12 \times 2\frac{7}{8}$.	28. $15\frac{4}{5} \times 36$.
9. $6\frac{2}{5} \times 9$.	19. $13 \times 3\frac{2}{3}$.	29. $16\frac{7}{8} \times 40$.
10. $3\frac{3}{10} \times 10$.	20. $16 \times 9\frac{1}{3}$.	30. $13\frac{5}{6} \times 27$.

DIVISION OF FRACTIONS.

120. When a product of two numbers is equal to 1, each of these two numbers is called the **reciprocal** of the other.

Thus, $5 \times \frac{1}{5} = 1$. Hence, the reciprocal of $\frac{1}{5}$ is 5, and the reciprocal of 5 is $\frac{1}{5}$. Again, $\frac{7}{5} \times \frac{5}{7} = 1$. Therefore, the reciprocal of $\frac{7}{5}$ is $\frac{5}{7}$, and the reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.

121. To multiply by the reciprocal of a number is the same as to divide by the number.

Thus, to multiply by $\frac{1}{3}$, means to separate the multiplicand into three equal parts, and to take one of the parts for the required product; and, to divide by 3 means to separate the dividend into three equal parts, and to take one of the parts for the required quotient.

To multiply by $\frac{2}{3}$ means to separate the multiplicand into three equal parts, and to take two of these parts; and to divide by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$, means to divide the dividend into three equal parts, and to take two of these parts.

Hence, to divide by a whole number or a fraction,

Multiply by its reciprocal.

Thus,

$$\tfrac{3}{5} \div 2 = \tfrac{1}{2} \times \tfrac{3}{5} = \tfrac{3}{10}.$$

$$\tfrac{3}{5} \div \tfrac{5}{6} = \tfrac{6}{5} \times \tfrac{3}{5} = \tfrac{18}{25}.$$

Mixed numbers must first be reduced to improper fractions.

Ex. 80. (*Oral.*)

Find the quotients of:

1. $\frac{5}{9} \div 5$.	7. $\frac{18}{27} \div 9$.	13. $\frac{3}{11} \div 6$.	19. $\frac{7}{8} \div 3$.
2. $\frac{8}{11} \div 4$.	8. $\frac{25}{30} \div 5$.	14. $\frac{7}{12} \div 3$.	20. $\frac{7}{8} \div 4$.
3. $\frac{3}{7} \div 2$.	9. $\frac{16}{17} \div 8$.	15. $\frac{12}{17} \div 6$.	21. $\frac{7}{8} \div 5$.
4. $\frac{75}{19} \div 3$.	10. $\frac{8}{10} \div 7$.	16. $\frac{12}{17} \div 3$.	22. $\frac{7}{8} \div 8$.
5. $\frac{14}{20} \div 2$.	11. $\frac{35}{90} \div 5$.	17. $\frac{12}{17} \div 4$.	23. $\frac{7}{8} \div 7$.
6. $\frac{4}{9} \div 5$.	12. $\frac{42}{50} \div 14$.	18. $\frac{12}{17} \div 12$.	24. $\frac{8}{18} \div 4$.

Ex. 81.

Find the quotients of :

1. $\frac{42}{45} \div 12$.
2. $\frac{350}{451} \div 25$.
3. $\frac{88}{95} \div 12$.
4. $\frac{78}{95} \div 13$.
5. $\frac{152}{209} \div 19$.
6. $\frac{7}{24} \div \frac{5}{11}$.
7. $\frac{3}{2} \div \frac{3}{8}$.
8. $2\frac{1}{2} \div \frac{3}{4}$.
9. $\frac{2}{3} \div \frac{4}{5}$.
10. $\frac{7}{8} \div 2\frac{1}{4}$.
11. $6\frac{1}{2} \div 4\frac{1}{6}$.
12. $\frac{4}{5} \div 7\frac{3}{10}$.
13. $3\frac{9}{10} \div 2\frac{3}{5}$.
14. $\frac{3}{4} \div \frac{2}{9}$.
15. $\frac{2}{7} \div \frac{5}{8}$.
16. $\frac{1}{2}$ of $\frac{3}{4} \div \frac{7}{8}$.
17. $\frac{3}{4}$ of $\frac{2}{9} \div \frac{5}{28}$.
18. $\frac{2}{5}$ of $3\frac{1}{2} \div \frac{27}{28}$.
19. $\frac{25}{39} \div \frac{10}{13}$.
20. $\frac{12}{49} \div \frac{2}{7}$.
21. $8\frac{3}{5} \div 6\frac{1}{7}$.
22. $7\frac{1}{9} \div 8\frac{2}{3}$.
23. $\frac{3}{7} \div \frac{15}{19}$.
24. $\frac{17}{63} \div \frac{85}{108}$.
25. $3\frac{31}{48} \div \frac{5}{12}$.
26. $3\frac{1}{6} \div 9\frac{1}{2}$.
27. $9\frac{1}{6} \div 3\frac{1}{2}$.
28. $8 \div \frac{14}{15}$.
29. $19 \div \frac{19}{20}$.
30. $3\frac{3}{7} \div 3\frac{1}{7}$.
31. $\frac{1}{2}$ of $\frac{3}{5} \div \frac{9}{10}$.
32. $1\frac{7}{8}$ of $7\frac{1}{5} \div 2\frac{4}{9}$.
33. $2\frac{1}{5}$ of $1\frac{3}{4} \div 2\frac{1}{8}$.
34. $1\frac{4}{5} \div \frac{3}{25}$ of $\frac{7}{8}$.
35. $\frac{3}{5}$ of $2\frac{1}{2} \div \frac{12}{19}$ of $6\frac{1}{3}$.
36. $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{7}{9}$ of $3\frac{3}{5}$.
37. $2\frac{2}{3}$ of $1\frac{1}{4} \div 5\frac{1}{3}$ of $3\frac{3}{4}$.
38. $2\frac{1}{5}$ of $2\frac{1}{2} \div \frac{3}{14}$ of $3\frac{2}{3}$.
39. $1\frac{1}{2} \div 1\frac{1}{8}$ of $\frac{5}{21}$ of $\frac{3}{23}$.
40. $3\frac{2}{3}$ of $5\frac{1}{4}$ of $7\frac{1}{2} \div 63$.
41. $3\frac{2}{5}$ of $7\frac{1}{7}$ of $1\frac{2}{5} \div 3\frac{1}{5}$.
42. $7\frac{1}{5}$ of $3\frac{1}{3} \div 1\frac{1}{10}$ of $1\frac{5}{7}$.
43. $9 \div \frac{2}{11}$ of $1\frac{5}{17}$ of $4\frac{4}{5}$.
44. $16 \div \frac{7}{15}$ of $2\frac{8}{11}$ of $\frac{11}{35}$.
45. $3\frac{3}{8}$ of $4\frac{4}{5} \div \frac{2}{5}$ of $6\frac{1}{2}$ of $\frac{3}{8}$.

122. When a mixed number is to be divided by a whole number, it is best to divide the integral part of the dividend first, and then the fractional part. If there is a remainder from dividing the integral part, this remainder may be put with the fraction, and the result reduced to an improper fraction, and *then divided* by the divisor.

Divide $16\frac{4}{5}$ by 4; $16\frac{2}{3}$ by 7.

OPERATION.	OPERATION.
$4\overline{)16\frac{4}{5}}$	$7\overline{)16\frac{2}{3}}$
$4\frac{1}{5}$	$2\frac{8}{21}$

In the first problem we simply divide the whole number 16 by 4, and then the fraction $\frac{4}{5}$ by 4, and obtain the result at once, $4\frac{1}{5}$.

In the second problem we divide the 16 by 7, and obtain the quotient 2 and a remainder 2. The remainder is joined with the $\frac{2}{3}$, making $2\frac{2}{3} = \frac{8}{3}$, and $\frac{8}{3} \div 7 = \frac{8}{21}$.

Ex. 82.

Find the quotients of:

1. $19\frac{1}{2} \div 3$.	6. $34\frac{3}{5} \div 17$.	11. $65\frac{4}{13} \div 9$.
2. $12\frac{4}{5} \div 5$.	7. $31\frac{2}{9} \div 11$.	12. $147\frac{5}{9} \div 13$.
3. $24\frac{2}{3} \div 8$.	8. $37\frac{5}{8} \div 18$.	13. $76\frac{5}{7} \div 19$.
4. $19\frac{3}{7} \div 6$.	9. $45\frac{3}{17} \div 7$.	14. $124\frac{7}{29} \div 6$.
5. $17\frac{5}{9} \div 9$.	10. $57\frac{5}{9} \div 16$.	15. $326\frac{5}{17} \div 15$.

Ex. 83.

1. What must be paid for 24 yards of cloth, at $\$\frac{5}{7}$ per yard?

2. A farmer bought 327 sheep, at $\$4\frac{2}{3}$ a head; required the cost of the flock.

3. At 25 cents a pound, what must be paid for $82\frac{1}{2}$ pounds of butter?

4. A merchant sold $15\frac{2}{7}$ yards of silk, at $\$4\frac{2}{3}$ per yard; what change should he give back from 8 ten-dollar bills?

5. If beefsteak cost 22 cents per pound, and mutton chops 21 cents, how much will a man pay for meat, who eats $\frac{1}{2}$ pound of beefsteak for breakfast, and $1\frac{2}{3}$ *pounds* of mutton chops for dinner?

6. At $\$\frac{4}{5}$ per yard, how much cloth can be bought for $25?

7. If $\$19\frac{1}{2}$ be paid for 9 yards of silk, what is the cost per yard?

8. A man walks $37\frac{4}{5}$ miles in 6 hours; how many miles does he walk an hour?

9. A farmer sells $19\frac{1}{2}$ acres of land for $375; what is the price per acre?

10. A lady pays $3 for $\frac{5}{8}$ of a yard of silk; what is the price per yard?

11. A man in one year pays $45.26 for cigars, the average price of which is $6\frac{1}{5}$ cents apiece. How many does he smoke in a year?

12. If $\frac{7}{8}$ of an acre of tillage land cost $125, what is the price per acre? How many acres can be bought for $1297?

13. Gideon Lyford earns $30 per week; what will remain at the close of the week when he has paid for $6\frac{3}{4}$ pounds of butter, at 33 cents a pound, $10\frac{1}{2}$ pounds of mutton, at 20 cents a pound, $8\frac{3}{5}$ pounds of beef, at 25 cents, 3 boxes of strawberries, at 16 cents, 150 pounds of ice, at $\frac{1}{2}$ cent, 20 loaves of bread, at 10 cents, fuel $2, vegetables $3?

14. Find the product of $17\frac{3}{5} \times 8\frac{1}{3}$ of $6\frac{3}{10}$.

15. If a man build $\frac{9}{10}$ of a rod of wall in one hour, how much will he build in $\frac{4}{5}$ of an hour?

16. If a ship costs $16,785, what will $\frac{4}{5}$ of it be worth?

17. If a water-pipe discharges $16\frac{1}{3}$ barrels of water in an hour, how many barrels will it discharge in $9\frac{3}{7}$ hours?

18. For 4 sheep $\$25\frac{3}{4}$ are paid; what is the price per *head?*

19. A coal dealer paid \$375 freight for transporting coal from Scranton to Hudson. If the price was $\$\frac{3}{4}$ per ton, how many tons were transported?

20. How many pounds of beef, at $18\frac{2}{5}$ cents per pound, can be bought for \$17.48?

21. A farmer hires an equal number of men and boys, and pays for a man and boy $\$2\frac{5}{8}$ a day. If the pay roll is \$84 a day, how many men and boys does he hire?

22. When $8\frac{78}{125}$ acres of land cost \$1297, what will $\frac{5}{8}$ of an acre cost?

23. A city speculator in land divided $\frac{3}{4}$ of an acre into lots of $\frac{1}{16}$ of an acre each, and sold them all for $\$13,426\frac{3}{4}$. What was the average price per lot?

24. For $\frac{5}{8}$ of $\frac{4}{9}$ of a ship the sum of \$6394 was received; what is the value of the ship?

25. A vessel sails $17\frac{3}{8}$ miles per hour; how many miles will she sail in $26\frac{3}{4}$ hours?

26. George is $13\frac{1}{2}$ years old, Henry is $\frac{3}{4}$ as old as George, and John's age is $1\frac{1}{3}$ that of Henry; what is the age of John?

27. There are $16\frac{1}{2}$ feet in one rod; how many feet are there in $84\frac{3}{8}$ rods?

28. How many feet around a field, each one of whose four sides measures $7\frac{3}{8}$ rods?

29. A schooner sails on the average $175\frac{5}{8}$ miles a day; how far will she sail in a week?

30. At the rate of $8\frac{3}{4}$ miles per hour, how many miles will a ship sail from a quarter past three A.M. to a quarter before six P.M.?

31. Reduce $\frac{3}{7}$ of $\frac{5}{11}$ of $\frac{7}{25}$ of $\frac{33}{49}$ to a simple fraction in its *lowest terms.*

32. George Ward inherited from his father $\frac{11}{13}$ of a farm containing 377 acres. He divided his share equally among his four sons; how many acres would each one of the sons receive?

33. How many pounds of sugar, at $9\frac{1}{2}$ cents a pound, can be bought for $1.52?

34. For $23\frac{2}{3}$ baskets of peaches, a grocer gave $20.59; what was the price per basket?

35. A farmer sold 42 bushels of potatoes for $26.58; what was the average price per bushel?

36. At $37\frac{1}{2}$ cents per yard, how many yards of lace can be bought for $\$5\frac{1}{4}$?

37. A farmer sold $6\frac{1}{2}$ bushels of apples for $\$4.87\frac{1}{2}$; what was the price per bushel?

38. When $6\frac{1}{2}$ bushels of apples bring $3.90, what are they worth a peck? (Four pecks make a bushel.)

39. At 60 cents a pound, how many pounds of tea can be bought for $4.65?

40. If $\frac{3}{10}$ of a yard of cloth cost 80 cents, what should be paid for $\frac{5}{12}$ of a yard?

41. The cost of fencing a lot $8\frac{1}{2}$ rods in circuit is $\$6\frac{4}{5}$, what is the rate per rod?

42. A roll of carpeting containing 202 yards is cut into pieces of $25\frac{1}{4}$ yards each, and each piece is sold for $\$32\frac{1}{2}$. Required the number of pieces, and the price per yard.

43. When $35\frac{3}{4}$ bushels of turnips cost $28.60, what should be paid for $\frac{1}{2}$ of a bushel?

44. How many yards of cloth can be bought for $10.80, if $\frac{7}{10}$ of a yard cost 63 cents?

LEAST COMMON DENOMINATOR.

123. A fraction is changed to *higher terms* by multiplication.

Reduce $\frac{8}{4}$ to twelfths.

Multiply both terms by 3; thus,

$$\frac{3\times 8}{3\times 4}=\frac{24}{12}$$

In either of the two forms $\frac{8}{4}$ or $\frac{24}{12}$ the value of the fraction is 2.

124. Hence, to reduce a fraction to higher terms,

Multiply both terms of the fraction by that number which will change the given denominator to the required denominator.

The required multiplier is found by dividing the required denominator by the denominator of the given fraction.

Ex. 84. (*Oral.*)

Reduce:

1. $\frac{1}{5}$ to 20ths.	8. $\frac{5}{9}$ to 27ths.	15. $\frac{3}{13}$ to 26ths.
2. $\frac{1}{2}$ to 10ths.	9. $\frac{5}{11}$ to 33ds.	16. $\frac{5}{18}$ to 36ths.
3. $\frac{2}{3}$ to 9ths.	10. $\frac{4}{7}$ to 28ths.	17. $\frac{5}{9}$ to 81sts.
4. $\frac{5}{7}$ to 14ths.	11. $\frac{7}{12}$ to 36ths.	18. $\frac{7}{12}$ to 96ths.
5. $\frac{5}{6}$ to 18ths.	12. $\frac{7}{10}$ to 20ths.	19. $\frac{8}{11}$ to 44ths.
6. $\frac{3}{4}$ to 12ths.	13. $\frac{4}{15}$ to 45ths.	20. $\frac{3}{4}$ to 16ths.
7. $\frac{5}{8}$ to 24ths.	14. $\frac{1}{20}$ to 100ths.	21. $\frac{5}{24}$ to 72ds.

125. Similar fractions are fractions that have a common denominator.

All fractions must be expressed as similar fractions before they can be added or subtracted, and in all cases it is best to express them with the least common denominator. (L.C.D.)

126. The least common denominator of two or more fractions is the least common multiple of their denominators.

Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ to similar fractions.

The least common multiple of the denominators 2, 3, 4 is 12. It is therefore necessary to reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ to 12ths, by the method explained in § 124, and we have $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$.

127. Hence, to reduce fractions to similar fractions,

Find the least common multiple of the denominators; this will be the required denominator. Divide this denominator by the denominator of each fraction.

Multiply the first numerator by the first quotient, the second numerator by the second quotient, and so on.

The products will be the numerators of the equivalent fractions.

Ex. 85. (*Oral.*)

Reduce to similar fractions:

1. $\frac{1}{2}$, $\frac{1}{3}$.	5. $\frac{1}{4}$, $\frac{1}{5}$.	9. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{21}$.	13. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$.
2. $\frac{1}{7}$, $\frac{1}{3}$.	6. $\frac{3}{4}$, $\frac{5}{7}$.	10. $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{14}$.	14. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{12}$.
3. $\frac{1}{2}$, $\frac{1}{4}$.	7. $\frac{5}{8}$, $\frac{5}{6}$.	11. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$.	15. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{18}$.
4. $\frac{1}{5}$, $\frac{1}{3}$.	8. $\frac{2}{3}$, $\frac{3}{8}$.	12. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$.	16. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$.

Ex. 86.

Reduce to similar fractions:

1. $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.	6. $\frac{2}{3}$, $\frac{5}{13}$, $\frac{12}{39}$, $\frac{25}{78}$.
2. $\frac{7}{12}$, $\frac{5}{36}$, $\frac{11}{48}$.	7. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{8}{15}$, $\frac{7}{25}$.
3. $\frac{3}{21}$, $\frac{2}{28}$, $\frac{5}{7}$.	8. $\frac{1}{5}$, $\frac{7}{8}$, $\frac{13}{16}$, $\frac{19}{20}$, $\frac{3}{40}$.
4. $\frac{4}{19}$, $\frac{3}{38}$, $\frac{2}{3}$.	9. $\frac{7}{9}$, $\frac{5}{12}$, $\frac{17}{18}$, $\frac{23}{24}$, $\frac{7}{36}$.
5. $\frac{4}{9}$, $\frac{5}{14}$, $\frac{5}{42}$.	10. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{8}{21}$, $\frac{15}{42}$.

Addition of Fractions.

128. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

These fractions reduced to similar fractions become $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, and
$\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$. $2\frac{1}{4}$. *Ans.*

129. Hence, to add fractions,

Reduce the fractions to similar fractions, and write the sum of the numerators over the common denominator.

Add $\frac{7}{8}$, $\frac{5}{12}$, $\frac{8}{15}$.

$$\begin{array}{r|rrr} 2 & 8 & 12 & 15 \\ \hline 2 & 4 & 6 & 15 \\ \hline 3 & 2 & 3 & 15 \\ \hline & 2 & 1 & 5 \end{array}$$

Hence, L. C. D. $= 2^3 \times 3 \times 5 = 120$.

$$\text{Numerators} \ldots \left\{\begin{array}{r} 105 \\ 50 \\ \underline{64} \end{array}\right.$$

Sum of numerators $= 219$

Therefore, sum of fractions $=\frac{219}{120}=\frac{73}{40}=1\frac{33}{40}$. $1\frac{33}{40}$. *Ans.*

130. If any of the expressions are integers or mixed numbers, add together separately the integers and the fractions, and find the sum of the results.

Find the sum of $2\frac{3}{20}$, $1\frac{7}{15}$, $5\frac{11}{12}$.

L. C. D. of the fractions $= 2^2 \times 3 \times 5 = 60$.

$$\text{Numerators} \ldots \left\{\begin{array}{r} 9 \\ 28 \\ \underline{55} \end{array}\right.$$

Sum of numerators $= 92$

Sum of fractions $=\frac{92}{60}=\frac{23}{15}=1\frac{8}{15}$

Sum of integers $= 2+1+5 = \underline{8}$

$9\frac{8}{15}$. *Ans.*

Ex. 87. (*Oral.*)

Find the sum of:

1. $\frac{5}{12}$, $\frac{8}{12}$.
2. $\frac{9}{16}$, $\frac{7}{16}$.
3. $\frac{8}{9}$, $\frac{4}{9}$.
4. $\frac{8}{15}$, $\frac{2}{15}$.
5. $\frac{11}{20}$, $\frac{9}{20}$.
6. $\frac{21}{80}$, $\frac{19}{80}$.
7. $7\frac{1}{4}$, $3\frac{1}{4}$.
8. $8\frac{3}{5}$, $4\frac{2}{5}$.

9. $5\frac{3}{8}$, $4\frac{7}{8}$.
10. $9\frac{3}{10}$, $5\frac{3}{10}$.
11. $8\frac{3}{4}$, $5\frac{3}{4}$.
12. $7\frac{7}{12}$, $3\frac{7}{12}$.
13. $\frac{5}{6}$, $\frac{2}{3}$.
14. $\frac{3}{4}$, $\frac{5}{8}$.
15. $\frac{9}{10}$, $\frac{3}{5}$.
16. $\frac{3}{8}$, $\frac{7}{24}$.
17. $\frac{2}{3}$, $\frac{7}{12}$.
18. $\frac{5}{6}$, $\frac{11}{18}$.
19. $3\frac{7}{10}$, $4\frac{3}{20}$.
20. $2\frac{5}{6}$, $3\frac{2}{3}$.
21. $8\frac{5}{12}$, $7\frac{3}{4}$.
22. $6\frac{3}{8}$, $5\frac{1}{2}$.
23. $7\frac{2}{3}$, $4\frac{5}{9}$.
24. $9\frac{3}{10}$, $8\frac{2}{5}$.

Ex. 88.

Find the sum of:

1. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$.
2. $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$.
3. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$.
4. $\frac{7}{8}$, $\frac{3}{5}$, $\frac{2}{3}$.
5. $\frac{7}{8}$, $\frac{5}{6}$, $\frac{4}{9}$.
6. $\frac{5}{6}$, $\frac{4}{9}$, $\frac{11}{15}$.
7. $4\frac{5}{6}$, $3\frac{2}{3}$, $6\frac{5}{12}$.
8. $7\frac{3}{4}$, $8\frac{11}{16}$, $9\frac{5}{8}$.
9. $8\frac{7}{15}$, $4\frac{4}{5}$. $3\frac{2}{3}$.
10. $4\frac{5}{9}$, $5\frac{3}{4}$, $6\frac{4}{5}$.
11. $9\frac{5}{6}$, $4\frac{3}{8}$, $8\frac{2}{9}$.
12. $4\frac{14}{15}$, $5\frac{13}{18}$, $6\frac{9}{10}$.
13. $\frac{7}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{11}{20}$.
14. $\frac{3}{8}$, $\frac{5}{6}$, $\frac{11}{24}$, $\frac{7}{12}$.
15. $\frac{7}{10}$, $\frac{8}{15}$, $\frac{11}{18}$, $\frac{3}{5}$.
16. $\frac{11}{16}$, $\frac{13}{20}$, $\frac{3}{4}$, $\frac{7}{15}$.
17. $\frac{5}{9}$, $\frac{7}{12}$, $\frac{8}{15}$, $\frac{9}{20}$.
18. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{13}{20}$, $\frac{11}{18}$.
19. $\frac{7}{8}$, $\frac{4}{9}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{11}{15}$.
20. $\frac{8}{9}$, $\frac{7}{10}$, $\frac{3}{8}$, $\frac{13}{15}$, $\frac{5}{12}$, $\frac{17}{20}$.
21. $\frac{2}{3}$, $\frac{5}{12}$, $\frac{17}{24}$, $\frac{13}{21}$, $\frac{11}{14}$, $\frac{15}{28}$.
22. $\frac{11}{15}$, $\frac{17}{20}$, $\frac{7}{12}$, $\frac{19}{24}$, $\frac{9}{10}$, $\frac{31}{36}$.
23. $\frac{14}{15}$, $\frac{17}{35}$, $\frac{23}{28}$, $\frac{19}{20}$, $\frac{13}{21}$, $\frac{19}{24}$.
24. $\frac{3}{4}$, $\frac{3}{8}$, $\frac{11}{20}$, $\frac{23}{28}$, $\frac{19}{21}$, $\frac{11}{14}$.
25. $24\frac{8}{9}$, $13\frac{4}{5}$, $36\frac{3}{8}$, $60\frac{5}{9}$, $47\frac{3}{5}$.
26. $35\frac{2}{3}$, $17\frac{1}{2}$, $25\frac{3}{4}$, $48\frac{3}{5}$, $18\frac{1}{2}$.
27. $54\frac{3}{5}$, $28\frac{2}{3}$, $16\frac{1}{2}$, $36\frac{3}{4}$, $64\frac{2}{5}$.
28. $36\frac{4}{7}$, $37\frac{2}{5}$, $59\frac{3}{4}$, $54\frac{2}{7}$, $16\frac{4}{5}$.
29. $23\frac{5}{8}$, $32\frac{8}{9}$, $18\frac{3}{4}$, $27\frac{5}{6}$, $28\frac{2}{3}$.
30. $74\frac{8}{15}$, $64\frac{13}{20}$ $48\frac{7}{12}$, $23\frac{11}{24}$, $27\frac{3}{16}$.

SUBTRACTION OF FRACTIONS.

131. From $\frac{19}{24}$ take $\frac{7}{18}$.

$$24 = 2^3 \times 3,$$
$$18 = 2 \times 3^2.$$

Hence, the L. C. D. $= 2^3 \times 3^2 = 72$.

$$\tfrac{19}{24} - \tfrac{7}{18} = \tfrac{57-28}{72} = \tfrac{29}{72}. \qquad \tfrac{29}{72}.\ \textit{Ans.}$$

132. Hence, to subtract one fraction from another,

Reduce the fractions to similar fractions.

Subtract the numerator of the subtrahend from the numerator of the minuend.

Write the result over the common denominator.

133. If the terms are mixed numbers, subtract separately the integers and fractions, and unite the results.

Subtract $5\frac{3}{8}$ from $15\frac{3}{4}$.

Here the L. C. D. $= 8$.

$$15\tfrac{3}{4} - 5\tfrac{3}{8} = 10\tfrac{6-3}{8} = 10\tfrac{3}{8}. \qquad 10\tfrac{3}{8}.\ \textit{Ans.}$$

Subtract $3\frac{5}{8}$ from $5\frac{5}{12}$.

$$5\tfrac{5}{12} - 3\tfrac{5}{8} = 2\tfrac{10-15}{24} = 1\tfrac{34-15}{24} = 1\tfrac{19}{24}.$$

$1\frac{19}{24}$. *Ans.*

The difference between $5\frac{5}{12}$ and $3\frac{5}{8}$ is $2\frac{10-15}{24}$. Since $\frac{15}{24}$ cannot be subtracted from $\frac{10}{24}$, 1 is taken from 2, and added to $\frac{10}{24}$, making $\frac{34}{24}$.

From 9 take $\frac{19}{24}$.

$$9 = 8\tfrac{24}{24}.$$

$$8\tfrac{24}{24} - \tfrac{19}{24} = 8\tfrac{5}{24}. \qquad 8\tfrac{5}{24}.\ \textit{Ans.}$$

Ex 89.

Find the value of :

1. $\frac{11}{12} - \frac{5}{12}$.
2. $\frac{58}{63} - \frac{44}{63}$.
3. $\frac{83}{84} - \frac{59}{84}$.
4. $\frac{67}{72} - \frac{49}{72}$.
5. $\frac{91}{96} - \frac{73}{96}$.
6. $\frac{5}{6} - \frac{3}{4}$.
7. $\frac{7}{8} - \frac{3}{5}$.
8. $\frac{4}{5} - \frac{1}{6}$.
9. $\frac{7}{10} - \frac{5}{12}$.
10. $\frac{2}{3} - \frac{3}{8}$.
11. $\frac{11}{12} - \frac{3}{4}$.
12. $\frac{5}{7} - \frac{5}{8}$.
13. $\frac{3}{8} - \frac{5}{24}$.
14. $\frac{1}{11} - \frac{1}{13}$.
15. $\frac{11}{12} - \frac{5}{16}$.
16. $\frac{25}{36} - \frac{13}{64}$.
17. $\frac{2}{5} - \frac{1}{8}$.
18. $\frac{14}{15} - \frac{6}{7}$.
19. $\frac{13}{14} - \frac{19}{21}$.
20. $\frac{14}{15} - \frac{41}{50}$.
21. $\frac{19}{30} - \frac{23}{90}$.
22. $\frac{11}{112} - \frac{13}{168}$.
23. $\frac{29}{56} - \frac{3}{14}$.
24. $\frac{16}{19} - \frac{13}{16}$.
25. $14 - \frac{7}{16}$.
26. $21 - \frac{19}{24}$.
27. $20 - \frac{13}{20}$.
28. $42 - \frac{13}{55}$.
29. $25 - \frac{13}{18}$.
30. $21 - \frac{13}{15}$.
31. $14 - \frac{15}{32}$.
32. $13 - \frac{5}{12}$.
33. $24 - 13\frac{5}{8}$.
34. $42 - 15\frac{7}{12}$.
35. $20 - 12\frac{7}{32}$.
36. $84 - 37\frac{17}{36}$.
37. $21\frac{5}{18} - \frac{13}{18}$.
38. $27\frac{5}{8} - \frac{7}{8}$.
39. $42\frac{5}{12} - \frac{7}{12}$.
40. $26\frac{7}{32} - \frac{19}{32}$.
41. $43\frac{17}{24} - \frac{11}{18}$.
42. $27\frac{5}{12} - \frac{13}{18}$.
43. $91\frac{4}{15} - \frac{7}{8}$.
44. $32\frac{11}{20} - \frac{13}{15}$.
45. $83\frac{7}{48} - \frac{13}{16}$.
46. $26\frac{3}{10} - \frac{7}{15}$.
47. $74\frac{5}{12} - \frac{11}{18}$.
48. $68\frac{4}{9} - \frac{5}{7}$.
49. $24\frac{5}{72} - 16\frac{11}{12}$.
50. $92\frac{1}{3} - 73\frac{4}{5}$.
51. $19\frac{5}{72} - 14\frac{13}{72}$.
52. $23\frac{1}{4} - 16\frac{3}{4}$.
53. $15\frac{2}{3} - 12\frac{1}{4}$.
54. $42\frac{1}{2} - 14\frac{1}{6}$.
55. $24\frac{9}{11} - 15\frac{3}{7}$.
56. $72\frac{8}{9} - 28\frac{5}{6}$.
57. $19\frac{1}{3} - 13\frac{1}{2}$.
58. $26\frac{3}{8} - 19\frac{2}{3}$.
59. $45\frac{5}{12} - 26\frac{7}{10}$.
60. $34\frac{3}{8} - 16\frac{8}{9}$.
61. $34\frac{3}{4} - 18\frac{5}{6}$.
62. $64\frac{3}{4} - 28\frac{13}{15}$.
63. $48\frac{2}{7} - 19\frac{5}{6}$.
64. $76\frac{7}{8} - 72\frac{9}{10}$.
65. $97\frac{1}{2} - 32\frac{15}{16}$.
66. $90\frac{1}{2} - 9\frac{5}{8}$.
67. $78\frac{3}{10} - 56\frac{8}{9}$.
68. $96\frac{1}{2} - 49\frac{3}{4}$.
69. $47\frac{3}{4} - 43\frac{11}{12}$.
70. $55\frac{3}{8} - 54\frac{2}{7}$.
71. $69\frac{1}{16} - 67\frac{23}{24}$.
72. $69\frac{3}{4} - 23\frac{17}{18}$.

Ex. 90.

1. A country merchant received on Monday \$$25\frac{3}{4}$, on Tuesday \$$19\frac{1}{8}$, on Wednesday \$$23\frac{2}{3}$, on Thursday \$$32\frac{3}{8}$, on Friday \$$29\frac{1}{2}$, on Saturday \$$37\frac{5}{8}$. What had he left after paying a freight bill of \$$19\frac{3}{8}$, and to his clerk \$$12\frac{1}{2}$?

2. A farmer sold two loads of hay, one for \$$13\frac{1}{2}$ and the other for \$$16\frac{2}{3}$, and received \$25 down. How much is still due?

3. A miner digs $17\frac{2}{3}$, $19\frac{1}{7}$, $18\frac{3}{5}$ ounces of gold. In washing there is a loss of $3\frac{2}{5}$ ounces. How much gold has he left?

4. Henry Cameron had three wheat-fields; the first produced $217\frac{3}{7}$ bushels, the second $309\frac{5}{9}$, the third $419\frac{1}{3}$. He sent $516\frac{2}{3}$ bushels to a flour mill, and sold 193 bushels. How many bushels had he left?

5. From a piece of cloth containing $47\frac{1}{2}$ yards, $22\frac{2}{3}$ yards were sold, and then $5\frac{3}{4}$ yards were sold. How many yards remained?

6. A grocer sold $2\frac{3}{8}$ pounds of tea to one man, $1\frac{1}{2}$ pounds more to a second man than to the first, and to a third man $1\frac{1}{4}$ pounds less than the amount he sold the first and second together. How many pounds did he sell to the second man, and how many to the third man?

7. Of the prismatic spectrum red occupies $\frac{1}{8}$, orange $\frac{3}{40}$, and yellow $\frac{2}{15}$. What part of the whole do these three colors together occupy?

8. What part of a piece of cloth has a merchant sold, who has cut off and sold $\frac{3}{16}$, $\frac{5}{32}$, $\frac{9}{64}$, and $\frac{7}{40}$ of it?

9. A treasurer has expended $\frac{11}{60}$, $\frac{7}{48}$, $\frac{13}{40}$, $\frac{7}{80}$, and $\frac{5}{24}$ of a given sum. What part of the whole has he left?

10. Of a pole $\frac{1}{5}$ is blue, $\frac{3}{7}$ red, and the rest white. What part of it is white?

11. A jeweller has used $\frac{3}{10}$, $\frac{7}{20}$, and $\frac{11}{40}$ of an ingot of gold. What part of it still remains?

12. A student has read $\frac{5}{21}$, $\frac{2}{7}$, and $\frac{1}{3}$ of a certain book. What part of it has he yet to read?

13. A traveller has gone $\frac{1}{9}$ of a journey on foot, $\frac{2}{15}$ on horseback, $\frac{1}{4}$ by rail, and the rest by coach. What part of the journey has he gone by coach?

14. Of the component elements of albumen $\frac{11}{20}$ is carbon, $\frac{7}{100}$ hydrogen, and $\frac{4}{25}$ nitrogen. What part of the whole do these elements constitute?

15. Add together the greatest and least of the fractions, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{19}{20}$, and subtract this sum from the sum of the other two fractions.

16. How many tons of ore must be raised from a mine so that, on losing $\frac{17}{40}$ in roasting, and $\frac{8}{19}$ of the remainder in smelting, there may be obtained 506 tons of pure metal?

17. A man invested $\frac{2}{5}$ of his capital in bank stock, $\frac{3}{4}$ of the remainder in real estate, and had left \$6000. Find his capital.

18. A man invests $\frac{1}{2}$ of his money in land, $\frac{1}{5}$ in bank stock, $\frac{1}{6}$ in railroad stock, and has \$8000 left. What is his fortune?

19. A owns $\frac{2}{5}$ of a ship, and B the remainder; and $\frac{3}{4}$ of the difference between their shares is \$1500. What *is the value of* the ship?

19. $7\frac{3}{4}$ is $2\frac{1}{3}$?

20. $7\frac{1}{2}$ is $\frac{15}{16}$?

21. $\frac{3}{4}$ of $10\frac{1}{3}$ is $\frac{31}{32}$?

22. $\frac{7}{8}$ of $25\frac{7}{8}$ is $\frac{181}{192}$?

23. $\frac{2}{3}$ of $12\frac{3}{4}$ is $\frac{51}{56}$?

24. $\frac{5}{7}$ of $3\frac{1}{6}$ is $\frac{12}{44}$?

25. $\frac{11}{35}$ of $7\frac{1}{2}$ is $3\frac{3}{8}$?

26. $\frac{7}{25}$ is $\frac{1}{4}$ of $\frac{14}{15}$?

27. $\frac{7}{8}$ is $\frac{1}{6}$ of $2\frac{3}{4}$?

28. 33 is $2\frac{1}{3}$ of $2\frac{1}{7}$?

29. $27\frac{13}{14}$ is $2\frac{43}{45}$ of $1\frac{1}{14}$?

30. 36 is $3\frac{3}{8}$ of $6\frac{3}{8}$?

137. To reduce a decimal to a common fraction.

Reduce 0.25 to a common fraction.

$$0.25 = \frac{25}{100} = \frac{1}{4}.$$

138. Hence, to reduce a decimal to a common fraction,

Write the figures of the decimal for the numerator; and 1, *with as many zeros as there are figures in the decimal, for the denominator.*

Ex. 93.

Reduce to common fractions:

1. 0.5.	9. 0.015.	17. 0.7168.	25. 1.6125.
2. 0.06.	10. 0.18.	18. 3.02.	26. 8.0396.
3. 0.15.	11. 0.125.	19. 5.85.	27. 2.18375.
4. 0.025.	12. 0.004.	20. 7.075.	28. 1.0725.
5. 0.7.	13. 0.032.	21. 0.15625.	29. 22.848.
6. 0.19.	14. 0.3125.	22. 0.46875.	30. 1.30125.
7. 0.135.	15. 0.0625.	23. 0.00256.	31. 17.875.
8. 0.005.	16. 0.0425.	24. 0.00375.	32. 2.9375.

9. $\dfrac{\frac{6}{7}}{\frac{14}{28}}$.

10. $\dfrac{\frac{15}{19}}{\frac{45}{76}}$.

11. $\dfrac{\frac{13}{14}}{7\frac{4}{5}}$.

12. $\dfrac{5\frac{3}{4}}{6\frac{2}{9}}$.

13. $\dfrac{19\frac{3}{4}}{28\frac{7}{18}}$.

14. $\dfrac{2\frac{8}{11}}{7\frac{3}{4}}$.

15. $\dfrac{23\frac{7}{9}}{24\frac{5}{6}}$.

16. $\dfrac{\frac{3}{4} \text{ of } 3\frac{1}{2}}{\frac{4}{5} \text{ of } 9\frac{2}{3}}$.

17. $\dfrac{\frac{2}{5} \text{ of } 13\frac{1}{4}}{\frac{4}{7} \text{ of } 7\frac{7}{8}}$.

18. $\dfrac{\frac{2}{11} \text{ of } 12\frac{6}{7}}{\frac{3}{8} \text{ of } \frac{5}{6\frac{2}{3}}}$.

19. $\dfrac{\frac{3}{4} \text{ of } \frac{2}{7}}{\frac{4}{7\frac{1}{2}} \text{ of } \frac{8}{11}}$.

20. $\dfrac{\frac{\frac{2}{3}}{\frac{3}{4}}}{5}$.

21. $\dfrac{2}{\frac{\frac{3}{4}}{\frac{5}{9}}}$.

136. To express one number as a fraction of another.

What fraction of 8 is 5?

Since $1 = \frac{1}{8}$ of 8,

$5 = 5 \times \frac{1}{8}$ of 8.

That is, $5 = \frac{5}{8}$ of 8.

The number which follows "of" is the denominator, and the other number the numerator of the required fraction.

Ex. 92.

What fraction of:

1. 8 is 7?
2. 7 is 8?
3. 6 is 2?
4. 5 is 3?
5. 7 is 15?
6. 15 is 7?
7. $2\frac{1}{2}$ is $\frac{1}{2}$?
8. $\frac{3}{4}$ is $4\frac{1}{4}$?
9. $2\frac{3}{4}$ is $1\frac{1}{4}$?
10. $2\frac{1}{3}$ is $5\frac{2}{3}$?
11. $2\frac{1}{3}$ is $5\frac{1}{4}$?
12. $5\frac{1}{4}$ is $2\frac{1}{3}$?
13. $3\frac{3}{4}$ is $\frac{7}{8}$?
14. $5\frac{1}{4}$ is $4\frac{2}{3}$?
15. $11\frac{3}{4}$ is $5\frac{2}{9}$?
16. $2\frac{17}{19}$ is $7\frac{6}{7}$?
17. $3\frac{11}{12}$ is $9\frac{2}{5}$?
18. $14\frac{2}{5}$ is $4\frac{4}{5}$?

Ex. 95.

Solve the following problems, first changing the common fractions to decimals:

1. A person owed \$24,560. When he has paid $\$8345\frac{3}{10}$, $\$7234\frac{3}{10}$, $\$6472\frac{9}{20}$, how much does he still owe?

2. A man sold $46\frac{17}{20}$ acres of land, at the rate of $\$9\frac{1}{2}$ an acre, and $54\frac{1}{2}$ acres at the rate of $\$2\frac{18}{25}$. How much did he receive for the whole?

3. A merchant purchases 346 pieces of cloth, each containing $32\frac{3}{4}$ yards, at $\$1\frac{3}{4}$ a yard, and sells the whole for $\$2\frac{9}{20}$ a yard. What does he gain?

4. A merchant purchased 8 yards of cloth at $\$6\frac{1}{2}$ a yard. What sum will he gain per yard if he sells the whole piece for $\$56\frac{24}{25}$?

5. A man bought a piece of land for $\$1046\frac{1}{4}$ at the rate of $\$15\frac{1}{2}$ an acre. He sells it for $\$17\frac{1}{5}$ an acre. How much does he gain on the whole?

6. A merchant purchased 15 casks of wine of 25 gallons each. He paid \$980 for the wine, $\$78\frac{3}{4}$ tax, $\$33\frac{3}{5}$ for transportation. He sold it for $\$3\frac{4}{5}$ a gallon. How much did he gain?

7. A speculator purchased 738 acres of land for \$21,294. He sells $\frac{2}{5}$ of his land at the rate of $\$34\frac{3}{4}$ an acre, and the rest at the rate of \$35 per acre. What does he gain?

8. A piece of cloth is $29\frac{3}{4}$ yards in length. How many pieces, each containing $1\frac{55}{64}$ yards, can be cut from *it?*

9. How many postage-stamps, each containing $\frac{18}{25}$ of a square inch, are in a sheet of $172\frac{4}{5}$ square inches?

10. Of a boat worth $5600, A, who has $\frac{17}{64}$, sells $\frac{3}{4}$ of his share to B, and B sells $\frac{4}{5}$ of his share to C. Find the value of C's share.

11. From Montreal to Toronto, by the Grand Trunk Railway, the distance is 332 miles. One-half a mile more than $\frac{3}{8}$ of this distance was opened in November, 1855, and the remainder in November, 1856. Find the number of miles opened in 1856.

12. The 36 Israelites who fell in the first assault on Ai, were $\frac{3}{250}$ of the force sent by Joshua. How many were sent by Joshua?

13. What number multiplied by $8\frac{2}{5}$ equals $3\frac{1}{2}+\frac{1}{4}+\frac{16}{25}+\frac{43}{50}$?

14. Multiply the sum of $\frac{9}{10}$ and $\frac{3}{4}$ by their difference.

15. Of the distance from Edinburgh to London by rail, that from Edinburgh to Carlisle is $\frac{1}{4}$, from Carlisle to Preston $\frac{9}{40}$, while that from Preston to London is 210 miles. Find the distance from Edinburgh to London.

16. How many times can a measure holding $\frac{7}{8}$ of a pint be filled from a vessel containing $63\frac{7}{8}$ pints?

17. Of a consignment of guano $\frac{207}{400}$ consisted of carbonate of lime and phosphates of lime and magnesia, and the phosphates made up $\frac{19}{40}$ of the guano. How many parts in a hundred of the guano were carbonate of lime?

18. Of the water of the Dead Sea $\frac{49}{1250}$ is muriate of lime, $\frac{41}{400}$ muriate of magnesia, $\frac{259}{2500}$ muriate of soda, $\frac{1}{2000}$ sulphate of lime. What part of the whole do these *ingredients constitute*?

[illegible]

1. [illegible] have been [illegible]

2. [illegible] of a yard must [illegible]

3. [illegible] his brother [illegible] himself. [illegible]

4. [illegible] home the [illegible] home?

5. [illegible]

6. [illegible]

7. [illegible]

8. [illegible] of 6 yards [illegible]

9. [illegible] of a ton of coal?

10. [illegible] weigh respectively 2¼, 3½, 4¼ pounds. What is the weight of the whole?

11. When poultry is worth 20 cents per pound, what must be paid for a turkey weighing 8½ pounds, and a chicken weighing 3¼ pounds?

12. From a jar of butter containing 15½ pounds there have been sold 7¾ pounds. How many pounds remain in the jar?

13. *Change to* mixed numbers $\frac{17}{4}$, $\frac{25}{2}$, $\frac{19}{8}$, $\frac{13}{7}$, $\frac{21}{5}$, $\frac{24}{11}$.

14. Express in lowest terms $\frac{12}{16}$, $\frac{8}{12}$, $\frac{16}{20}$, $\frac{56}{63}$, $\frac{36}{42}$, $\frac{84}{108}$.

15. Change to improper fractions $5\frac{1}{2}$, $6\frac{3}{5}$, $8\frac{5}{9}$, $9\frac{4}{7}$, $13\frac{1}{4}$.

16. Reduce $\frac{3}{5}$ to 10ths; to 15ths; to 20ths; to 25ths.

17. A lady gave $\frac{1}{2}$ a dollar to her daughter, and $\frac{1}{5}$ of a dollar to her son. What fraction of a dollar did the daughter receive more than the son?

18. At $\frac{4}{5}$ of a dollar per bushel, how many bushels of apples can be bought for \$3?

19. Four pecks make a bushel. If $2\frac{3}{8}$ pecks be sold from a bushel of cranberries, how many pecks remain?

20. A gentleman bought 2 pairs of gloves at $\$1\frac{1}{2}$ a pair, and 3 pairs of slippers at $\$1\frac{3}{4}$ per pair. He gave a ten-dollar bill in payment. What change should he receive?

21. What part of 2 is 1? of 7 is 3? of 9 is 2? of 12 is 4?

22. A farmer planted 3 bushels of potatoes, and harvested 50 bushels. What fraction of the crop was the seed?

23. From a piece of cloth containing 81 yards there were sold 45 yards. What part of the piece was sold?

24. What part of $\frac{4}{5}$ is $\frac{2}{3}$? of $\frac{1}{2}$ is $\frac{1}{3}$? of $\frac{1}{2}$ is $\frac{1}{5}$?

HINT. Reduce the fractions to similar fractions.

25. What part of 8 is $\frac{1}{2}$? of 7 is $\frac{2}{3}$?

26. In a year there are 365 days. What part of a year are 30 days? 50 days? 75 days? 105 days?

27. Three-fourths of a cord of oak wood costs \$6. What is the cost of $\frac{1}{4}$ of a cord? of a cord?

28. Seven-eighths of a yard of cloth cost 42 cents. Find the cost of $\frac{1}{8}$ of a yard; of 1 yard; of $2\frac{1}{4}$ yards.

29. Four-fifths of a load of wood is sold for \$8. Required the cost of $\frac{1}{5}$ of the load; of the whole load; of $4\frac{3}{4}$ such loads.

30. What is the price of a bushel of turnips, when $\frac{3}{4}$ of a bushel are sold for 30 cents?

31. A farmer divided among his 4 sons $\frac{3}{4}$ of his farm. What part of the farm did each son receive?

32. At the rate of \$10 per week, what is the cost of board per day?

33. How many bushels of carrots, at \$$\frac{3}{4}$ per bushel, can be bought for \$$3\frac{3}{4}$?

34. How many cows are $\frac{4}{5}$ of 20 cows? 16 sheep are $\frac{4}{5}$ of how many sheep?

35. Five-sixths of 12 hens are $\frac{5}{8}$ of how many hens?

36. Three-fourths of a cord of wood, at \$7 per cord, will pay for what part of a ton of coal, at \$9 per ton?

37. The captain of a vessel owns $\frac{1}{4}$ of it, the first mate $\frac{3}{8}$, and the captain's wife $\frac{1}{2}$ of the remainder. What part of the vessel does she own?

38. John Rogers sold to Henry Cook $\frac{1}{2}$ of his woodland, and then bought back $\frac{1}{3}$ of what he had sold. What part of the land did each have then?

39. From a bin of potatoes containing 30 bushels, $5\frac{3}{4}$ bushels, $2\frac{1}{2}$ bushels, $4\frac{3}{4}$ bushels were sold. How many bushels were left in the bin?

40. A bushel of wheat weighs 60 pounds. If a miller takes 3 pounds from each bushel for toll, what part of a bushel does he take?

41. At 20 cents per yard, how many yards of ribbon can be bought for \$2.20?

42. Four-fifths of \$20 is $\frac{2}{3}$ of how much money?

43. Two-thirds of a yard of silk can be bought for \$$\frac{3}{4}$. What is the price per yard? How many yards can be bought for \$$3\frac{3}{8}$?

44. If 5 bushels of oats cost \$2, what will be the cost of 9 bushels at the same rate?

45. If \$$1\frac{1}{2}$ are paid for $\frac{3}{4}$ of a yard of velvet, what will be the cost of $\frac{5}{8}$ of a yard?

46. If $\frac{2}{3}$ of the distance between two towns is $6\frac{4}{5}$ miles, what is the whole distance?

47. If $2\frac{3}{4}$ bushels of apples make a barrel, how many barrels will 11 bushels make?

48. A carpet dealer sold $\frac{2}{3}$ of $\frac{1}{2}$ of a roll of carpet. What part of the roll was left?

49. How many pigs can be bought for \$20, at \$$2\frac{1}{2}$ each?

50. Four quarts make a gallon. When $2\frac{1}{2}$ quarts have been taken from a gallon of vinegar, what part of the gallon has been taken?

51. A drover puts $\frac{1}{2}$ of his cattle in a field, $\frac{2}{5}$ of them in another field, and 10 in a barn. How many cattle has he?

53. [illegible] of it, and [illegible] 12 pounds, [illegible] at first?

54. [illegible] pipe 3 gallons of [illegible] and by the [illegible]. If the cistern [illegible] open, in how [illegible]?

55. [illegible] of work in 4 hours, [illegible] work in 6 hours. What [illegible] 1 hour? What [illegible] both together [illegible] both together [illegible]?

56. [illegible] in 6 hours. C and D [illegible] an acre can [illegible] What part can [illegible] D plant [illegible] take D to [illegible]?

[illegible] greyhound. The [illegible] greyhound 65. In [illegible] overtaken?

57. By [illegible] of their cost is [illegible] at which they [illegible] their cost.

58. By selling a farm for $2400, the owner lost $\frac{1}{[illegible]}$ of what it cost him. How much did he pay for the farm?

59. How many flowers can be planted along the borders of a flower-bed 12 feet long and 10 feet wide, if the flowers are $\frac{1}{2}$ of a foot apart?

Ex. 97.

1. Reduce to simple fractions, $8\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{7}{8}$, $\dfrac{9\frac{1}{6}}{\frac{1}{2} \text{ of } 7}$, $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2} \text{ of } \frac{2}{3}}$.

2. Find the values of

   $169 - 14\frac{3}{7}$; $1\frac{9}{11} - \frac{1}{3}$ of 4; $76\frac{1}{4} - \frac{2}{3}$ of 19.

3. Six pieces of cloth measure respectively $23\frac{1}{2}$ yards, $19\frac{2}{3}$ yards, $21\frac{5}{6}$ yards, $24\frac{7}{8}$ yards, $35\frac{11}{12}$ yards, $18\frac{3}{4}$ yards. After $39\frac{5}{24}$ yards have been sold from their sum, how much remains?

4. The remainder being 4, the quotient 51, and the divisor 25, it is required to find the dividend.

5. Find the value of $\frac{3}{8}$ of a chest of tea weighing $57\frac{1}{2}$ pounds, at $\$1\frac{1}{2}$ per pound.

6. If a man work $8\frac{1}{2}$ hours in a day he can finish a piece of work in $12\frac{1}{2}$ days. How many hours per day must he work to complete it in $10\frac{5}{8}$ days?

7. A confectioner sells $\frac{2}{3}$ of $\frac{1}{2}$ of a bushel of walnuts. What part of the bushel remains, and what will it bring at 15 cents per quart?

8. What is the value of a basket of 588 eggs, worth 25 cents per dozen?

9. A man starts on a journey 5 hours before the mail coach. How many miles will the coach be ahead of the man after it has run for 12 hours, supposing that he travels at the rate of 3 miles an hour, and the coach 10 miles an hour?

10. If $\frac{5}{7}$ of $\frac{2}{3}$ of a piece of land cost $\$420$, what is the *value of the whole?*

11. A farmer sold at market 15 sheep at $\$2\frac{5}{8}$ each, and bought 7 yards of cloth at $\$1\frac{4}{5}$ per yard. How much money did he take home?

12. A man walked a distance of 60 miles; for the first 5 hours, at the rate of 3 miles an hour, and during the remainder of the journey he walked at the rate of 4 miles an hour. In how many hours did he complete the journey?

13. The circumference of a fore wheel of a wagon is $6\frac{2}{3}$ feet; that of the hind wheel $8\frac{4}{5}$ feet. In a distance of 20 miles, of 5280 feet each, how many more turns will be made by the former than by the latter?

14. A young man received $1200 from his father. He spent $\frac{1}{9}$ of the money for clothes, $\frac{1}{2}$ of it in travelling, and invested the remainder in a mortgage. What fraction of the whole was the sum invested?

15. A baker paid $32 for $\frac{4}{7}$ of a hogshead of molasses. What was the value of $\frac{1}{3}$ of the remainder?

16. A gentleman paid $125 for keeping 2 horses 12 weeks. What would it cost, at the same rate, to keep one horse $\frac{2}{3}$ of a week?

17. If $\frac{5}{9}$ of a yard of ribbon cost $\$\frac{7}{8}$, what will be the value of $5\frac{2}{3}$ yards?

18. Reduce $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{18}$ to decimal fractions, and add the results.

19. The contents of a chest of tea weighing 87.5 pounds are made up into 1 pound, $\frac{1}{2}$ pound, $\frac{1}{4}$ pound packages, an equal number of each. How many packages of each kind?

20. In five successive days a farmer puts into his bin $37\frac{7}{8}$ bushels of potatoes, and on each of these days he sells $19\frac{2}{3}$ bushels. How many bushels have been put into the bin? How many more are in the bin at the end than at the beginning of the five days?

21. A man's weekly income is $\$18\frac{1}{2}$, and his weekly expenses are $\$23\frac{2}{3}$. If he have $\$75\frac{1}{4}$ in reserve, how many weeks can he live without incurring debt?

22. By a leak $87\frac{5}{9}$ barrels of water enter the hold of a boat in 1 hour; the pumps will discharge $58\frac{2}{3}$ barrels in an hour. If she can carry only 875 barrels, in how many hours will she sink?

23. A can mow a field in 10 days, B in 8 days, and C in 5 days. When working together, how many days will they need?

24. A carpenter alone can build a shop in 15 days, and with the help of his son he can build it in 10 days. In how many days will the son alone build the shop?

25. Wales Edwards and George Peters hire a pasture for \$14. Edwards puts in 8 horses; Peters puts in 50 sheep. If 21 sheep will eat as much as 2 horses, what must each pay?

26. A flour dealer bought 125 barrels of flour at $\$6\frac{2}{3}$. He sold 97 barrels at $\$7\frac{3}{4}$, and the remainder, being injured, brought only $\$5\frac{4}{5}$. What did he gain?

27. A lady bought $\frac{2}{3}$ of $\frac{4}{5}$ of a yard of ribbon for $\$1\frac{3}{7}$. What was the cost per yard?

28. From two fields 482 bushels of corn are gathered. The first field yields $\frac{1}{2}$ as much as the second. How many bushels does each field yield?

29. A farmer brought to market 3 jars of butter, weighing 26 pounds, 37 pounds, 19 pounds. The empty jars weighed $3\frac{1}{8}$ pounds, $4\frac{1}{3}$ pounds, $5\frac{5}{8}$ pounds. The butter brought $30. What was the price per pound?

30. From 120 acres of land $32\frac{1}{8}$ acres are sold to one man, and $\frac{1}{3}$ of the remainder to another. How many acres are unsold?

31. If the rent of 3 acres of land for $\frac{3}{4}$ of a year be $9, what will be the rent of 45 acres for 1 year?

32. If $\frac{5}{8}$ of a ton of coal cost $4, how many tons can be bought for $$145\frac{3}{4}$?

33. If 12 horses eat $65\frac{5}{8}$ bushels of oats in 3 months, how many bushels will 7 horses eat in 2 years?

34. The agent for a line of steamers sells $\frac{1}{3}$ of a steamship to one company, $\frac{1}{4}$ of the remainder to a second, and $\frac{1}{2}$ of what is left to a third. What part of the whole ship has the third company?

35. A farmer exchanged 13 loads of oats, of 18 bags each, every bag containing $2\frac{1}{2}$ bushels, for 150 sheep, at $2.925. What was the price of the oats per bushel?

36. Two men 95.784 miles apart approached each other until they met. One travelled 7.476 miles more than the other. How many miles did each travel?

37. A teacher spent $\frac{3}{5}$ of his salary in board for himself and family, and $\frac{1}{10}$ of it in clothing for himself. The clothing of his wife and child cost $\frac{7}{8}$ as much as his own. At the end of the year $187 remained. What was the salary?

38. A road to the top of a hill has a rise of $\frac{1}{6}$ of a foot in 100 feet. How many feet is the total elevation of the hill, if the length of the road is 2 miles?

39. A man bequeathes to his wife $\frac{1}{3}$ of his estate; to his daughter, $\frac{1}{5}$ of it; to his son, $\frac{1}{2}$ of the daughter's share; he divides the remainder equally between a hospital and a public library. What part is received by the hospital?

40. If the above estate is worth \$150,784, what is the amount received by the hospital?

41. A can build a wall in 7 days, B in 6 days, and C in 5 days. A and B worked together for 2 days, when they were joined by C. How many days will they need to complete the remainder of the work?

42. Find the cost of 75,849 bricks, at \$9.75 per M.

43. A lumberman exchanged 50,495 feet of round timber, at \$$4\frac{1}{2}$ per M, for pork, at \$$20\frac{3}{4}$ per barrel. How many barrels of pork did he receive?

44. For $\frac{7}{8}$ of a bushel of apples \$$\frac{3}{4}$ are paid. What will $4\frac{5}{8}$ bushels be worth?

45. Henry Jones bought at a saw-mill 3485 ft. boards, at \$7.50 per M; 9872 feet laths, at \$0.25 per C; 6492 feet flooring, at \$$8\frac{3}{4}$ per M; 8975 feet cherry boards, at \$15.05 per M. He paid \$152.75 in cash, and the balance in flour, at \$9.25 per barrel. Required the number of barrels of flour.

46. A merchant mixed 7 pounds of black tea at 68 cents with 9 pounds of green tea at 75 cents. At what price per pound must he sell the mixture to gain \$3.69?

47. Nine men working 10 hours per day will harvest a piece of grain in 8 days. How many days will be needed for the same work by 6 men working 9 hours per day?

48. At $8.75 per M, how many bricks can be bought for $393.75?

49. When 1000 bricks cost $7.20, what is the cost of a single brick?

50. If $437.645 be paid for 6500 feet of rosewood, what is the cost per M?

51. A sea captain who owned $\frac{3}{5}$ of a ship and cargo, gave to his wife $\frac{1}{8}$ of his share, to his daughter $\frac{1}{2}$ of what his wife received, to his son $\frac{3}{5}$ of the remainder, and equally divided what was still left between two nieces. What part of the whole had each niece?

52. Peter Knowlton sold a farm for $9786, which was $\frac{5}{8}$ of the sum paid for it. Required the original cost of the farm.

53. A merchant bought a bag of coffee, containing 60 pounds, for $15. At what advance per pound must he sell it to buy with the gain on the coffee 3 yards of velvet at $3 per yard?

54. After selling $\frac{2}{5}$ of his sheep to a drover, and $\frac{1}{3}$ of the remainder to his neighbor, a farmer has 150 left. How many were there in the flock at first?

55. A stock broker bought 9 shares in the Northern Pacific Railroad, at $99\frac{1}{2}$, and 12 shares in the Illinois Central Railroad, at $102\frac{1}{2}$. He sold them all at $103\frac{1}{4}$. How much did he gain?

56. A bankrupt's available property can be sold for \$19,780, which will pay $62\frac{1}{2}$ cents on every dollar he owes. How much does he owe?

57. A loaf of bread weighing 2 pounds, when flour is worth \$9.80 per barrel, is sold for 10 cents. What should it bring when flour is worth \$7.84?

58. Divide 0.75 of $17\frac{5}{8}$ by $\frac{4}{5}$ of 0.035.

59. An army of 7844 men has 490,250 pounds of beef. If for every man $1\frac{1}{4}$ pounds daily be allowed, in how many days will the beef be consumed?

60. A seedsman bought $37\frac{2}{3}$ bushels of lawn grass-seed for \$226. He sold 25 bushels at a profit of $\$1\frac{2}{5}$ per bushel. For what price per bushel must he sell the remainder to make his whole gain \$73?

61. The cost of 50 gallons of molasses is \$25. By leakage $\frac{1}{5}$ of it is lost; 20 gallons are sold at $62\frac{1}{2}$ cents. At what rate must the remainder be sold to gain \$5 on the whole?

62. For $\frac{2}{3}$ of a yard of broadcloth at $\$6\frac{1}{2}$ per yard, $1\frac{3}{4}$ yards of cassimere and 50 cents in money were given in exchange. What was the price per yard of the cassimere?

63. A owns $\frac{3}{5}$ of a ship and cargo worth \$25,748, B $\frac{1}{4}$ of the remainder, C $\frac{1}{8}$ of the amount belonging to A and B, and D owns what is still left. Required the amount of D's share.

64. A farmer gives to his eldest son $\frac{13}{25}$ of a farm, and the remainder to his daughter. The difference between their shares is 780 acres. How many acres does the daughter receive?

65. If 1200 pounds can be carried 36 miles for \$14, how many pounds can be carried 24 miles for \$14?

66. If $2\frac{1}{4}$ acres of land cost \$500, what will 460 acres cost?

67. Four and four-sevenths tons of cannel coal cost \$64. Required the cost of $13\frac{3}{4}$ tons.

68. Of a certain estate $\frac{1}{3}$ is pasture, $\frac{3}{5}$ land suitable for cultivation, and the remainder, woodland, is 50 acres. How many acres in the estate?

69. If 1.4 bushels of walnuts cost \$1.50, find the value of 7 bushels.

70. If a man breathes 17 times a minute, and takes in at each breath $\frac{4}{7}$ of a quart of air, how many quarts of air does he need in 1 hour?

71. If the crop of potatoes from an acre is on the average 255 bushels, but the potato beetle destroys $\frac{2}{3}$ of the crop, how many bushels will $3\frac{3}{4}$ acres produce?

72. If a miller takes $\frac{1}{16}$ for toll, and a bushel of wheat produces 40 pounds of flour, how many bushels must be carried to the mill to obtain 196 pounds of flour?

73. An expressman carried 100 vases, on the condition that he was to receive $\frac{1}{4}$ of a dollar for every one he carried without breaking, and pay $1\frac{1}{4}$ dollars for every one he broke. He received 16 dollars. How many did he break?

HINT. The expressman loses \$9 on the lot, and he loses \$$1\frac{1}{2}$ on each vase broken.

74. A man who rows 4 miles an hour in still water takes $1\frac{1}{5}$ hours to row 4 miles up a river. How many minutes will it take him to row 4 miles down the river?

HINT. The man rows $4\frac{4}{5}$ miles in $1\frac{1}{5}$ hours. Hence the current sets him back $\frac{4}{5}$ of a mile in $1\frac{1}{5} = \frac{6}{5}$ hours, or $\frac{4}{5} + \frac{6}{5} = \frac{2}{3}$ of a mile in 1 hour. In rowing down the river he rows 4 miles *an hour, and the* current carries him $\frac{2}{3}$ of a mile an hour.

CHAPTER IX.

COMPOUND QUANTITIES.

142. A quantity expressed with reference to a *single unit* is called a **simple quantity**; but a quantity expressed with reference to *different units* is called a **compound quantity.**

Thus, $20\frac{1}{4}$ pounds is a simple quantity, but 20 pounds 4 ounces is a compound quantity.

143. The process of changing the *unit* in which a quantity is expressed, without changing the *value* of the quantity, is called **reduction.**

144. If the change be from a higher denomination to a lower, it is called **reduction descending**; if from a lower to a higher, it is called **reduction ascending.**

Thus, 1 yard = 36 inches is an example of reduction descending; and 24 inches = 2 feet is an example of reduction ascending.

UNITS OF LENGTH.

145.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet,	= 1 rod (rd.).
320 rods, 1760 yards, or 5280 feet,	= 1 mile (mi.).

NOTE. A line = $\frac{1}{12}$ in.; a barleycorn = $\frac{1}{3}$ in.; a hand (used in measuring the height of horses) = 4 in.; a palm = 3 in.; a span = 9 in.; a cubit = 18 in.; a military pace = $2\frac{1}{2}$ ft.; a chain = 4 rds.; a link = $\frac{1}{100}$ chain; a furlong = $\frac{1}{8}$ mi.; a knot (used in navigation) = 6086 ft.; a nautical league = 3 knots; a fathom (used in measuring depths at sea) = 6 ft.; a cable length = 120 fathoms.

Ex. 98. (*Oral.*)

1. How many inches in 1 yd.? in $\frac{1}{2}$ yd.? in $\frac{1}{4}$ yd.?
2. How many yards in 180 in.? in 48 in.? in 45 in.?
3. How many yards in 3 rds.? in 4 rds.? in 5 rds.?
4. How many feet in 2 yds.? in 2 rds.? in 2 rds. 2 yds.?
5. How many rods in 33 ft.? How many yards in 33 ft.?
6. In $\frac{1}{2}$ mi. how many rods? yards? feet?
7. How many rods in 0.4 of a mile? in 0.3? in 0.7?
8. What part of a mile are 160 rds.? 80 rds.? 40 rds.?
9. What part of a foot are 4 in.? 3 in.? 6 in.? 8 in.?
10. What part of a yard are 2 ft.? 1 ft. 6 in.? 2 ft. 6 in.?

REDUCTION DESCENDING.

146. Change 10 mi. 40 rds. to feet.

```
10 mi. 40 rds.
  320
 3200
   40
 3240
  16½
 1620
19440
 3240
53460
```

10×320 rds. = 3200 rds., to which the 40 rds. are added.

Again, $3240 \times 16\frac{1}{2}$ ft. = 53,460 ft.

The multiplicand and multiplier are interchanged in the operation.

Ex. 99.

Reduce to feet:

1. 8 mi. 5 yds. 2 ft.
2. 40 mi. 5 rds. $2\frac{1}{2}$ yds.
3. 2 mi. 52 rds. 1 ft.

Reduce to inches:

4. 18 mi. 252 rds. 2 yds.
5. 11 mi. 6 rds. 4 yds.
6. 18 mi. 230 rds. 8 ft.

7. 2 yds. 1 ft. 9 in.

8. 5 yds. 2 ft. 7 in.

9. 170 rds. 3 yds. 9 in.

10. 8 mi. 96 rds. 4 yds.

11. 2 mi. 80 rds. 2 ft.

12. 200 rds. 115 yds. 5 in.

Reduction Ascending.

147. Change 53,463 ft. to a compound quantity.

```
16½) 53463 ft.
         2
 33) 106926 . . . . . half-feet.
 320) 3240 rds. . . 6 half-feet = 3 ft.
        10 mi. . . 40 rds.
```

10 mi. 40 rds. 3 ft. *Ans.*

There are 16½ ft., or 33 half-feet, in a rod; so the 53,463 ft. are changed to half-feet, and the half-feet to rods, by dividing by 33. The remainder is 6 half-feet = 3 ft.

3240 rds. are changed to miles by dividing by 320, the number of rods in a mile. The remainder is 40 rds.

Reduce 376,985 in. to higher denominations.

```
 12 | 376985 in.
  3 |  31415 ft. . . 5 in.
 5½ |  10471 yds. . 2 ft.
  2 |      2
 11 |  20942 half-yards.
320 |   1903 rds. . 9 half-yards = 4½ yds.
    |      5 mi.. . 303 rds.
```

The ½ yd. of the 4½ yds. should be reduced to lower denominations, and the result, 1 ft. 6 in., added to the 2 ft. 5 in. Thus,

mi.	rds.	yds.	ft.	in.
5	303	4	2	5
			1	6
5	303	5	0	11

5 mi. 303 rds. 5 yds. 0 ft. 11 in. Ans.

Ex. 100.

Reduce to higher denominations:

1. 211 in.	5. 125,899 in.	9. 348,164 in.
2. 33,777 in.	6. 179,875 in.	10. 247,391 in.
3. 142,737 in.	7. 87,476 ft.	11. 99,204 ft.
4. 33,000 ft.	8. 97,378 yds.	12. 11,220 ft.

Compound Addition and Subtraction.

148. Add:

mi.	rds.	yds.	ft.	in.
6	120	3	2	2
18	15	1	1	6
3	215	2	2	8
28	31	$2\frac{1}{2}$	0	4
		$\frac{1}{2}$	=1	6
28	31	2	1	10

28 mi. 31 rds. 2 yds. 1 ft. 10 in. *Ans.*

Write the numbers so that units of the same denomination shall be in the same column. The sum of the inches is 16. Divide the 16 in. by 12 (12 in. = 1 ft.). The result is 1 ft. 4 in. Write 4 under the column of inches, and add 1 to the column of feet.

The sum of the feet, including the 1 ft. from the 16 in., is 6. Divide by 3 (3 ft. = 1 yd.). The result is 2 yds. 0 ft. Write 0 under the column of feet, and add 2 to the yards.

The sum of the yards, including the 2 yds. from the 6 ft., is 8. Divide by $5\frac{1}{2}$ ($5\frac{1}{2}$ yds. = 1 rd.). The result is 1 rd. $2\frac{1}{2}$ yds. Write $2\frac{1}{2}$ under the column of yards, and add 1 to the rods.

The sum of the rods, including the 1 rd. from the 8 yds., is 351. Divide by 320 (320 rds. = 1 mi.). The result is 1 mi. 31 rds. Write 31 under the column of rods, and add 1 to the miles.

The sum of the miles, including the 1 mi. from the 351 rds., is 28. The $\frac{1}{2}$ yd. is changed to 1 ft. 6 in., and added to 0 ft. 4 in.

149. Take 4 mi. 110 rds. 5 yds. 2 ft. from 6 mi. 25 rds. 4 yds. 2 ft.

mi.	rds.	yds.	ft.	
6	25	4	2	
4	110	5	1	
1	234	$4\frac{1}{2}$	1	
		$\frac{1}{2}=$	1	6 in.
1	234	4	2	6

1 mi. 234 rds. 2 ft. 6 in. *Ans.*

Write the numbers so that units of the same denomination shall be in the same column.

Since 5 yds. are more than 4 yds., 1 rd. is taken from the 25 rds., and reduced to yards, and the result added to 4 yds., making $9\frac{1}{2}$ yds. Then $9\frac{1}{2}$ yds. – 5 yds. = $4\frac{1}{2}$ yds.

The $4\frac{1}{2}$ is written under the column of yards.

Since 110 rds. are more than 24 rds., 1 mi. is taken from the 6 mi., and reduced to rods, and the result added to 24 rds., making 344 rds. Then 344 rds. – 110 rds. = 234 rds.

The 234 is written under the column of rods. The 4 mi. are subtracted from 5 mi. and the $\frac{1}{2}$ yd. is changed to 1 ft. 6 in.

Ex. 101.

Add:

1.

yds.	ft.	in.
15	1	7
23	2	9
35	0	6
7	2	11

2.

rds.	yds.	ft.
23	3	1
18	4	2
27	0	2
6	4	0

3.

mi.	rds.	yds.
17	23	4
9	17	2
23	0	3
11	35	1

4.

mi.	rds.	yds.
37	14	2
28	16	2
19	10	4
10	56	3

5.

mi.	rds.	ft.
23	119	15
19	173	11
8	65	12
32	147	8

6.

mi.	rds.	ft.	in.
7	95	8	9
8	96	7	8
3	98	9	9
6	87	8	7

Find the difference between:

	yds.	ft.	in.		rds.	yds.	ft.		mi.	rds.	ft.
7.	14	1	4	8.	22	2	0	9.	23	76	1
	10	2	11		19	3	2		6	157	2

	mi.	rds.	ft.	in.		mi.	rds.	yds.	ft.		mi.	rds.	yds.
10.	17	125	1	10	11.	7	0	0	0	12.	13	33	2
	8	187	2	11		3	64	3	2		9	32	4

Compound Multiplication and Division.

150. Multiply 37 yds. 2 ft. 11 in. by 4.

yds.	ft.	in.
37	2	11
		4
151	2	8

4×11 in. = 44 in. = 3 ft. 8 in. Write the 8 in. under the column of inches.

4×2 ft. = 8 ft.; 8 ft. with the 3 ft. added are 11 ft. = 3 yds. 2 ft. Write the 2 ft. under the column of feet.

4×37 yds. = 148 yds.; and 148 yds. with the 3 yds. added = 151 yds.

151 yds. 2 ft. 8 in. *Ans.*

NOTE. When the multiplier is the product of two factors, multiply by one of the factors, and the resulting product by the other.

151. Divide 121 yds. 2 ft. by 73.

```
    yds.  ft.
73) 121   2(1 yd. 2 ft.
     73
     48
      3
    144
      2
    146
    146
```

The remainder from dividing 121 yds. by 73 is 48 yds., which are reduced to feet by multiplying by 3 (3 ft. = 1 yd.). The result with the 2 ft. added is 146 feet.

There is no remainder from dividing 146 ft. by 73.

1 yd. 2 ft. *Ans.*

Divide 10 ft. 11 in. by 2 ft. 8 in.

Reduce both quantities to inches.

10 ft. 11 in. = 131 in. $\frac{131}{32} = 4\frac{3}{32}$.
2 ft. 8 in. = 32 in.

$4\frac{3}{32}$. *Ans.*

Ex. 102.

1. Multiply 33 yds. 2 ft. 11 in. by 17.
2. Multiply 23 rds. 3 yds. 2 ft. by 100.
3. Divide 15 yds. 1 ft. 9 in. by 3.
4. Divide 289 yds. 2 ft. 9 in. by 213.
5. Divide 150 mi. 178 rds. 3 yds. by 9.
6. Multiply 3 mi. 72 rds. 9 ft. by 11.
7. Multiply 150 rds. 2 yds. 1 ft. by 235.
8. Divide 33 mi. 40 rds. by 200.
9. Divide 200 mi. 56 rds. 3 yds. 2 ft. by 121.
10. Multiply 11 mi. 200 rds. by 14.
11. Multiply 52 mi. 1021 yds. by 47.
12. Divide 43 mi. 280 rds. by 24.

Fractions of Simple and Compound Quantities.

152. Express $\frac{2}{3}$ of a mile in rods, feet, and inches.

$\frac{2}{3}$ mi. = $\frac{2}{3}$ of 320 rds. = $213\frac{1}{3}$ rds.
$\frac{1}{3}$ rd. = $\frac{1}{3}$ of $16\frac{1}{2}$ ft. = $5\frac{1}{2}$ ft.
$\frac{1}{2}$ ft. = $\frac{1}{2}$ of 12 in. = 6 in.

213 rds. 5 ft. 6 in. Ans.

Express 0.6275 of a mile in rods, feet, and inches.

0.6275			
320			
12 5500	0.6275 mi. = 0.6275 of 320 rds.	= 200.8 rds.	
188 25	0.8 rd. = 0.8 of $16\frac{1}{2}$ ft.	= 13.2 ft.	
200.8 rds.	0.2 ft. = 0.2 of 12 in.	= 2.4 in.	
$16\frac{1}{2}$			
13.2 ft.			200 rds. 13 ft. 2.4 in. *Ans.*
12			
2.4 in.			

Find the value of $\frac{5}{9}$ of 3 rds. 14 ft. 7 in.

	rds.	ft.	in.
	3	14	7
			5
9)	19	6	11
	2	2	$7\frac{8}{9}$

Here we multiply by the numerator of the fraction, and divide the product by the denominator.

2 rds. 2 ft. $7\frac{8}{9}$ in. *Ans.*

NOTE. When the multiplier is a mixed number, multiply by the integer and the fraction separately, and add the resulting products.

Ex. 103.

Find the value of:

1. $\frac{5}{9}$ of a mile.
2. $\frac{2}{7}$ of a mile.
3. $\frac{5}{9}$ mi. $-$ $\frac{5}{7}$ rd.
4. $\frac{3}{16}$ mi. + $\frac{3}{5}$ of 40 rds. + $\frac{3}{8}$ yd.
5. 0.475 of a mile.
6. 0.3975 of a mile.
7. 0.01284 of 14 miles.
8. 3.726 mi. $-$ 33.57 rds.
9. Find $\frac{3}{5}$ of 5 mi. 89 rds. 3 yds. 2 ft.
10. Take $\frac{3}{5}$ of 4 mi. from $\frac{7}{8}$ of 3 mi. 18 rds. 3 yds. 2 ft.
11. Add 0.525 mi., 0.125 rd., 0.5 yd., 0.16 ft.

To express one Quantity as the Fraction of another.

153. Express 145 rds. 2 yds. 1 ft. 6 in. as the fraction of a mile.

$$6 \text{ in.} = \tfrac{6}{12} \text{ ft.} = \tfrac{1}{2} \text{ ft.}$$

$$1\tfrac{1}{2} \text{ ft.} = \frac{1\frac{1}{2}}{3} \text{ yds.} = \tfrac{1}{2} \text{ yd.}$$

$$2\tfrac{1}{2} \text{ yds.} = \frac{2\frac{1}{2}}{5\frac{1}{2}} \text{ rds.} = \tfrac{5}{11} \text{ rd.}$$

$$145\tfrac{5}{11} \text{ rds.} = \frac{145\frac{5}{11}}{320} = \frac{1600}{320 \times 11} \text{ mi.} = \tfrac{5}{11} \text{ mi.}$$

$\frac{5}{11}$ of a mile. *Ans.*

Express 120 rds. 3 yds. 1 ft. 6.72 in. as the decimal of a mile.

$$\begin{array}{r|l} 12 & 6.72 \text{ in.} \\ 3 & 1.56 \text{ ft.} \\ 5.5 & 3.52 \text{ yds.} \\ 320 & \underline{120.64 \text{ rds.}} \\ & 0.377 \text{ mi.} \end{array}$$

6.72 in. ÷ 12 = 0.56 ft., and this added to the 1 ft. gives 1.56 ft. 1.56 ft. ÷ 3 = 0.52 yds., and this added to 3 yds. gives 3.52 yds. 3.52 yds. ÷ 5.5 gives 0.64 rds., and this added to 120 rds. gives 120.64 rds. 120.64 rds. ÷ 320 gives 0.377 mi.

0.377 of a mile. *Ans.*

Note. The quotient in any case need not be carried beyond the *fifth* decimal place, and the required answer will be sufficiently accurate for all practical purposes.

154. Express 1 yd. 2 ft. 3 in. as the fraction of 5 yds. 1 ft. 3 in.

1 yd. 2 ft. 3 in.:

$$3 \text{ in.} = \tfrac{3}{12} \text{ ft.} = \tfrac{1}{4} \text{ ft.}$$

$$2\tfrac{1}{4} \text{ ft.} = \frac{2\frac{1}{4}}{3} \text{ yd.} = \tfrac{3}{4} \text{ yd.}$$

$1\frac{3}{4}$ yds.

5 yds. 1 ft. 3 in.:

$$3 \text{ in.} = \tfrac{3}{12} \text{ ft.} = \tfrac{1}{4} \text{ ft.}$$

$$1\tfrac{1}{4} \text{ ft.} = \frac{1\frac{1}{4}}{3} \text{ yd.} = \tfrac{5}{12} \text{ yd.}$$

$5\frac{5}{12}$ yds.

$$\frac{1\frac{3}{4}}{5\frac{5}{12}} = \tfrac{21}{65}.$$

$\frac{21}{65}$. *Ans.*

NOTE. If the answer to the last problem is to be expressed as a decimal fraction, first find the answer as a common fraction, and reduce this common fraction to a decimal fraction.

Ex. 104.

Express:

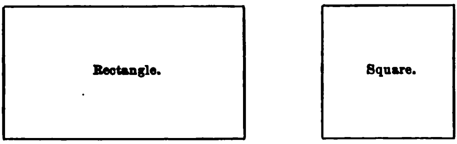
1. 125 rds. 4 yds. 2 ft. 6 in. as the fraction of a mile.
2. 1 yd. 2 ft. 3 in. as the fraction of 5 yds.
3. 51 rds. 1 yd. 3.6 in. as the decimal of a mile.
4. $\frac{1}{2}$ rd. + $\frac{1}{4}$ yd. as the fraction of a mile.
5. 3 mi. 53 rds. 4 yds. 1.2 ft. as the decimal of 5 mi. 89 rds. 3 yds. 2 ft.
6. 2 mi. 138 rds. 1 yd. as the fraction of 3 mi. 265 rds. $3\frac{1}{2}$ yds.
7. 233 rds. 9 ft. 10.8 in. as the decimal of a mile.
8. 3 mi. 242 rds. $2\frac{3}{4}$ yds. as the decimal of 7 mi. 160 rds.
9. 2 ft. $7\frac{1}{2}$ in. as the decimal of 100 yds.
10. 11 rds. 4 yds. $4\frac{1}{2}$ in. as the fraction of a mile.
11. $\frac{7}{8}$ rd. + $\frac{3}{8}$ yd. + $\frac{3}{16}$ ft. as the fraction of a rod.
12. 195 yds. 1 ft. 8 in. as the fraction of $\frac{1}{8}$ of a mile.
13. 1 mi. 232 rds. 4 yds. 1 ft. 6 in. as the fraction of 8 mi. 204 rds. 0 yd. 1 ft. 6 in.
14. 127 rds. 3 ft. 3.6 in. as the decimal of a mile.
15. 261 rds. 4 yds. 1 ft. 6 in. as the fraction of a mile.
16. $\frac{1}{5}$ of the difference between 3 yds. 2 ft. 11 in. and 10 yds. 7 in. as the fraction of 16 yds.
17. 7 rds. 1 ft. 3.17 in. as the decimal of 76 rds. 2 yds. 5 in.
18. 248 rds. 4 yds. 2 ft. 8 in. as the fraction of 2 mi.

MEASURES OF SURFACE.

155. A surface has two dimensions, *length* and *breadth*.

156. If a surface is flat and has four square corners, it is called a **rectangle.**

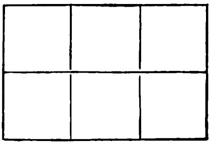
157. If a rectangle has its four sides equal, it is called a **square.**



158. The unit of surface is a square each side of which is a linear unit.

159. The area of a surface is the number of square units it contains.

160. Suppose the rectangle in the margin is 3 in. long and 2 in. wide. If lines are drawn as represented in the figure, the surface will be divided into **square inches.** There will be 2 horizontal rows of 3 square inches each; that is, in all, 2×3 square inches. Hence,



Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express its area in **square units** *of the same name as the linear unit of the sides.*

Conversely, the number of square units in a rectangle divided by the number of linear units in one side will give the number of linear units in its adjacent side.

UNITS OF SURFACE.

161.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet,	= 1 square rod (sq. rd.).
160 square rods, or 10 square chains,	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

A square of flooring or roofing	= 100 sq. ft.
A section of land	= 1 mile square.
A township	= 36 sq. mi.

The units of surface measure are obtained by squaring the units of linear measure. Thus,

$$144 = 12^2;\ 9 = 3^2;\ 30\tfrac{1}{4} = (5\tfrac{1}{2})^2;\ 272\tfrac{1}{4} = (16\tfrac{1}{2})^2$$

Ex. 105. (*Oral.*)

1. How many square feet of surface in a blackboard 4 ft. wide and 9 ft. long?

2. If a slate is 8 in. wide and has a surface of 80 sq. in., what is the length of the slate?

3. How many square inches in $\frac{1}{4}$ of a square foot? in $\frac{3}{4}$? in $\frac{1}{3}$? in $\frac{5}{6}$?

4. How many square feet in 3 sq. yds.? in 5 sq. yds.?

5. How many square inches in a board 4 in. long and 3 in. wide?

6. A square yard of carpet is 3 ft. long and 3 ft. wide; how many square feet in it?

7. How many square feet in a yard of carpet 2 ft. wide? $2\frac{1}{2}$ ft. wide?

8. How many square feet in a room 12 ft. by 15 ft.?

9. How many yards of carpet 2 ft. wide will be required to cover the floor of the above room, if the strips run lengthwise of the room?

10. How many square yards in 81 sq. ft.?

11. How many square rods in $\frac{5}{8}$ of an acre?

12. What part of an acre are 40 sq. rds.? 80? 100?

Ex. 106.

1. Reduce 5 A. 147 sq. rds. to square rods.

2. How many square inches in 9 sq. yds. 7 sq. ft.?

3. Reduce 33,796 sq. in. to square yards.

4. Reduce 153 A. 87 sq. rds. to square inches.

5. In 67,413 sq. yds. how many acres?

6. In a rectangular field 49 yds. long and 16 yds. wide, how many square feet?

7. How many tiles 1 ft. square will be needed to pave a hall 20 ft. long and 9 ft. wide?

8. How much greater is the area of a lot 50 rds. square than that of a lot containing 50 sq. rds.?

9. How many square yards in a square lot measuring 142 ft. on a side?

10. Ingrain carpet is 3 ft. wide. How many yards will be required for a room 27 feet long and 18 ft. wide?

11. From each corner of a square, the side of which is 2 ft. 5 in., a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.

12. Find the value of 0.45 of an acre.

13. Reduce $\frac{17}{24}$ of a square mile to lower denominations.

14. Reduce 80 sq. rds. 2.42 sq. yds. to the decimal of an acre.

15. Add $\frac{4}{5}$ of an acre, $\frac{5}{9}$ sq. rd., and $\frac{2}{3}$ sq. yd.

16. Add $\frac{2}{3}$ of an acre and $\frac{4}{9}$ of a square rod.

17. From $\frac{2}{11}$ of a square rod take $\frac{3}{4}$ of a square yard.

18. Find $\frac{5}{7}$ of 9 A. 70 sq. rds. 15 sq. yds. 7 sq. ft. 19 sq. in.

19. A side of Russell Square in London is 660 ft. How many acres does it contain?

20. The area of a rectangular field is 33 sq. rds. 1 sq. yd. 6 sq. ft. 108 sq. in., and the length is 9 rds. 1 ft. 6 in. What is the width?

The area of a circle is found by multiplying the square of its radius by 3.1416. (The radius is half the diameter.)

21. Find the area of a circular pond if its radius is 300 ft.

22. Find the area of the bottom of a round cistern if its diameter is 11 ft.

23. The radius of the rotunda of the Pantheon of Rome is 71 ft. 6 in. Find the area of the floor in square feet.

24. The two dials of the clock of St. Paul's, London, are each $18\frac{1}{7}$ ft. in diameter. Find the area of each in square feet.

Carpeting Rooms.

In determining the number of yards of carpeting required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of yards in a strip, including the waste in matching the pattern, multiplied by the number of strips will give the required number of yards.

25. How many yards of carpet $2\frac{1}{4}$ ft. wide will cover a floor 18 ft. by 15 ft., if the strips run across the room?

Hint. $18 \div 2\frac{1}{4} = 8$. Hence 8 strips are required. 15 ft. = 5 yds., and 8×5 yds. = 40 yds.

26. How many yards of carpet $\frac{3}{4}$ of a yard wide will be required for a floor 26 ft. by $15\frac{3}{4}$ ft., if the strips run lengthwise? If the strips run across the room? How much will be turned under in each case?

27. How many yards of carpeting $\frac{7}{8}$ of a yard wide will be required for a room $8\frac{1}{2}$ yds. long and 17 ft. wide, if the strips run lengthwise and there is a waste of $\frac{1}{16}$ of a yard in each strip in matching patterns?

28. Find the cost of carpet 30 inches wide, at $1.25 per yard, for a room 18 ft. by 14 ft., if the strips run lengthwise. If the strips run across the room.

29. Find the cost of carpeting $\frac{3}{4}$ of a yard wide, at $2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there is a waste of $\frac{1}{4}$ of a yard on each strip in matching the pattern.

30. Which way must the strips of carpet $\frac{3}{4}$ of a yard wide run in order to carpet most economically a room 20 ft. 6 in. long and 19 ft. 6 in. wide, if there is no waste for matching the pattern?

Ex. 108.

1. Reduce 38 chains 80 links to the decimal of a mile.

2. The four sides of a field are 23 chains 19 links, 17 chains 34 links, 6 chains 85 links, and 24 chains 62 links. How many yards around the field?

3. One field contains 3 sq. chains, and another is 3 chains square. How many acres in both fields together?

4. A field is crossed by a driveway 15 links wide, and 13 chains 43 links long. How many square rods in the driveway?

5. From a field of 4 A. a rectangular piece 3 chains 25 links long and 2 chains 75 links wide is reserved. How much of the field is left?

6. The sides of a triangular field measure $21\frac{1}{2}$ chains, 14 chains 11 links, and 8 chains 10 links respectively. By how many yards is the longest side less than the sum of the other two?

Board Measure.

163. Boards one inch or less in thickness are sold by the square foot.

Boards more than one inch in thickness, and all squared lumber, are sold by the number of square feet of boards one inch in thickness to which they are equivalent.

Thus, a board 16 ft. long, 1 ft. wide, and 1 in. thick, contains 16 ft. *board measure.* If only $\frac{7}{8}$, $\frac{3}{4}$, or $\frac{1}{2}$ of an inch thick, it still contains *16 ft.; but, if* $1\frac{1}{4}$ in. thick, it contains 20 ft. board measure.

Ex. 109.

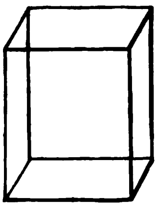
How many feet board measure in :

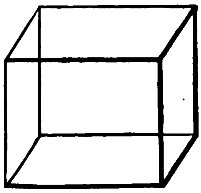
1. A board 18 ft. long, 6 in. wide, $\frac{7}{8}$ in. thick?
2. A board 16 ft. long, 12 in. wide, 1 in. thick?
3. Forty boards 14 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick?
4. A stick of square timber 8 in. by 9 in., and 30 ft. long?
5. Six joists, each 3 in. by 4 in., and 11 ft. long?
6. Ten joists, each 6 in. by 4 in., and 14 ft. long?
7. A stick of square timber 10 in. by 12 in., and 36 ft. long?
8. Ten 2-in. planks, each 13 ft. long, 15 in. wide?
9. Thirty 3-in. planks, each 12 ft. long, 10 in. wide?
10. A board 24 ft. long, 23 in. wide at one end, and 17 in. at the other, and $1\frac{1}{2}$ in. thick?

    HINT. The average width is $\frac{23+17}{2}$ in.
11. In a stick of timber 40 ft. long and 15 in. square?
12. Ten 4-in. planks 16 ft. long and 10 in. wide?

MEASURES OF VOLUME.

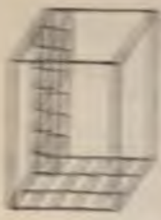
164. A rectangular solid is a solid bounded by six rectangles.





165. If the rectangles are all squares, the solid is called a cube.

166. In the figure represented in the margin, let the length contain 5, the breadth 3, and the height 7 in.



The base may be divided into square inches; there will be three rows of 5 sq. in. each; in all 15 sq. in. Upon each square inch may be placed a pile of 7 cu. in., so that the solid will contain 15 × 7 cu. in.; that is, 3 × 5 × 7 cu. in.

Hence, to find the volume of a rectangular solid,

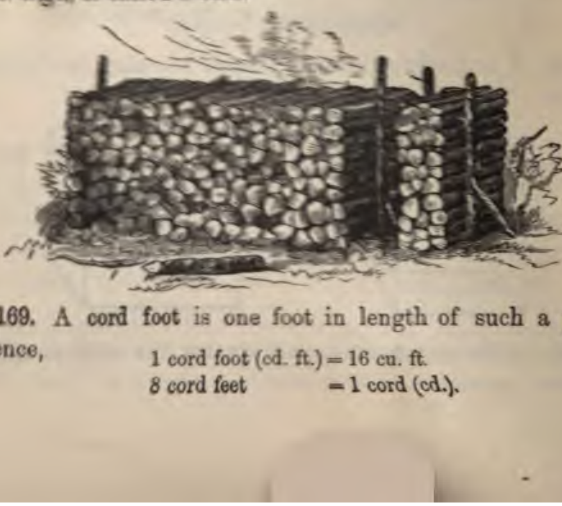
Express its length, breadth, and height in the same linear unit; the product of these numbers will express its volume in cubic units of the same name as the linear unit.

UNITS OF VOLUME.

167. 1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

The units of volume are cubes of the linear units. Thus, $1728 = 12^3$, $27 = 3^3$.

168. In measuring wood, a pile 8 ft. long, 4 ft. wide, and 4 ft. high, is called a cord.



169. A cord foot is one foot in length of such a pile. Hence,

1 cord foot (cd. ft.) = 16 cu. ft.
8 cord feet = 1 cord (cd.).

Ex. 110. (*Oral.*)

1. A brick is 2 by 4 by 8 in. Find its volume.
2. How many cubic feet in 2 cu. yds.?
3. How many cubic yards in 81 cu. ft.?
4. Twelve cubic feet are what part of a cubic yard?
5. How many cords in 40 cd. ft.? in 16 cd. ft.?
6. How many cords in a pile of wood 32 ft. long, 4 ft. wide, and 4 ft. high?
7. How many loads of earth, each 1 cu. yd., must be removed in digging a ditch 21 ft. long, 3 ft. wide, and 3 ft. deep?
8. How many cubic feet in a stick of timber 12 in. wide, 9 in. thick, and 24 ft. long?
9. How many feet board measure in a cubic foot?
10. How many cubic feet in a stick of timber 16 in. wide, 9 in. thick, and 21 ft. long?

Ex. 111.

1. Reduce 15 cu. yds. 13 cu. ft. to cubic inches.
2. Reduce 150,000 cu. in. to cubic yards.
3. Subtract 28 cu. yds. 25 cu. ft. 1500 cu. in. from 47 cu. yds. 13 cu. ft. 1236 cu. in.
4. Multiply 17 cu. yds. 17 cu. ft. 187 cu. in. by 11.
5. Divide 22 cu. yds. 10 cu. ft. 933 cu. in. by 7.
6. How many cubic inches can be cut out of a cubic foot?
7. How many cubic feet of water will a cistern hold whose *three dimensions* are each 4 ft.?

8. How many cubic inches in a rectangular stone post 3 ft. high, 1 ft. wide, and 1 ft. thick?

9. Find the value of 0.975 of a cubic yard.

10. Reduce 13 cu. ft. 864 cu. in. to the decimal of a cubic yard.

11. A man bought 52 cu. yds. 18 cu. ft. 984 cu. in. of stone for the cellar of a house, and $\frac{3}{4}$ as much for the cellar of a second house. How much did he buy for both?

12. How many cords of wood in a pile 50 ft. long, 6 ft. high, and 4 ft. wide?

13. How many cords of wood in a pile 42 ft. long, $6\frac{1}{2}$ ft. high, and 8 ft. wide?

14. What must be the length of a load of wood that is 4 ft. wide, $5\frac{1}{3}$ ft. high, to contain 2 cds.?

15. A cubic foot of wood weighs 20 pounds. Find the weight of 10 boards, each 30 ft. long, 1 ft. wide, and 1 in. thick.

Units of Capacity.

Dry Measure.

170.

2 pints (pt.)	=	1 quart (qt.).
8 quarts	=	1 peck (pk.).
4 pecks	=	1 bushel (bu.).

Liquid Measure.

171.

4 gills (gi.)	=	1 pint (pt.).
2 pints	=	1 quart (qt.).
4 quarts	=	1 gallon (gal.).

$31\frac{1}{2}$ gallons	=	1 barrel (bbl.).
2 barrels	=	1 hogshead (hhd.).

NOTE. The gallon of liquid measure contains 231 cu. in. The bushel of dry measure contains 2150.42 cu. in. Therefore the quart of liquid measure contains $57\frac{3}{4}$ cu. in., and the quart of dry measure $67\frac{1}{5}$ cu. in.

Ex. 112. (*Oral.*)

1. How many pints in 16 gi.? in 37 gi.?
2. How many pints in 2 qts.? in 7 qts. 1 pt.?
3. How many quarts in 2 pks.? in 3 pks. 3 qts.?
4. How many quarts in 3 bu.?
5. How many baskets holding $2\frac{1}{2}$ pks. each will 10 bu. of apples fill?
6. If a pint of milk cost 4 cts., what will a gallon cost?
7. How many times will a gallon of water fill a half-pint cup?
8. How many pint bottles will be required to hold 5 gals. 2 qts. of cider?
9. If 4 qts. of blueberries cost 32 cts., what will a bushel cost at the same rate?
10. A 2-gal. measure of molasses lacks 3 pts. of being full. What is the molasses worth at 80 cts. a gallon?
11. If a horse eats 4 qts. of corn a day, how many days will a bushel last him?
12. If a quart of berries is worth 10 cts., what is a peck worth?

Ex. 113.

1. Reduce 440 pts. to pecks, and 109 pts. to gallons.
2. Reduce 2024 pts. to bushels.

Add:

	gals.	qts.	pts.		bu.	pks.	qts.		bu.	pks.	qts.
3.	13	2	1	4.	5	1	3	5.	17	2	4
	2	3	0		6	1	1		11	3	4
	15	0	0		2	0	0		3	0	0
	7	1	1		3	0	2		18	3	4

Subtract:

	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]		bu.	pks.	qts.
6.	[illegible]	[illegible]	[illegible]	7.	[illegible]	[illegible]	[illegible]	8.	27	1	1
	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]		18	1	3

9. [illegible] by 1308.

10. [illegible] by 9.

11. How many [illegible] bottles will be required to hold 63 [illegible]?

12. How many [illegible] packages can a seedsman make from [illegible]?

13. In [illegible] a market-gardener sold 3758 baskets of strawberries averaging 1 pt. each. How many [illegible] did he sell?

14. A [illegible] flock of [illegible] fowls feeds them daily [illegible] of grain. What is the average amount for each fowl?

15. A [illegible] her 4 canaries $\frac{3}{4}$ of a [illegible] $1\frac{1}{4}$ [illegible] and to 3 mocking-birds [illegible]. How much seed did she give to all the birds together?

16. A [illegible] $\frac{3}{4}$ of a bushel of Lima beans, and $\frac{1}{4}$ [illegible] beans. He sold 3 pks. [illegible] How many had he left?

17. A merchant receives [illegible] boxes of oranges, amounting to [illegible] 3 pks. 7 qts. Only $\frac{4}{5}$ of the fruit was fit to sell; how many bushels had to be thrown away?

18. A tank is [illegible] ft. [illegible] in. long, 16 ft. 4 in. wide, and 6 ft. 4 in. deep. Find how many gallons it will hold.

19. Find the number of bushels in a bin that is 6 ft. long, 5 ft. wide, 4 ft. deep.

20. How many gallons will a cistern hold that is 5 ft. square and 6 ft. deep?

Units of Weight.

Avoirdupois Weight.

172.

16 drams (drs.)	= 1 ounce (oz.).
16 ounces	= 1 pound (lb.).
100 pounds	= 1 hundred-weight (cwt.).
20 hundred-weight	= 1 ton (t.).
112 pounds	= 1 long hundred-weight.
2240 pounds	= 1 long ton.

Note. Avoirdupois weight is used for weighing all articles except gold, silver, and jewels.

In the United States custom house and in wholesale transactions in coal and iron the long ton is used.

The pound avoirdupois contains 7000 grains.

Ex. 114. (*Oral.*)

1. How many ounces in 2 lbs.? in 5 lbs.?
2. How many ounces in $\frac{1}{4}$ of a pound? in $\frac{3}{4}$ of a pound? in $\frac{1}{8}$ of a pound? in $\frac{5}{8}$ of a pound? in $\frac{7}{8}$ of a pound?
3. What part of a pound are 4 oz.? 2 oz.? 8 oz.? 6 oz.? 12 oz.?
4. How many pounds in 48 oz.? in 36 oz.? in 24 oz.?
5. How many hundred-weight in 2 t.? in 3 t.?
6. How many 4-oz. packages of nutmegs can be put up from $2\frac{1}{4}$ lbs. of nutmegs?
7. If hay is \$20 a ton, how many pounds can be bought for \$5? \$7? \$10?
8. If hay is \$16 a ton, what are 750 lbs. worth?
9. What part of a pound is $\frac{3}{8}$ of an ounce?

10. **If butter is 25 cts. a pound, and hay is $16 a ton, how many pounds of butter will it take to pay for 1500 lbs. of hay?**

Ex. 115.

1. **Reduce 12,484 oz. to higher denominations.**

2. **Reduce 7 cwt. 64 lbs. to ounces.**

3. **Reduce 95,784 oz. to higher denominations.**

4. **A bushel of wheat weighs 60 lbs. How many bushels in $1\frac{3}{4}$ t.?**

5. **A cubic foot of water weighs 1000 oz. In 1800 cu. ft. of water how many tons?**

6. **What is the difference in pounds between 27 long tons of coal and 27 short tons of coal?**

7. Find the value of $\frac{4}{15}$ of a ton.

8. What fraction of a pound is 0.00006 of a ton?

9. Add $\frac{4}{5}$ t., $\frac{1}{4}$ cwt., $\frac{3}{4}$ lb.

10. Reduce 8 cwt. 34 lbs. to the decimal of a ton.

11. Find the value of 0.472875 of a ton.

12. Reduce 12 cwt. 80 lbs. 6 oz. to the decimal of a ton.

13. A farmer sold in one week 5.825 t. of hay. On Monday he sold 1350 lbs.; on Tuesday, $\frac{1}{2}$ t.; on Wednesday, $1\frac{1}{8}$ t.; on Thursday, 1.415 t.; on Friday, $1\frac{3}{4}$ t. What part of a ton did he sell on Saturday?

14. A grocer sold in one day 17 cwt. 83 lbs. 6 oz. of loaf sugar, 13 cwt. 95 lbs. 12 oz. of coffee sugar, 15 cwt. 78 lbs. 15 oz. of brown sugar. Required the whole amount sold.

15. A grocer has 7 cwt. 57 lbs. 12 oz. of Java coffee, 5 cwt. 39 lbs. 10 oz. of Mocha. After mixing the two kinds of coffee, he sells from the mixture 10 cwt. 97 lbs. 9 oz. How much coffee has he left?

16. A butcher receives from the West every day, Sundays excepted, 9 cwt. 81 lbs. 7 oz. of beef. How much does he receive per week?

17. A man puts into his cellar 17 loads of coal, averaging 1 t. 387 lbs. a load. Required the whole amount.

18. Divide 19 t. 17 cwt. 58 lbs. by 9.

19. A farmer sells 4 oxen whose united weight is 2 t. 7 cwt. 29 lbs. 13 oz. What is their average weight?

20. Find $\frac{3}{4}$ of 8 t. 16 cwt. $24\frac{3}{4}$ lbs.

21. Divide 15 t. 17 cwt. 29 lbs. 7 oz. by $\frac{4}{5}$.

Troy Weight.

173.

24 grains (grs.)	=	1 pennyweight (dwt.).
20 pennyweights	=	1 ounce (oz.).
12 ounces	=	1 pound (lb.).

NOTE. Troy weight is used for weighing gold, silver, and jewels. The pound Troy contains 5760 grs.

Ex. 116. (*Oral.*)

1. How many grains in 2 dwt.? in 2 dwt. 9 grs.? in 3 dwt. 7 grs.?

2. How many pennyweights in 24 grs.?

3. How many pennyweights in 1 oz.? in 2 oz.? in 2 oz. 8 dwt.? in 5 oz. 17 dwt.?

4. How many ounces in 40 dwt.? in 100 dwt.? in 60 *dwt?*

5. How many ounces in 1 lb.? in 5 lbs.? in 10 lbs.? in 3 lbs. 6 oz.? in 4 lbs. 9 oz.?

6. How many pounds in 12 oz.? in 48 oz.? in 72 oz.? in 80 oz.? in 90 oz.?

7. If 1 dwt. of gold is worth $1.50, find the value of 1 oz. of gold; 1 lb. of gold.

8. How many spoons weighing 25 dwt. each can be made from 1 lb. 3 oz. of silver?

9. If 10 dwt. of silver are worth 70 cts., find the value of 1 lb. of silver.

Ex. 117.

1. Reduce 3 lbs. 9 oz. 18 dwt. 17 grs. to grains.
2. Reduce 25 lbs. 9 oz. 5 dwt. to pennyweights.
3. Reduce 3420 dwt. to higher denominations.
4. What is the difference in weight between 3 doz. silver tablespoons weighing 5 lbs. 9 oz. 8 dwt. and 3 doz. silver teaspoons weighing 1 lb. 9 oz. 16 dwt. 18 grs.?
5. Required the weight of 8 silver teapots, each weighing 3 lbs. 9 oz. 18 dwt. 13 grs.
6. When 12 tankards weigh 36 lbs. 8 oz. 14 dwt. 16 grs., what is their average weight?
7. Find the value of $\frac{8}{9}$ of a pound.
8. Reduce $\frac{4}{9}$ of a grain to the fraction of an ounce.
9. Reduce 7 oz. 10 dwt. to the fraction of a pound.
10. Add 0.475 lbs., 0.75 dwt., 0.125 oz., 0.374 lb.
11. From 0.675 lb. subtract 5.25 oz.
12. Reduce 1 oz. 7 dwt. 18 grs. to the decimal of a pound.

13. Reduce $\frac{3}{4}$ dwt. to the fraction of a pound.

14. Reduce 4 oz. 4 dwt. to the fraction of a pound.

15. What decimal of a pound is $\frac{7}{25}$ lb. — $\frac{3}{4}$ oz.?

174. In preparing medicines, apothecaries use the following:

Apothecaries' Weight.

20 grains (grs.)	= 1 scruple (℈).
3 scruples	= 1 dram (ʒ).
8 drams	= 1 ounce (℥).
12 ounces	= 1 lb.

Apothecaries' Measure.

60 minims (♏)	= 1 dram (♏ lx.).
8 drams	= 1 ounce (fl. drm. viij.).
16 ounces	= 1 pint (fl. oz. xvj.).

Ex. 118.

1. In 4 lbs. 8 ℥ 4 ʒ 2 ℈ how many grains?

2. In 7864 grs. how many pounds?

3. A patient is required to take daily 2 ʒ 2 ℈ of bark. How many weeks will 7 lbs. of bark last him?

4. Find the amount of 0.4 lb. 0.25 ℥ 0.375 ʒ 0.648 ℈ 2.147 grs.

5. Subtract 3 ℥ 7 ʒ 12 grs. from 9 ℥ 6 ʒ 1 ℈ 16 grs., and reduce the result to the decimal of a pound.

6. How many grains in 1 lb. of apothecaries' weight?

7. What part of a pound avoirdupois is a pound troy or a pound apothecaries' weight?

8. What part of an ounce avoirdupois is an ounce troy or an ounce apothecaries' weight?

UNITS OF TIME.

175.	**60 seconds (sec.)**	**= 1 minute (min.).**
	60 minutes	**= 1 hour (hr.).**
	24 hours	**= 1 day (dy.).**
	7 days	**= 1 week (wk.).**
	365 days (or 52 wks. 1 dy.)	**= 1 common year (yr.).**
	366 days	**= 1 leap-year.**
	100 years	**= 1 century.**

The names of the months called calendar months, and the number of days in each are:

	dys.		dys.
1. January (Jan.)	31	7. July	31
2. February (Feb.) .	28 or 29	8. August (Aug.)	31
3. March	31	9. September (Sept.) . . .	30
4. April	30	10. October (Oct.)	31
5. May	31	11. November (Nov.) . . .	30
6. June	30	12. December (Dec.) . . .	31

NOTE. The number of days in each month may be easily remembered by committing the following lines:

> "Thirty days hath September,
> April, June, and November;
> All the rest have thirty-one,
> Except the second month alone,
> Which has but twenty-eight, in fine,
> Till leap-year gives it twenty-nine."

A solar year is 365 dys. 5 hrs. 48 min. 50 sec.; that is, nearly $365\frac{1}{4}$ days. As there are 365 days in a common year, a common year lacks nearly $\frac{1}{4}$ of a day of being a solar year, and this defect is made up by reckoning for some years (leap-years) 366 days.

The leap-years are the years whose dates are exactly divisible by 4; except in the case of complete hundreds, and these are leap-years if divisible by 400. Thus, 1600 and 1884 were leap-years; 1800 and 1885 were not; 1900 will not be a leap-year.

Ex. 119. (*Oral.*)

1. How many seconds in 2 min.? in 3 min.?
2. How many minutes in 2 hrs.? in 3 hrs.? in 60 sec.? in 120 sec.?
3. How many hours in 2 dys.? in 3 dys.? in 120 min.? in 360 min.?
4. How many days from Jan. 1 to Feb. 17, both days inclusive?
5. How many months from Aug. 9 to Nov. 9? from March 5 to Sept. 5? from April 4 to Oct. 4?
6. If a man can do a piece of work in 30 min., how many hours will it take him to do four times as much?
7. If a man can walk a mile in 15 min., how many hours will it take him to walk 24 mi.?
8. At the rate of 3 mi. an hour, how far will a man walk in 45 min.?
9. If a man earns $12 a week, and pays for expenses $12 per month of 4 wks., how much will he save in 20 wks.?
10. If a man walks $\frac{1}{4}$ of a mile in 5 min., how many hours, at that rate, will it take him to walk 4 mi.?

Ex. 120.

1. Reduce 4 yrs. 39 dys. 17 hrs. 22 min. 18 sec. to seconds.
2. In 48,967,349 sec. how many years?
3. Find the exact length of the lunar month which contains 2,551,443 sec.

4. How many seconds more are there in the 3 spring months than in the 3 autumn months?

5. Reduce $\frac{4}{5}$ of a year to days.

6. Find the value of 0.375 yr. 0.142 dy. 0.27 min.

7. What part of a day are 12 hrs. 15 min. 25 sec.?

8. What part of 2 dys. 7 hrs. 18 min. are 1 dy. 3 hrs. 15 min.?

9. How much greater is the quotient of 100 yrs. 25 dys. 12 hrs. 27 min. 28 sec. divided by 4 than the product of 4 yrs. 17 dys. 9 hrs. 12 min. 18 sec. multiplied by 5?

10. Find the number of days, reckoning from noon of the one to noon of the other, between Feb. 24 and June 23, 1884; also between Dec. 25, 1884, and May 25, 1885.

11. How many hours from noon of the 4th to midnight of the 7th of July, 1885?

12. Divide 11 wks. 6 dys. 18 hrs. by 9.

13. Divide 2 yrs. 135 dys. 17 hrs. by 72.

14. From 5 yrs. 17 hrs. take 2 yrs. 138 dys. 22 hrs.

15. Find the value of 3.1725 days.

16. Find the value of 21.325 years.

17. Express 9 dys. 3 hrs. as the decimal of a week.

18. Express 13 hrs. 15 min. 17 sec. as the fraction of 6 dys. 1 hr. 48 min. 7 sec.

19. Express 3 dys. 20 hrs. 35 min. 33 sec. as the decimal of 27 dys. 13 hrs. 22 min. 30 sec.

20. Find the value of 5.58 years.

DIFFERENCE BETWEEN TWO DATES.

176. Find the difference between April 3, 1885 and May 7, 1837.

yrs.	mos.	dys.
1885	4	3
1837	5	7
47	10	26

In finding the difference between long dates, 30 days are considered a month. As April is the fourth, and May the fifth, month, we write 4 and 5 instead of the names of these months.

In finding the difference between short dates, the exact number of days is generally counted.

Ex. 121.

1. On the 1st of January, 1885, how much time had passed since the discovery of the Island of San Salvador, Oct. 12, 1492?
2. At the birth of Lafayette, Sept. 6, 1757, what was the age of George Washington, born Feb. 22, 1732?
3. If a note is dated March 5, 1885, and has 3 mos. 3 dys. to run, when is the note due?
4. If a note is discounted Feb. 1, and is due April 22, how many months and days has it to run?
5. Find the exact number of days from Sept. 23 to Jan. 11.

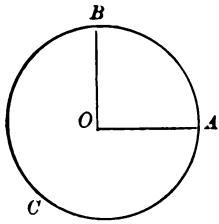
   NOTE. In finding the difference of these dates, the 23d of September is not counted, but the 11th of January is.
6. Find the exact number of days from March 5 to July 4.
7. Find the exact number of days from June 3 to Nov. 1.
8. Find the exact number of days from Feb. 3 to June 3, of a common year.
9. Find the difference between June 7, 1885, and July 4, 1776.

Angular Measure.

177. A circle is a plane figure bounded by a curved line called the **circumference**, all points of which are equally distant from a point within called the **centre**. A part of the circumference is called an **arc**.

178. A line drawn through the centre and terminated by the circumference is called a **diameter**; and half the diameter is called the **radius**.

If a straight line fixed at one end is revolved, the other end will describe the circumference of a circle; and the amount of rotation of the straight line from its position at the start to any other given position, is the angular magnitude described by the moving straight line. Thus, if OA revolve about O as a fixed point, the extremity A will describe the circumference ABC. When OA has reached the



position OB, the part of the circumference between A and B is described by A, and the part of the angular magnitude about the point O, between OA and OB, is described by OA. The angle AOB is such a part of the angular magnitude about O as AB is of the circumference.

The circumference of every circle is divided into 360 equal parts, called *arc-degrees*, and corresponding to every one of these equal parts is an angle at the centre of the circle. Hence the whole angular magnitude about any point in a plane is divided into 360 equal parts called *angle-degrees*, and the number of degrees in the angle formed by two lines drawn from the centre of a circle is the same as the number of degrees in the arc which is intercepted between these two lines.

179. An angle described by a line making one-fourth of a revolution contains 90° and is called a **right angle**, as AOB; and OA and OB are said to be **perpendicular** to *each other*.

UNITS OF ANGULAR MEASURE.

180. 60 seconds ($''$) = 1 minute ($'$).
60 minutes = 1 degree ($^\circ$).
360 degrees = 1 revolution.

NOTE. A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

Ex. 122.

1. Reduce $49^\circ\ 37'\ 29''$ to seconds.
2. In $13{,}978''$ how many degrees?
3. Find the value of $\frac{4}{7}$ of 360°.
4. What part of the whole angular magnitude about a point is $\frac{3}{5}$ of a second?
5. Find the sum of 45.425°, $0.115'$, $0.255''$.
6. Change 0.471 of a minute to the decimal of a degree.
7. What part of $7^\circ\ 35'\ 15''$ are $3^\circ\ 20'\ 45''$?
8. Divide $17^\circ\ 27'\ 13''$ by 5; multiply $8^\circ\ 19'\ 47''$ by 8; and find the difference between the results.
9. From $7^\circ\ 0'\ 18''$ subtract $3^\circ\ 47'\ 36''$.
10. The latitude of New York is $40^\circ\ 42'\ 43''$ North; the latitude of Boston is $42^\circ\ 21'\ 30''$ North. Find their difference in latitude.
11. The latitude of New Orleans is $29^\circ\ 57'\ 46''$ North; the latitude of Rio Janeiro is $22^\circ\ 56'$ South. Find their difference in latitude.

    HINT. Their difference in latitude is found by taking the *sum of their latitudes.*

187. *Weights.*

A bushel of corn or rye	= 56 lbs.
A bushel of corn meal, rye meal, or cracked corn,	= 50 lbs.
A bushel of wheat	= 60 lbs.
A bushel of potatoes	= 60 lbs.
A bushel of beans	= 60 lbs.
A bushel of oats	= 32 lbs.
A bushel of barley	= 48 lbs.
A bushel of timothy-seed	= 45 lbs.
A stone of iron or lead	= 14 lbs.
A pig of iron or lead	= 21½ stone.
A fother of iron or lead	= 8 pigs.

The weight of a bushel of potatoes, corn, etc., varies slightly in different States, but the weights here given are those generally adopted in business transactions.

A barrel of flour	= 196 lbs.
A barrel of pork or beef	= 200 lbs.
A cask of lime	= 240 lbs.
A cental of grain	= 100 lbs.
A quintal of fish	= 100 lbs.

Books.

188. A book formed of sheets folded in

2 leaves is a folio;
4 leaves is a quarto;
8 leaves is an octavo;
12 leaves is a duodecimo;
16 leaves is a 16mo.

Ex. 125.

1. How many barrels in 75 t. of beef?
2. In a car-load of 36,000 lbs. of wheat, how many bushels?
3. Find the weight of 27 bu. of potatoes.
4. How much paper will be used by an author who sends to a semi-weekly paper 6 sheets of manuscript twice a week for a year?

5. Reduce $\frac{7}{8}$ of a quire to the fraction of a bundle.

6. Reduce 2 bundles 6 quires 6 sheets to the fraction of 2 bales 1 bundle.

7. Reduce 3 bundles 7 quires 18 sheets to the decimal fraction of a bale.

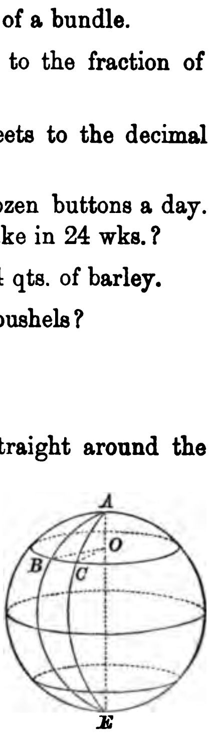
8. A button manufactory makes 96 dozen buttons a day. How many great gross will it make in 24 wks.?

9. Find the weight of 103 bu. 3 pks. 4 qts. of barley.

10. In 5 t. 624 lbs. of oats, how many bushels?

Longitude and Time.

189. A **meridian** is any line drawn straight around the earth, and passing through both poles.



190. The **longitude** of a place is the angle of inclination of the two planes which are supposed to pass through the centre of the earth, and contain, the one the meridian of that place, and the other the standard meridian. Thus, the longitude of *C*, reckoned from meridian *ABE*, is the angle *BOC*, *OB* and *OC* being both perpendicular to the diameter of the earth *AE* at the point *O*. Places on the Eastern Hemisphere are in East Longitude; on the Western Hemisphere, in West Longitude.

191. As the earth turns upon its axis once in twenty-four hours, a point on the earth's surface will describe a circumference (360°) in twenty-four hours. Therefore longitude may be reckoned in *time* as well as in degrees.

In one hour a point on the earth's surface describes $\frac{1}{24}$ of

$360° = 15°$; in one minute $\frac{1}{60}$ of $15° = 15'$; and in one second $\frac{1}{60}$ of $15' = 15''$.

Again, since it requires one hour (60 min.) for a point to pass over 15°, to pass over 1° it requires $\frac{1}{15}$ of 60 min. = 4 min.; and to pass over 1′ it requires $\frac{1}{60}$ of 4 min. = 4 sec.

192. Express 20° 36′ 15″ of longitude in time.

$$\begin{array}{r} 15\underline{)20°\ 36'\ 15''} \\ 1\text{ hr. } 22\text{ min. } 25\text{ sec.} \end{array}$$

Since 15° longitude give 1 hr. in time, 15′ longitude 1 min., and 15″ longitude 1 sec., divide 20° 36′ 15″ by 15, as in compound division, and the quotient will be the time required.

193. Express 1 hr. 4 min, 4 sec. in degrees.

$$\begin{array}{r} 1\text{ hr. } 4\text{ min. } 4\text{ sec.} \\ \underline{15} \\ 16°\ 1'\ 0'' \end{array}$$

Since 1 hr. of time equals 15° of longitude, 1 min. of time 15′, and 1 sec. of time 15″, multiply 1 hr. 4 min. 4 sec. by 15, as in compound multiplication, and the product will be the longitude required.

194. Hence, if longitude is expressed in **degrees, divide** by 15; the quotient gives the longitude in *hours, minutes, and seconds.*

195. If longitude is expressed in **time, multiply** by 15; the product gives the longitude in *degrees, minutes, and seconds.*

Ex. 126.

1. The difference in time between New York and Paris is 5 hrs. 5 min. 20 sec. What is the difference in longitude?
2. Boston is 71° 3′ and San Francisco 122° 26′ west of Greenwich. What is the difference in clock-time between the two cities?

3. The difference in clock-time between New York and Canton is 12 hrs. 28 min. 12 sec. Find the difference in longitude.

4. The difference in longitude between Cincinnati and Boston is 13° 26′. Find the difference in time.

5. New York is 74° and Cincinnati is 84° 30′ west longitude. Find the difference in time.

6. The difference in time between Canton and Cincinnati is 13 hrs. 10 min. 8 sec. Find the difference in longitude.

7. The difference in longitude between New York and Canton is 187° 3′. What is the difference in time?

8. Find the difference in time between Philadelphia, longitude 75° 10′ West, and Buenos Ayres, longitude 58° 22′ West.

9. The difference of time between St. Petersburgh and New Orleans is 8 hrs. 1 min. 16 sec. What is the difference in longitude?

10. Find the difference in time between the Cape of Good Hope, longitude 18° 28′ East, and Halifax, longitude 63° 36′ West.

196. Since the sun *appears* to move from east to west, sunrise will occur earlier at all points east, and later at all points west, of a given place. Hence, clock-time will be later in all places east, and earlier in all places west, of a given meridian.

Therefore, if the time of a place be given,

To find the time of a place **east, add** to the given time the difference of time between the two places.

To find the time of a place **west, subtract** from the given time the difference of time between the two places.

To Find the Difference in Clock-Time when the Difference in Longitude is known.

When it is noon at Boston (long. 71° 3′ 30″ West), what is the time at Paris (long. 2° 20′ 22″ East)?

$$\begin{array}{l} 71° \ 3' \ 30'' \text{ W.} \\ \underline{2° \ 20' \ 22'' \text{ E.}} \\ 73° \ 23' \ 52'' \ldots \text{ difference in longitude.} \end{array}$$

$$15\underline{)73° \ 23' \ 52''}$$
$$4 \text{ hrs. } 53 \text{ min. } 35.47 \text{ sec.}$$

6 min. 24½ sec. before 5 P.M. *Ans.*

Since Boston is west and Paris is east of the meridian of Greenwich, the difference between their longitudes is found by taking the sum of their longitudes.

Their difference in longitude, 73° 23′ 52″, is equivalent to 4 hrs. 53 min. 35.47 sec., and as Paris is *east* of Boston, the time at Paris is found by *adding* the 4 hrs. 53 min. 35.47 sec. to the time at Boston.

Ex. 127.

1. When it is noon at Chicago, what is the hour at New York, the difference in longitude being 13° 37′?
2. What is the time in London when it is half-past 3 in the afternoon at Constantinople, Constantinople being 29° east of London?
3. The longitude of New York is 74° West, that of Paris is 2° 20′ East. When it is 15 minutes past 10 A.M. in New York, what is the time in Paris?
4. The longitude of Boston is 71° 3′ and that of New York 74° West. What is the time in Boston when it is midnight in New York?

The difference in longitude between San Francisco and Chicago is 34° 49′. What time is it at San Francisco when it is 9 o'clock P.M. at Chicago?

6. Paris is 45° 10′ east of Rio Janeiro. What time is it at Rio Janeiro when it is 7 o'clock P.M. at Paris?

7. If the sun rises at half-past 4, when it is sunrise at Richmond, Va., what is the time at Rouen, France, the difference of longitude being 78° 46′?

8. The French residents in Calcutta wish to unite with the people of Paris in a celebration to occur at 3 o'clock P.M. Paris is 2° 20′ East, Calcutta, 88° 27′ East. At what hour must the festivities begin in Calcutta?

NOTE. **Standard Time** is the clock-time of some selected meridian. Eastern standard time is the clock-time of the meridian 75° west of Greenwich, and is five hours slower than Greenwich time. Central standard time is the clock-time of 90° west of Greenwich, and is just one hour slower than Eastern standard time. Mountain standard time is the clock-time of the meridian of 105°, and is one hour slower than that of 90°. Western standard time is the clock-time of the meridian of 120°, and is one hour slower than that of 105°. The railroads and many cities and towns of the United States have adopted standard time.

MISCELLANEOUS EXAMPLES.

Ex. 128.

1. Find the amount of the following bill:

To 5 yds. ribbon @ $62\frac{1}{2}$ cts. . .	$	
" 3 yds. satin @ $4.375 . . .		
" 4 papers pins @ $12\frac{1}{2}$ cts. . .		
" 1 sun-hat @ 75 cts. . . .		

How much change out of a 20-dollar bill should the *purchaser* receive?

Make out the bills for:

2.	27	yds. of flannel	at	80 cts.	a yard;
	32	yds. of calico	at	11 cts.	a yard;
	3½	doz. of stockings	at	$2	per dozen;
	6	pairs of gloves	at	84 cts.	a pair;
	4	collars	at	35 cts.	each.
3.	10	lbs. of sugar	@	10 cts.	a pound;
	6	lbs. of tea	@	88 cts.	a pound;
	8	lbs. of coffee	@	32 cts.	a pound;
	12	lbs. of currants	@	11½ cts.	a pound;
	10	lbs. of rice	@	9 cts.	a pound.
4.	18½	lbs. of beef	@	22 cts.	a pound;
	10½	lbs. of mutton	@	21 cts.	a pound;
	7½	lbs. of pork	@	17 cts.	a pound;
	16	lbs. of veal	@	16 cts.	a pound;
	14¾	lbs. of ham	@	20 cts.	a pound.
5.	5½	lbs. of soap	@	9½ cts.	a pound;
	3½	lbs. of candles	@	13 cts.	a pound;
	2	lbs. of butter	@	35 cts.	a pound;
	56	lbs. of rice	@	4½ cts.	a pound.
6.	7	doz. and 4 eggs	@	18 cts.	per dozen;
	19	lbs. of soap	@	11 cts.	per pound;
	18	lbs. of butter	@	28 cts.	per pound;
	13½	lbs. of cheese	@	15 cts.	per pound;
	½	lb. pepper	@	2½ cts.	per ounce.
7.	12½	t. of hay	@	$18	a ton;
	66	bu. of rye	@	$1.26	a bushel;
	102	bu. of barley	@	78 cts.	a bushel;
	5	bbls. of flour	@	$6.60	a barrel.

8. A man walks 1 mi. 47 rds. in 20 min. How many hours will it take him to walk 41 mi. 92 rds.?

9. Required the cubic feet of a box 6 ft. 6 in. long, 4 ft. 9 in. wide, and 3 ft. 3 in. deep.

10. What will be the weight of a wall of brick-work 10 ft. long, $1\frac{1}{2}$ ft. thick, $4\frac{1}{6}$ ft. high, if each cubic foot weighs 120 lbs.?

11. How many cubic yards of earth will be cut out of a drain 420 ft. long, 2 ft. wide, and 4 ft. deep?

12. What will be the expense of glazing a window of 16 squares, each $1\frac{1}{6}$ ft. long and $\frac{5}{6}$ ft. wide, at $1.08 per square foot?

13. What length must be cut off an inch-board 9 in. wide to obtain 4 ft. board measure?

14. How many boards each $11\frac{1}{2}$ ft. long, and 10 in. wide will be required for the flooring of a room 23 ft. long and 17 ft. 6 in. wide?

15. A farm of $22\frac{1}{2}$ acres is divided into house-lots measuring 75 yds. in length by 33 yds. in breadth. How many lots are there?

16. At 9 cts. per cubic foot, what will be the cost of a block of stone 9 ft. long, $5\frac{1}{2}$ ft. wide, and 4 ft. thick?

17. At 50 cts. an ounce, what is the value of a silver cup weighing 15 oz. 12 dwt. 12 grs.?

18. If the cost of making a barrel of flour into bread is $2.20, and flour is worth $9 a barrel, what should a baker receive for a loaf containing $1\frac{3}{4}$ lbs. of flour?

19. At $30 per M., what is the value of a stick of timber 24 ft. long, and 2 ft. square at the end?

20. A schoolroom is 44 ft. long, $28\frac{1}{2}$ ft. wide, and 13 ft. high. What will be the cost of painting the four walls and the ceiling, at the rate of 18 cents a square yard, making no allowance for doors and windows?

21. A druggist pays 50 cts. a pound avoirdupois for chloride of potash, and retails it in powders containing 1 ℈ 5 grs., at 5 cts. each. How much will he gain on $5\frac{1}{2}$ lbs.?

22. Find the entire surface of a block of marble 8 ft. long, 2 ft. wide, $1\frac{1}{2}$ ft. thick.

23. How many revolutions will be made by a wheel $3\frac{3}{4}$ yds. in circumference in passing over 198 mi.?

24. When an ounce of gold is worth $16.25, what must be paid for $\frac{1}{25}$ of a pound?

25. If candles $8\frac{1}{2}$ in. long are worth 9 cts. a half-dozen, and candles $10\frac{1}{4}$ in. long are worth 11 cts. a half-dozen, which is the better kind to buy?

26. How many silver spoons weighing 1 oz. 18 dwt. 12 grs. each can be made from 23 oz. 2 dwt. of silver?

27. An apprentice 14 yrs. 11 mos. 14 dys. old is to serve his employer until he is 21 yrs. of age. How long is he to stay with his employer?

28. What is the rate per hour of a horse that travels 18 mi. 1620 yds. in 3 hrs. 45 min.?

29. At 15 cts. a yard, what will be the cost of fencing a rectangular field 325 yds. long and 215 yds. wide?

30. What will be the width of carpeting, if 120 yds. are necessary to cover a floor 30 ft. long and $22\frac{1}{2}$ ft. wide?

31. When the mercury in the tube of a barometer is 30 in. high, the pressure of the atmosphere is about 15 lbs. for every square inch. What will be its pressure when the mercury stands 25 in. high?

32. A cistern containing 60 hhds. of water has two pipes open, by one of which 3 gals. of water per minute run in, and by the other 9 gals. run out. In how many hours will the cistern be emptied?

33. How many pounds of cement will be required to plaster an open cistern whose dimensions are $4\frac{1}{2}$ ft. long, $3\frac{1}{8}$ ft. wide, and $2\frac{3}{8}$ ft. deep, if the cement on a square foot weighs $6\frac{3}{4}$ lbs.?

34. How many tons of water will the cistern in example 33 hold, if a cubic foot of water weighs 1000 oz.?

35. What will be the cost of covering with paper $\frac{1}{2}$ yd. wide the four walls of a room 21 ft. long, 16 ft. wide, and 10 ft. high, if the cost of the paper is $12\frac{1}{2}$ cts. per yard, and no allowance is made for doors and windows?

36. What is the breadth of a rectangular field containing $7\frac{1}{2}$ A., if the length is 242 yds.?

37. A certain watch gains $3\frac{1}{2}$ sec. in 24 hrs., and another loses $2\frac{1}{2}$ sec. in the same time. If both be set right on Monday at noon, what will be the difference between them at 6 o'clock (true time) the next Saturday evening?

38. A milkman paid a farmer $3.20 for ten 2-gal. cans of milk. He lost 20 qts. At what price per quart must he retail the remainder to gain 8 cts. a gallon?

39. A man who had $\frac{3}{8}$ of a square mile of woodland sold $12\frac{1}{2}$ *sq. rds.* How much had he left?

40. **How many yards of Florence silk $\frac{5}{8}$ yd. wide will be required to line 19 yds. of camel's hair cloth $1\frac{1}{4}$ yds. wide?**

41. **How many days from Sept. 16, 1882, to Feb. 12, 1884?**

42. **A miller makes 154 bbls. of flour from $885\frac{1}{2}$ bu. of wheat. How many bushels on the average are required for a barrel of flour?**

43. **What must be the length of a walk $2\frac{1}{2}$ ft. wide to contain 38 sq. ft.?**

44. **A cistern 7 ft. long and 5 ft. wide contains 105 cu. ft. What is its depth? how many gallons of water will it hold?**

45. **What must be paid for a pile of wood 25 ft. long, 3 ft. high, and 4 ft. wide at $5.50 per cord?**

46. Each person on the average breathes 28 cu. ft. of air in an hour. How many hours will the air in a room 14 ft. long, 12 ft. wide, and 6 ft. high last 12 men?

47. Sound travels at the rate of 1130 feet a second. How long after the flash will the clap of thunder come when the cloud is 2 mi. 1000 yds. distant?

48. There are 9 oz. of iron in the blood of 1 man. How many men would furnish iron enough in their veins to make a ploughshare weighing $22\frac{1}{2}$ lbs.?

49. The fore wheel of a carriage is 4 ft. 7 in. in circumference, and the hind wheel 5 ft. 6 in. How many more times will the fore wheel turn than the hind wheel in going a distance of 1 mi.?

50. A boarding-house uses 3 pks. of potatoes daily. At $87\frac{1}{2}$ cts. per bushel, what will be the expense for potatoes during October, November, and December?

CHAPTER X.

PERCENTAGE.

197. **1.** A boy gave away 6 marbles out of every hundred he had. How many did he give away out of 400? of 600? of 1000?

2. A man had 300 sheep, and sold 10 out of every hundred. How many did he sell?

3. A man sold 7 tons of coal out of every hundred he had. How many did he sell out of 900 tons?

4. A man sells 11 lbs. of sugar out of every hundred he has. How many pounds will he sell out of 500? How many pounds will he have left?

198. In considering the increase or decrease of quantities, we usually employ the **number 100 as the representative of the quantity considered.**

199. Instead of using the phrases 6 *in every hundred*, 10 *in every hundred*, 7 *in every hundred*, 11 *in every hundred*, we say 6 *per cent*, 10 *per cent*, 7 *per cent*, 11 *per cent*. The words *per cent* therefore mean *hundredths.*

200. The symbol % is used for the words *per cent.*

How many hundredths of a number are:

2%?	$6\frac{1}{4}$%?	20%?	$12\frac{1}{2}$%?
5%?	7%?	25%?	$33\frac{1}{3}$%?

How many per cent of a number is:

0.20?	0.75?	$0.12\frac{1}{2}$?	1.40?
0.15?	$0.06\frac{1}{4}$?	0.50?	2.25?

What common fraction of a number (in its lowest terms) is:

10%?	25%?	$8\frac{1}{3}$%?	$12\frac{1}{2}$%?	125%?
20%?	50%?	$6\frac{1}{4}$%?	$66\frac{2}{3}$%?	160%?
$16\frac{2}{3}$%?	75%?	$33\frac{1}{3}$%?	100%?	175%?

Express as hundredths and as per cent:

$\frac{1}{2}$;	$\frac{2}{3}$;	$\frac{4}{5}$;	$\frac{3}{8}$;	$\frac{7}{12}$;	$\frac{5}{16}$;	$\frac{7}{40}$;
$\frac{1}{3}$;	$\frac{3}{4}$;	$\frac{5}{6}$;	$\frac{5}{8}$;	$\frac{1}{16}$;	$\frac{7}{20}$;	$\frac{7}{50}$;
$\frac{1}{4}$;	$\frac{3}{5}$;	$\frac{3}{7}$;	$\frac{9}{16}$;	$\frac{5}{16}$;	$\frac{7}{25}$;	$\frac{7}{100}$.

201. Express $\frac{1}{2}$% as hundredths and as a common fraction.

$$\tfrac{1}{2}\% = 0.00\tfrac{1}{2} = \tfrac{1}{200}.$$

In like manner express as hundredths and also as common fractions:

$\frac{1}{4}$%;	$\frac{3}{5}$%;	$\frac{1}{7}$%;	$\frac{3}{8}$%;	$\frac{1}{10}$%;
$\frac{1}{5}$%;	$\frac{4}{5}$%;	$\frac{3}{7}$%;	$\frac{5}{8}$%;	$\frac{3}{10}$%;
$\frac{3}{4}$%;	$\frac{1}{6}$%;	$\frac{5}{7}$%;	$\frac{1}{9}$%;	$\frac{7}{10}$%;
$\frac{2}{5}$%;	$\frac{5}{6}$%;	$\frac{1}{8}$%;	$\frac{4}{9}$%;	$\frac{9}{10}$%.

202. Find 8% of 250 bu. of corn.

8% of a number is $\frac{8}{100}$ of the number; and $\frac{8}{100}$ of 250 bu. = 20 bu.

20 bu. *Ans.*

Find 20% of 80 yds. of cloth.

$20\% = \frac{20}{100} = \frac{1}{5}$; and $\frac{1}{5}$ of 80 yds. is 16 yds.

16 yds. *Ans.*

Ex. 129. (*Oral.*)

Find:

1. 4% of 400 sheep.
2. 5% of 1000 bricks.
3. 8% of 200 ft. of boards.
4. 6% of 90 dys.
5. 10% of 150 cds. of wood.
6. 20% of 250 prs. of gloves.
7. 25% of 120 horses.
8. 80% of 400 A.
9. $6\frac{1}{4}\%$ of 320 rds.
10. $12\frac{1}{2}\%$ of 400 melons.
11. $66\frac{2}{3}\%$ of 300 oranges.
12. $8\frac{1}{3}\%$ of 1 doz. eggs.
13. 75% of 40 hens.
14. 60% of 20 girls.
15. $16\frac{2}{3}\%$ of 60 lbs. of butter.
16. $37\frac{1}{2}\%$ of 120 gals. of syrup.
17. $62\frac{1}{2}\%$ of 800 soldiers.
18. $\frac{4}{5}\%$ of 500 bu. of wheat.
19. $\frac{1}{2}\%$ of 4000 yds. of cloth.
20. $\frac{1}{5}\%$ of 100 dollars.

Ex. 130.

Find:

1. 9% of 1297.
2. $2\frac{1}{2}\%$ of 4300.
3. $\frac{1}{2}$ of 1% of 1346.
4. 12% of 6072.
5. $\frac{1}{2}\%$ of 150,975.
6. $1\frac{1}{10}\%$ of 1984.
7. 150% of 1050.
8. 100% of 7968.
9. A farmer having a flock of 1200 sheep lost 37% of them. What per cent of them, and how many sheep, had he left?
10. If copper ore yields 6% of pure metal, how many pounds of copper will be obtained from 1 t. of ore?

11. If a man buys 24 A. of land at $84 an acre, what must be the annual income that the investment may yield 10% ?

12. A grocer bought 40 cwt. of sugar for $240. 4% of it is wasted, and the remainder is retailed so that there is neither loss nor gain. What is the retail price per pound?

13. A stone-mason contracted to dig a cellar 45 ft. long, 36 ft. wide, and 6 ft. deep at 25 cts. a cubic yard. He lost 5% of his contract price. What was his loss?

14. A coal-dealer bought 25,784 t. of coal at $5 a ton. He sold 40% of it at $7, 20% at $8.50, and the remainder at $4.50. How much did he gain?

15. A gentleman owns 2 farms. The first contains 360 A., and the number of acres in the second is 150% of the number of acres in the first. Find the number in the second farm.

203. What per cent of 20 is 5?

$5 = \frac{5}{20}$ or $\frac{1}{4}$ of 20; and $\frac{1}{4} = 4)\underline{1.00}$

Therefore $5 = 25\%$ of 20. $0.25 = 25\%$.

25%. *Ans.*

Ex. 131. (*Oral.*)

What per cent of:

1. 16 is 8?
2. 20 is 5?
3. 25 is 15?
4. 48 is 8?
5. 100 is $12\frac{1}{2}$?
6. 2 is $\frac{1}{2}$?
7. 3 is $\frac{2}{3}$?
8. $\frac{4}{9}$ is $\frac{2}{9}$?
9. $72 are $18?
10. $52 are $39?
11. 50 qts. are 5 qts.?
12. 66 gals. are 6 gals.?
13. 480 dys. are 24 dys.?
14. 90 cds. are 9 cds.?
15. 80 men are 50 men?

Ex. 132.

1. From a school of 150 scholars, 50 are absent. What per cent of the whole is the number present?

2. In a school numbering 200 the daily average attendance is 160. What is the per cent of attendance? The number absent on the average is what per cent of the number present?

3. A person bought a house and lot for $6000, paying $5000 for the house. The value of the lot is what per cent of the value of the house?

4. From a peck of corn a crop of $48\frac{1}{4}$ bu. was raised. What per cent was the increase?

5. From $67\frac{1}{2}$ bu. of corn, 6 bu. 3 pks. are sold. What per cent of the whole is sold?

6. A house worth $8000 rents for $720 a year. What per cent of its value does it rent for?

7. From Delhi to Bombay the distance is 720 miles, and from Delhi to Madras 1080 miles. What per cent of the distance to Madras is the distance to Bombay?

8. Westminster Hall is 270 ft. long and 75 ft. broad. What per cent of the length is the breadth?

9. The Peak of Teneriffe is 12,232 ft. high. What per cent of a mile is its height?

10. The Danube is 1630 miles long, and the Missouri from its source to the Gulf of Mexico is 4000 miles long. What per cent of the length of the Missouri is the length of the Danube?

11. What per cent of 7 hrs. and 30 min. are $6\frac{3}{4}$ min.?

12. What per cent of 3 wks. and 4 dys. are 3 dys. and $10\frac{1}{2}$ hrs.?

[illegible]. What is the number of which 15 is 5%?

[illegible] 15 is 5%, [illegible] 15 is $\frac{1}{20}$ of the number, and if 15 is $\frac{1}{20}$ of [illegible] number, the number [illegible] will be 20 × 15 = 300.

300. *Ans.*

Ex. [illegible]. (Oral.)

What is the number of which

1. [illegible] 6. [illegible] 11. $\frac{7}{8}$ is 175%?
2. [illegible] 7. [illegible] 12. 17 is 34%?
3. [illegible] 8. [illegible] 13. 50 is $62\frac{1}{2}$%?
4. [illegible] 9. [illegible] 14. 300 is 0.3%?
5. [illegible] 10. [illegible] 15. 30 is $\frac{1}{2}$%?

Ex. [illegible].

1. [illegible] is [illegible]% of what number?
2. [illegible] is [illegible]% of what number?
3. [illegible] is [illegible]% of what number?
4. [illegible] is [illegible]% of what number?
5. [illegible] is [illegible]% of what number?
6. [illegible] is [illegible]% of what number?
7. [illegible] is [illegible]% of what number?
8. [illegible] is [illegible]% of what number?
9. [illegible] is [illegible]% of what number?

10. A city in 5 yrs. increased 12000 in population, a gain of 25%. What was the population at the beginning and end of the 5 yrs.?

11. A schoolboy in one week read 450 lines of Latin, which was 75% of the number in the book. How many lines had he still to read?

12. A boy sold chestnuts at 12½ cts. a quart, which was 200% of their cost. What did they cost a bushel?

13. A clerk spent 60% of his salary for board, 20% of it for clothes, 11% for books, and saved $117. What was his salary?

14. At Christmas a lady gave her daughter an atlas worth $27, and ⅔ of the cost of the atlas was 90% of the sum paid for an engraving. What was the sum paid for the engraving?

15. A sea-captain owning 60% of a vessel gave to his son 50% of his share, which was worth $6000. What was the value of the vessel?

16. A gentleman worth $50,000 gave 30% of his property to his son, and this gift was 80% of the property which the son already owned. Find the amount the son was worth after receiving his father's gift.

205. By selling a horse for $90, a man gains 20% of its cost. Find the cost.

He gets the cost (100%) and 20% of the cost, or 120% of the cost. The question therefore is, $90 is 120% of how many dollars?

A man sold a horse for $90, and lost 25% of the cost. What did the horse cost?

He got the cost (100%) minus 25% of the cost, or 75% of the cost. The question therefore is, $90 is 75% of how many dollars?

Ex. 135. (*Oral.*)

1. 36 is 12½% more than what number?
2. 65 is 6¼% less than what number?
3. 68 is 6¼% more than what number?
4. 75 is 12½% less than what number?
5. By selling a hat for $5.40, I sell it for 20% more than the cost. What was the cost?

6. A manufacturer sells mowing-machines at \$125 apiece, and gains 40%. What do they cost?

7. Sold a carriage for \$240, which was 20% more than the cost. What was the cost?

8. 64 is $33\frac{1}{3}$% more than what number?

9. What number diminished by 5% of itself equals 190?

10. What number diminished by 10% of itself equals 180?

Ex. 136.

1. 874 is $33\frac{1}{3}$% less than what number?

2. 1740 is 20% more than what number?

3. 40% of 4000 is 20% less than what number?

4. What number diminished by 15% of the number equals 5100?

5. What fraction increased by 25% of itself equals $\frac{15}{16}$?

6. 7500 is $33\frac{1}{3}$% less than what number?

7. A drover sold 250 sheep for \$1150, which was 15% more than they cost. Find the cost of the sheep per head.

8. At a forced sale, a bankrupt sold his house for \$8000, which was 20% less than its real value. If the house had been sold for \$12,000, what per cent above its real value would it have brought?

9. A flock of sheep has been increased by 250% of its number, and now numbers 1050. What is the original number?

10. If 20% be lost on a ton of rye-straw sold for \$19.20, what is the cost of the straw?

PROFIT AND LOSS.

206. The difference between the buying and selling prices of goods is called **profit** or **loss,** according as the selling-price is more or less than the buying-price.

Ex. 137.

1. A horse which cost $80 was sold for $60. Find the actual loss and the loss per cent.

   NOTE. Gain or loss is so much per cent on the **cost** of the goods.

2. Flour that cost $10 per barrel was sold for $12 per barrel. Find the gain per cent.

3. If milk is bought at 4 cts. a quart, and sold at 6 cts., what is the gain per cent?

4. Goods that cost $40 were sold at 20% below cost. What was the actual loss?

5. Velvet is sold for $3.75 per yard, at a gain of 25%. Find the cost of the velvet.

6. By selling cloth at $1.60 a yard, a merchant loses 20%. What is the cost?

7. Five cords of wood costing $20 were sold at $7 per cord. What was the gain per cent?

8. A carpenter paid $5000 for a house; spent in repairs a sum equal to 80% of the purchase-price; and then sold the house for $12,000. How much did he gain, and what per cent of the whole cost?

9. In selling 32 yds. of cloth, a merchant made $6.40, which was 16% of the cost. What did the cloth cost a yard?

10. Goods were sold for \$1615.12½, at a gain of 9½%. What did they cost?

11. If tea sold at 84 cts. a pound gives a profit of 20%, what would be the profit per cent if it were sold at 75 cts. a pound?

12. A trader's profits were \$1980 in the year 1880. This sum was 20% more than his profits in 1881. Find his profits in 1881.

13. A cord of wood costing \$4.50 sold for \$9. What was the gain per cent?

14. A house-lot was sold for \$1850, at an advance of 15% on its cost. What would have been the gain per cent if it had been sold for \$2210?

15. A manufacturer owning ⅘ of a factory sold 12½% of his share, at 10% above cost, for \$1100. What is the cost of the factory?

16. What per cent is made in buying coal by the long ton, at \$5 a ton, and selling it by the short ton, at the same price?

17. Corn cultivated at an expense of 28 cts. a bushel is sold at 1 ct. a pound. What is the gain per cent?

18. What per cent advantage is there in buying opium by the pound avoirdupois, and selling it by the pound apothecaries' weight?

19. A grocer lost 5% in selling a 50-lb. tub of butter for \$15.20. What did the butter cost per pound?

20. Ten cows were sold for \$690, at a gain of 15%. For how much per head, on the average, should they been sold to gain 20%?

21. For what price per dozen must gloves be bought in order that, by selling them at $1.75 per pair, there may be a gain of 25%?

22. A merchant lost 25% by selling flour at $6 per barrel. If he had sold it at $9 per barrel, what would have been the gain per cent?

23. A fruit-grower sent to New York 300 peck baskets of peaches, valued at 75 cts. each. Sixty baskets were spoiled on the journey. At what rate per basket must he sell the remainder to make 20% proft on the entire value of his fruit?

24. Sold goods at a loss of 20%, and actual loss of $57.50. What was the first cost?

25. Find the selling-price of goods by which there is a loss of 2% and an actual loss of $54.50.

26. How many pounds of cheese bought at 9 cts. a pound must be sold at 12 cts. a pound to gain $30?

27. Sold steel at $25.44 a ton with a profit of 6% and a total profit of $103.32. What quantity was sold?

Commission and Brokerage.

207. The **commission** paid to an agent for his services is generally reckoned at a rate per cent.

208. The sum left after the payment of the commission and other expenses is called **net proceeds.**

209. Commission paid to a broker is called **brokerage.**

210. In **selling**, the commission is reckoned on **the money received**; in **buying**, the commission is reckoned on **the money paid.**

(1) A real-estate agent sold a house for $7000. Find the amount of his commission, at $1\frac{1}{2}\%$.

$$\begin{array}{r} \$7000 \\ 0.01\frac{1}{2} \\ \hline 3500 \\ 7000 \\ \hline \$105.00 \end{array}$$

$105. *Ans.*

(2) A jockey receives $32 as his commission, at 4%, for purchasing a pair of horses. What did he pay for the horses?

Commission on $1 invested at 4% is $0.04. Therefore the sum invested to obtain a commission of $32 is

$$\$\frac{32}{0.04} = \$800.$$

$800. *Ans.*

(3) If a commission of $212.94 is paid for selling wool to the amount of $6552, what is the rate per cent allowed?

If the commission on $6552 is $212.94, the commission on $1 will be

$$\$\frac{212.94}{6552} = \$0.03\tfrac{1}{4}.$$

Therefore the commission is at the rate of $3\frac{1}{4}\%$. $3\frac{1}{4}\%$. *Ans.*

(4) A speculator in New York sent $18,360 to his agent in Chicago, with which to buy wheat. If the agent charges 2% for buying, how many bushels of wheat can he buy at 90 cents a bushel?

Commission on $1, at 2%, is $0.02. Hence out of every $1.02 sent, there is invested in wheat $1.

Hence, out of $18,360 sent there is invested in wheat

$$\$\frac{18360}{1.02} = \$18,000,$$

and the number of bushels of wheat bought is $\frac{18000}{0.90} = 20,000.$

20,000. *Ans.*

Ex. 138.

1. A commission-merchant sold 90 bbls. of flour at \$6 a barrel, and received 5% commission. What was his commission?

2. A commission of \$121.29 was charged for selling \$1866 worth of goods. What was the rate of commission?

3. A grain-dealer charged 7% commission for selling a quantity of wheat, and received for his commission \$109.20. What was the total amount received for the wheat?

4. A real-estate broker sold a house on $6\frac{1}{4}$% commission, and sent to the owner as net proceeds \$3060. What was the broker's commission, and what sum was received for the house?

   HINT. The broker received $6\frac{1}{4}$%, and the owner $93\frac{3}{4}$%, of the sum the house sold for. Hence the question is, \$3060 is $93\frac{3}{4}$% of what sum?

5. A New York merchant sent \$1295.32 to New Orleans to be expended in cotton. The broker in New Orleans charged 6% commission. What sum was paid for cotton?

   HINT. The broker received 6% commission on the money invested in cotton. Therefore, the question is: \$1295.32 is 106% of what sum?

6. If \$5125 include the amount expended for wool and $2\frac{1}{2}$% commission to the purchasing agent, how much money does the agent lay out in wool?

7. A lawyer collected 75% of a debt of \$1260, and charged 5% commission on the sum collected. What did the creditor receive?

8. An agent sold 420 bu. of corn at 60 cts. a bushel, and the commission was $7.56. What rate of commission was charged for selling?

9. A land agent charged 4% for selling 750 A. of land at $20 an acre. What was his commission?

10. How many yards of cloth, at 45 cts. a yard, can an agent buy with the commission received from the sale of 180 bu. of potatoes at 50 cts. a bushel, his rate of commission for selling the potatoes being $1\frac{1}{2}$%?

11. A man bought a horse for $225, which sum was half of his commission, at $2\frac{1}{2}$%, on the sale of a farm. What did the farm bring?

12. A young man selling tea on $2\frac{3}{4}$% commission sent to his employer $875.25 as the net proceeds of one week's sales. What did the average daily sales amount to?

13. A St. Louis merchant received $150 as his commission, at $2\frac{1}{2}$% for purchasing 1200 bbls. of flour. What was the price paid per barrel?

14. A broker sold for a farmer 12,000 lbs. of pork, at $8\frac{1}{8}$ cts. per pound. He charged 3% commission for selling, and paid $37.60 for freight. How many feet of pine boards, at $25 per M., can the broker buy with the net proceeds, if he charges 1% commission for buying?

15. A broker is offered a commission of $5\frac{1}{2}$% for selling wool and guaranteeing payment, or a commission of $3\frac{3}{4}$% without guaranteeing payment. He accepts the $5\frac{1}{2}$% commission, and guarantees the payment. The sales amount to $8500, and the bad debts to $147.75. How much did he gain by his choice?

INSURANCE.

211. In insurance a payment called a **premium of insurance** is made for a guaranty of a specified sum of money in the event of loss from fire or accident, and is reckoned at a rate per cent on the amount insured.

212. In life-insurance an annual payment is made in order to secure a specified sum of money in the event of death, or at the end of a fixed period of time.

213. The written contract is called the **policy of insurance.**

(1) A house worth \$8000 is insured for three years for $\frac{3}{4}$ of its value, at 1%. Find the premium.

$\frac{3}{4}$ of \$8000 = \$6000, and 1% of \$6000 = \$60. \$60. *Ans.*

(2) The premium for insurance on a store, at $1\frac{1}{2}\%$, is \$150. Find the amount of the insurance.

The premium on \$1 insurance, at $1\frac{1}{2}\%$, is \$0.015.

Hence the amount of insurance is $\$\frac{150}{0.015} = \$10,000$.

\$10,000. *Ans.*

(3) A man pays \$27.50 premium for having his house insured for five years, at $1\frac{1}{4}\%$ on $\frac{2}{3}$ of its value. What is the value of the house?

The premium on \$1 insured at $1\frac{1}{4}\%$ is \$0.0125.

Hence the amount of the insurance is $\$\frac{27.50}{0.0125} = \$2200$, and the value of the house is $\$2200 \div \frac{2}{3} = \$3300$. \$3300. *Ans.*

(4) For what sum must a cargo worth $24,500 be insured, at 2%, so that, in case of loss, the owner may recover both the value of the cargo and the premium paid?

Premium on $1 at 2% is $0.02.
Insurance on $0.98 worth of cargo = $1.
Hence insurance on $24,500 worth of cargo $= \$\frac{24500}{0.98} = \$25,000.$

$25,000. *Ans.*

Ex. 139.

1. Find the cost of insuring property worth $15,000, if $\frac{2}{3}$ of the value is insured at $\frac{4}{5}$%.
2. Find the cost of insuring $\frac{5}{6}$ of the value of 6000 bbls. of flour worth $9.60 a barrel, the insurance being reckoned at $\frac{1}{2}$%.
3. A stock of goods worth $12,000 was insured for $\frac{7}{8}$ of its value at $\frac{3}{4}$%. If the whole stock were burned, what would be the loss to the owner, including the premium paid for insurance?
4. After three annual payments of $337.50, premium at $1\frac{1}{2}$% on $\frac{3}{4}$ of the value of a mill, it was burned. Find the loss to the insurance company.
5. At $\frac{4}{5}$%, how much insurance can be effected upon a store for $108?
6. What annual premium at $1\frac{1}{3}$% must be paid on a life-insurance of $6000?
7. At the rate of $17 upon $1000, what annual premium will be paid on a life-insurance of $6700?
8. The annual premium paid for life-insurance at $1\frac{3}{4}$% is $70. What is the sum insured?
9. For what sum should a cargo worth $74,496 be insured, at 3%, so that, in case of loss, the owner may recover both the value of the cargo and the premium paid?

Taxes and Duties.

214. *Taxes* on property are reckoned at a rate per cent on the assessed value of the property; and *duties* on imported goods are sometimes reckoned at a rate per cent on the cost in the country from which they are imported.

Ex. A tax of \$18,000 is levied upon a town which contains 800 polls, assessed at \$1.50 each, and which has taxable property valued at \$1,100,000. It is estimated that the town will receive from the state \$3600 as its share of the railroad tax, etc. Find the rate of taxation and the tax paid by Brown, whose property is assessed at \$5960, and who pays for 1 poll.

The amount of poll-taxes = 800 × \$1.50 = \$1200
The amount from the state = \$3600
The sum from state and polls = \$4800
Sum levied on property = \$18,000 − \$4800 = \$13,200.
The rate = \$13,200 ÷ \$1,100,000 = \$0.012.
That is, the tax is 12 mills on a dollar, or \$12 on \$1000.
Therefore Brown's property-tax is 0.012 of \$5960 = \$71.52.
Total tax = \$71.52 + \$1.50 = \$73.02.

Ex. 140.

1. If the assessed valuation of a town is \$784,750, and the town has 260 polls, paying \$1.25 each, what is the rate when the tax levy is \$16,020 besides the estimated amount to be received from the state?

2. A district schoolhouse is to cost \$3500, and the property of the district is assessed at \$210,000. What is the rate, and what tax must be paid on property assessed at \$3798.60?

3. In a city of 2000 polls, each paying $1.50, the sum of $111,000 is to be raised by taxation on property assessed at $9,000,000. What is the tax of a man who pays for 4 polls, and tax on property assessed at $25,670?

4. What is the rate of taxation when $710.92 is the tax upon $50,780?

5. If a tax of $12,350 is to be raised, and the collector receives 5% for collecting the taxes, what sum must be levied?

6. A town-hall is to be built at a cost of $11,400. What sum must be assessed if the collector receives 5% for collecting the taxes, and what will be the rate if the assessed valuation of the town is $800,000?

7. Find the duty, at 15%, on 95 cases of indigo, each weighing 190 lbs., and invoiced at 75 cts. per pound.

8. After deducting 20% for leakage, what will be the duty on 40 hhds. of molasses, of 84 gals. each, if the molasses is invoiced at 90 cts. a gallon, and the duty is 30%?

9. On 15 doz. bottles of sherry wine there is paid $1.25 per dozen for transportation, and $1.50 per dozen for duty. What is the whole cost of importation?

10. A Boston merchant received from Paris:

325 yds. of silk	@	$2.25 a yard;
296 yds. of lace	@	1.50 a yard;
480 yds. of ribbon	@	0.50 a yard;
45 doz. gloves	@	15.00 a doz.

If the duty on silk, ribbon, and lace is 35%, and on gloves 25%, what is the whole amount of the duties?

11. If the duty is \$2.50 a gallon on cologne-water, what must be paid on 75 doz. pint bottles, if there is an allowance of 5% for breakage?

12. What is the invoice cost of goods upon which \$625 duty is paid, if the duty is reckoned at 25%?

13. What will be paid by a grocer importing 120 chests of tea, containing 79 lbs. each, invoiced at 75 cts. per pound, if the duty is 15%?

Ex. 141.

Miscellaneous Examples.

1. Of what number is 450 nine per cent?
2. What is the excess of 5% of 1500 over $\frac{1}{2}$% of 7000?
3. What per cent of 9000 is 45?
4. Five hundred and sixty is 12% more than what number?
5. Seven hundred and fifty-two is 6% less than what number?
6. There is a difference of 893 between a certain number and 6% of the number. Find the number.
7. What per cent of 25 lbs. are 3 lbs. 4 oz.?
8. The difference between 50$\frac{1}{2}$% and 75$\frac{1}{4}$% of a number is 99. Find the number. Let the example be proved.
9. A merchant sold cloth at \$4.20 per yard, and gained 20%. If it had been sold at \$3.60, what actual gain, and what gain per cent, would have been made?
10. By how much does $\frac{6}{7}$% exceed $\frac{4}{5}$%?

11. At an average price of 55 cts. per bushel, and a charge of $2\frac{1}{2}\%$ commission, how many bushels of grain can be bought for \$4510?

HINT. First find the cost of 1 bu., including commission.

12. A landau was sold for \$488, at a gain of 22%. Required the cost.

13. A milkman's gallon measure was too small by $\frac{1}{2}$ gi. What was the rate per cent of fraudulent gain?

14. A merchant paid \$112.50 for 75 yds. of silk, of which 15 yds. were worthless. At what price per yard must the remainder be sold to gain 20% on the purchase-price of the whole?

15. For selling goods, an agent received \$106.83 commission, $2\frac{1}{4}\%$ for selling, $2\frac{3}{4}\%$ for guaranteeing payment. What sum was received for the goods?

16. A dealer bought 70 bags of wool at \$32 a bag; 10% of it proved unsalable. For what price per bag must he sell the rest to realize 15% on his purchase?

17. A lady paid for investing money $\$9.37\frac{1}{2}$ brokerage, rate $\frac{1}{8}\%$. Required the amount invested.

18. From a stack of hay, 7 t. 11 cwt. were sold, which was $75\frac{1}{2}\%$ of the whole. What did the stack contain before the sale?

19. A carriage worth \$250 was bought for \$50 less, and sold for \$25 more, than its value. What was the rate of gain on the price paid?

20. A man left 30% of his estate to his wife, 50% of the remainder to his son, 75% of the residue to his daughter, and the balance, \$546, to a family servant. Required the value of the estate.

21. What per cent of $\frac{9}{11}$ is $\frac{3}{22}$? of $\frac{7}{13}$ is $\frac{14}{39}$?

22. A man sold 36 horses for \$200 each: on half of them he gained 20%, and on half he lost 10%. What was his gain per cent on the whole sale?

23. A gentleman sent to a broker \$1281.25 to be invested in land at \$62.50 an acre. A commission of $2\frac{1}{2}$% being charged for buying, how many acres were bought?

24. The dimensions 10, 8, and 6, of a rectangular bin being increased 10%, what will be the rate per cent of increase in capacity?

25. One-half of a stock of goods valued at \$612.60 was sold for $\frac{7}{8}$ of the value of the whole stock. What was the gain per cent?

26. A roll of 140 yds. of carpet was sold for \$72, at a loss of 10%. What should it have brought per yard to insure a gain of 15%?

27. A railroad company with \$9,000,000 capital declares a dividend of \$360,000. What sum will be received on 120 shares of \$100 each?

28. Ten per cent of a roll of carpet having been sold to one man, 10% of the remainder to another, 30.375 yds. are left. How many yards were there at first?

29. At an annual premium of \$405, rate $1\frac{1}{2}$%, $\frac{3}{4}$ of the value of a mill is insured. What is the entire value of the mill?

30. A broker buying cotton at $\frac{3}{4}$% commission retained \$75 for his commission, and paid \$25 for storage. What sum was sent by his employers to cover the whole expense of investment?

31. What sum must be insured upon a library to cover its entire value, $18,000, and the premium at $1\frac{4}{5}\%$?

HINT. If 100 be taken to represent the sum to be insured, then $1\frac{4}{5}$ will represent the premium; and $100 - 1\frac{4}{5}$, that is, $98\frac{1}{5}$, will represent the value of the library. Hence the sum to be insured will be $\$18,000 \div 0.98\frac{1}{5} = \$18,329.94$.

32. A merchant placed 80% of his year's profits in a bank; having drawn out 20% of this deposit, $2880 remained. What were his profits for the year?

33. Required the tax-rate, in a city appropriating for public expenses $147,000, to be assessed on property worth $35,000,000.

34. A lady bought a house for $7965, which rented for $841.85. The taxes were $50; repairs, $75. What rate per cent did the investment yield?

35. A premium of $960 was paid for full insurance on a ship and cargo, at $1\frac{1}{2}\%$. The cost of the cargo was 60% of the cost of the ship. What was the value of each?

36. Find the entire cost of 4000 bbls. of flour purchased by an agent, at $7 a barrel, who charged 3% commission, and paid $315 for freight.

37. How many barrels of flour can be bought for $5924.38 by an agent who pays $7 a barrel for the flour, charges 3% commission, and pays $315 for the freight?

38. The insurance on $\frac{2}{3}$ the value of a hotel and furniture cost $300. The rate being 75 cts. on $100, what was the value of the property?

39. What is the duty, at $25\frac{3}{4}$ cts. per gallon, on 48 bbls. of turpentine, 31 gals. making a barrel, and 5% being allowed for leakage?

CHAPTER XI.

INTEREST AND DISCOUNT.

215. **Interest** is the payment made for the use of money.

The interest to be paid for the use of a given sum of money differs from the payments considered in the last chapter, in that it depends upon the *time* for which the sum is loaned as well as on the *rate per cent* charged.

216. The sum loaned is called the **principal**. The principal and interest together is called the **amount**.

SIMPLE INTEREST.

217. If 100 be taken as the representative of the principal, the rate will represent *the interest for one year;* the product of *the rate by the number of years* will represent *the whole interest.*

Thus, if the time be 4 yrs., and the rate per cent 5, the interest will be represented by 20, and the amount by 120.

Find the interest on \$512 for 2 yrs. 4 mos., at 6%.

$$\begin{array}{rl}
\$512 & \\
0.06 & \\
\hline
\$30.72 & = \text{interest for 1 yr.} \\
2\frac{1}{3} & = \text{2 yrs. 4 mos.} \\
\hline
1024 & \\
6144 & \\
\hline
\$71.68 &
\end{array}$$

\$71.68. *Ans.*

218. In most business transactions the time for which interest is required is 1, 2, 3, or 4 months (30 dys. being

reckoned 1 mo.), and the rate of interest is 6%, that is, $\frac{1}{2}$% a month.

Hence the interest at 6% on a given sum for 2 mos. (or 60 dys.) is found by moving the decimal-point two places to the left; for 1 mo., 3 mos., 4 mos., by moving the decimal-point two places to the left, and multiplying by $\frac{1}{2}$, $1\frac{1}{2}$, and 2 respectively.

Thus, the interest on \$2500 for 2 mos. is \$25.00; for 1 mo., \$12.50; for 3 mos., \$37.50; for 4 mos., \$50.

Find the interest on \$1120 for 3 yrs. 2 mos. 18 dys., at 6%.

The interest at 6% for 1 yr. = 0.06 of the principal.
The interest for 1 mo. is $\frac{1}{12}$ of 0.06 = 0.005 of the principal.
The interest for 1 dy. is $\frac{1}{30}$ of 0.005 = $\frac{1}{6}$ of 0.001 of the principal.
Hence the interest for

3 yrs.	= 3 × 0.06	= 0.18
2 mos.	= 2 × 0.005	= 0.01
18 dys.	= 18 × $\frac{1}{6}$ of 0.001	= 0.003
3 yrs. 2 mos. 18 dys.		= 0.193 of the principal.

And 0.193 of \$1120 = \$216.16.

\$216.16. *Ans.*

219. The **six per cent method** may be employed for any rate per cent by first finding the interest at 6%, and then taking such a part of the interest as the given rate is of six per cent.

Thus, the interest at $4\frac{1}{2}$% = $\frac{4\frac{1}{2}}{6}$ = $\frac{3}{4}$ of the interest at 6%. In this case, we should diminish the interest at 6% by $\frac{1}{4}$ of itself. The interest at 8% is $\frac{8}{6}$ = $\frac{4}{3}$ of the interest at 6%. In this case, we increase the interest at 6% by $\frac{1}{3}$ of itself.

220. To compute interest for days at 6%, we move the decimal-point in the principal *three places to the left*, **and** multiply by one-sixth of the number of days.

Find the interest for \$8080 for 93 dys., at 6%.

$$\begin{array}{r} \$8.08 \\ 15\frac{1}{2} \\ \hline 404 \\ 4040 \\ 808 \\ \hline \$125.24 \end{array}$$

By moving the decimal-point three places to the left, we have \$8.08; and $\frac{1}{6}$ of 93 dys. = $15\frac{1}{2}$. Therefore, multiplying \$8.08 by $15\frac{1}{2}$, we obtain the required interest.

\$125.24. *Ans.*

221. For any other rate, find the interest at 6%, and then take such a part of the interest as the given rate is of 6%.

For years and months, call a year 360 days and a month 30 days.

Ex. 142.

Find the interest of:

1. \$51.25 for 30 dys., at 6%.
2. \$2581 for 60 dys., at 6%.
3. \$1261 for 90 dys., at 6%.
4. \$1250.60 for 4 mos., at 6%.
5. \$3020 for 3 mos., at 6%.
6. \$2300 for 3 mos., at 6%.
7. \$275 for 2 mos., at 6%.
8. \$5000 for 1 mo., at 6%.
9. \$1361 for 2 yrs., at 5%.
10. \$675.90 for 5 yrs., at $3\frac{1}{2}$%.
11. \$775.83 for 3 yrs. 9 mos., at $2\frac{1}{2}$%.
12. \$533.33$\frac{1}{3}$ for 10 mos., at $4\frac{1}{2}$%.
13. \$250.60 for 3 yrs. 6 mos., at $4\frac{1}{4}$%.
14. \$575.87$\frac{1}{2}$ for 1 yr. 10 mos. 15 dys., at 5%.
15. \$760 for 2 yrs. 11 mos. 27 dys., at $4\frac{1}{2}$%.

16. $725.40 for 5 mos. 27 dys., at $5\frac{1}{4}\%$.

17. $547.60 from Feb. 20 to Dec. 5, at $3\frac{1}{4}\%$.

18. $1750 from May 5, 1884, to June 21, 1885, at $5\frac{1}{2}\%$.

19. $1517 from Jan. 5 to July 1, at $4\frac{1}{2}\%$.

20. $476.50 from July 5, 1884, to Feb. 9, 1885, at 4%.

21. $319.20 from April 7 to Aug. 31, at $3\frac{1}{4}\%$.

22. $6460 from June 15, 1883, to May 7, 1885, at $4\frac{1}{4}\%$.

23. $150 from Aug. 5, 1883, to March 17, 1885, at 7%.

24. $527.20 from Jan. 1 to Nov. 20, at $4\frac{1}{2}\%$.

25. $1250 from Nov. 15, 1884, to March 1, 1885, at 5%.

26. $624.36 from March 5 to Dec. 20, at $7\frac{3}{10}\%$.

Find the amount of:

27. $1100 for 3 yrs. 4 mos., at 5%.

28. $1290.50 for 60 dys., at 6%.

29. $1275 for 3 yrs. 2 mos. 15 dys., at 8%.

30. $250.80 for 10 mos. 10 dys., at $3\frac{1}{2}\%$.

31. $377.65 for 1 yr. 3 mos., at 5%.

32. $7234.25 for 22 yrs. 2 mos. 20 dys., at $4\frac{1}{2}\%$.

33. $6130 from May 6 to Oct. 24, at $3\frac{3}{4}\%$.

34. $258.85 from March 6 to June 24, at 5%.

35. $25.62 for 33 dys., at 6%.

36. $85.85 for 1 yr. 7 mos. 21 dys., at 6%.

37. $600 for 93 dys., at 4%.

38. $350 from Sept. 21, 1884, to March 5, 1885, at 4%.

39. $1226 from Oct. 4, 1884, to May 6, 1885, at 5%.

40. $342.42 from Feb. 5, 1884, to March 15, 1885, at 7%.

41. $360.50 from Aug. 1, 1884, to March 3, 1885, at $6\frac{1}{2}$%.

42. $504.25 from Jan. 8 to March 10, at $6\frac{1}{4}$%.

43. $1240 from Mar. 3 to Aug. 28, at 7%.

NOTE. In business, a year is reckoned at 360 days in computing interest. But national governments take the exact number of days from the date of the note or bond to and including the day on which the interest is due, and reckon for the interest such a part of a year's interest as this number of days is of 365 days.

222. It is often required to find the rate, time, or principal, when two of these and the interest (or amount) are given.

223. When the principal, interest (or amount), and time are given, to find the rate per cent.

At what rate per cent will $320 produce $48 in 3 yrs.?

Interest on $320 for 3 yrs. is $48.
Interest on $320 for 1 yr. is $\frac{1}{3}$ of $48.
Interest on $1 for 1 yr. is $\frac{1}{320}$ of $\frac{1}{3}$ of $48 = $0.05.
But $0.05 = 5% of $1. 5%. *Ans.*

At what rate per cent will $8000 amount to $9277.78 in 2 yrs. 6 mos. 20 dys.?

Interest is $9277.78 − $8000 = $1277.78.
Time is 2 yrs. 6 mos. 20 dys. = $2\frac{5}{9}$ yrs.
Interest on $8000 for $2\frac{5}{9}$ yrs. = $1277.78.

Interest on $8000 for 1 yr. $= \frac{\$1277.78}{2\frac{5}{9}}$.

Interest on $1 for 1 yr. $= \frac{\$1277.78}{2\frac{5}{9} \times 8000} = \$0.06\frac{1}{4}$.

But $0.06¼ = 6¼% of $1. $6\frac{1}{4}$%. *Ans.*

Ex. 143.

Find the rate per cent:

1. When the interest on $500 for 1 yr. 6 mos. is $67.50.
2. When the interest on $250 for 2 yrs. is $52.50.
3. When $500 amount to $754 in 9 yrs.
4. When the interest on $725 for 12 yrs. is $141.37½.
5. When $880 amount to $899.25 for 7 mos.
6. When the interest on $424 for 2 yrs. 6 mos. is $26.50.
7. When the interest on $255.50 from April 1 to June 20 is $2.80.
8. When $175 amount to $203.35 for 3 yrs. 7 mos. 6 dys.
9. When a sum of money is doubled in 16 yrs.
10. When an investment for 6 yrs. produces a sum equal to ⅜ of the capital.

224. When the principal, interest (or amount), and rate per cent are given, to find the time.

In what time will the interest on $793.87½ be $11.96½, at 5½%?

Interest on $793.875 at 5½% for 1 yr. = $43.663.

Therefore the number of years will be $\frac{11.965}{43.663} = 0.274$.

And 0.274 yr. = 3 mos. 9 dys.

3 mos. 9 dys. *Ans.*

Ex. 144.

Find the time in which:

1. The interest on $225 will be $36, at 4%.
2. $440 will amount to $505.45, at 4¼%.

3. $250 will double itself, at $2\frac{1}{2}\%$.
4. $225 will amount to $256.50, at $3\frac{1}{2}\%$.
5. $50 will amount to $85, at 6%.
6. The interest on $4260 will be $873.30, at 6%.
7. $1005.34 will amount to $1156.14, at $4\frac{1}{2}\%$.
8. $1587.75 will amount to $1611.68, at $5\frac{1}{2}\%$.
9. A sum of money will double itself, at 6%.
10. $1000 will amount to $1125, at 4%.

225. When the interest, time, and rate are given, to find the principal.

What principal will in 8 yrs. 6 mos. produce $100 interest, at 5%?

8 yrs. 6 mos. = 8.5 yrs.

Interest for 1 yr. $= \frac{\$100}{8.5} = \$11.765.$

Interest on $1 for 1 yr. at 5% = 0.05 of $1.

Hence principal required $= \frac{\$11.765}{0.05} = \$235.30.$

$235.30. *Ans.*

Ex. 145.

Find the principal that will:

1. Produce $180 interest in 3 yrs., at 4%.
2. Produce $189 interest in 3 yrs., at $6\frac{1}{4}\%$.
3. Produce $3493.20 interest in 3 yrs. 5 mos., at 6%.
4. Produce $10.70 interest in 5 mos., at 4%.
5. Produce $75.40 interest in 3 yrs. 4 mos., at $4\frac{1}{2}\%$.
6. Produce $75.05 interest in 3 mos. 2 dys., at $4\frac{1}{4}\%$.

7. Produce $1746.60 interest in 3 yrs. 5 mos., at 6%.

8. Produce $64.46 interest in 6 yrs., at 4½%.

9. Produce $80.62½ interest in 3 yrs. 9 mos., at 4%.

10. Produce $669.64 interest in 2 yrs. 7 mos. 24 dys., at 6%.

226. When the amount, time, and rate are given, to find the principal.

Find the principal that will amount to $748.12½ in 3 yrs. 6 mos., at 4%.

3 yrs. 6 mos. = 3½ yrs.
Let the principal be represented by 100.
The interest will be represented by 3½ × 4 = 14.
The amount will be represented by 100 + 14 = 114.
Hence the principal = $\frac{100}{114}$ of $748.125 = $656.25.

$656.25. *Ans.*

Ex. 146.

Find the principal that will amount:

1. To $1680 in 3 yrs., at 4%.
2. To $962 in 4½ yrs., at 4½%.
3. To $725.47 in 2 yrs. 3 mos., at 3½%.
4. To $3215.83 in 4 yrs. 6 mos., at 3%.
5. To $595.20 in 8 mos., at 6%.
6. To $1275.75 in 1 yr. 1 mo., at 5%.
7. To $2053.32 in 3 yrs. 5 mos., at 6%.
8. To $131.88 in 2 yrs. 11 mos. 15 dys., at 6%.
9. To $37.02 in 2 yrs. 3 mos. 18 dys., at 5%.
10. To $2359.38 in 2 yrs. 7 mos. 24 dys., at 4½%.

BANK DISCOUNT.

227. When the holder of a promissory note sells the note to a bank, or other purchaser, the sum paid by the bank is called the **proceeds** or **avails** of the note, and the difference between the sum named in the note and the proceeds is called the **discount**.

228. Discount is reckoned at so much per cent, and the per cent is called the **rate of discount**.

229. Questions in bank discount are calculated like questions in simple interest; *discount* being used for *interest*, and *rate of discount* for *rate of interest*.

NOTE. The sum named in the note should be written in words, and is called the *face* of the note. The person signing the note is called the *maker*; a person who writes his name on the back of the note is called an *indorser*, and is responsible for the payment of the note, unless he writes above his signature the words "without recourse."

A note should contain the words "*value received*." A note, to be negotiable, must be made payable to the *bearer*, or *to the order* of some person who must indorse the note.

When a note bears interest, the discount is computed on the *amount* of the note.

A note is *nominally* due at the expiration of the time named in the note, but it does not *mature*, that is, become *legally* due, until three days after this time. These three days are called **days of grace**. And the discount is computed on the time between the day the note is discounted and the day of its maturity.

When the time is expressed in *days*, the day of maturity is found by counting forward from the date of the note the *number of days* named in the note, and the three days of grace. When the time is in *months*, the day of maturity is found by counting the *number of calendar months*, and the odd days plus the three days of grace. When a note falls due on Sunday, or a legal holiday, it is payable on the previous business day.

A **protest** is a notice in writing by a notary public to the indorsers that a note has not been paid. If a note be not protested on the *last day of grace* the indorsers are released from their obligation.

230. Find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds* of the following notes:

$610.25 BOSTON, June 12, 1885.

Sixty days after date I promise to pay to Edwin Ginn six hundred ten and $\frac{25}{100}$ dollars, for value received.

Discounted at 6%, July 1. SAMUEL HALE.

Counting 60 dys. from June 12, we have 18 in June, 31 in July, and 11 in August. Therefore the note becomes due Aug. $^{11}/_{14}$ (11 denotes the day it is nominally due, and 14 the day it is legally due).

The time to run is 44 dys., 30 dys. in July and 14 in August.

The discount is the interest on $610.25 for 44 dys., at 6%. Therefore (§ 220) the discount is $7\frac{1}{3} \times \$0.61025 = \$4.48$.

The proceeds = $610.25 − $4.48 = $605.77.

Due Aug. 14; discount, $4.48; proceeds, $605.77. *Ans.*

$1050. CHICAGO, Feb. 13, 1885.

Six months from date we jointly and severally promise to pay to the order of George Hall ten hundred and fifty dollars, for value received, with interest at six per cent.

Discounted at 8%, May 13. JAMES BLAKE.
HENRY SHAW.

Interest on note for 6 mos. 3 dys. = $32.03.

Amount of note when due is $1050 + $32.03 = $1082.03.

Day of maturity, Aug. $^{13}/_{16}$. Time to run, 3 mos. 3 dys.

Discount on $1082.03, at 8%, for 3 mos. 3 dys. = $22.36.

Proceeds, $1082.03 − $22.36 = $1059.67.

Due Aug. 16; discount, $22.36; proceeds, $1059.67. *Ans.*

NOTE. In Delaware, Maryland, Missouri, Pennsylvania, and the District of Columbia, banks reckon the *day of discount* in the time for which they charge interest; that is, they charge interest on a 30-day, 60-day, or 90-day note for 34, 64, 94 days, respectively.

In New York days of grace are abolished by statute.

Ex. 147.

Find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds*, on the following notes:

1. $2250. CONCORD, N.H., Jan. 1, 1885.

Four months from date I promise to pay to the order of George Marston twenty-two hundred and fifty dollars, for value received.

Discounted at 7%, Jan. 12. SIMON STEVENS.

2. $432.55. DES MOINES, Jan. 3, 1885.

Sixty days from date I promise to pay James Wilson, or order, four hundred thirty-two and $\frac{55}{100}$ dollars, value received.

Discounted at $6\frac{1}{2}$%, Jan. 6. JOHN ALLEN.

3. $670.35. ST. LOUIS, Jan. 6, 1885.

Ninety days from date I promise to pay to the order of Peter Holmes six hundred seventy and $\frac{35}{100}$ dollars, value received.

Discounted at 7%, Jan. 26. ROBERT DAY.

4. $1304.90. CINCINNATI, Jan. 25, 1885.

Five months after date I promise to pay to the order of John Shannon thirteen hundred four and $\frac{90}{100}$ dollars, for value received, with interest at six per cent.

Discounted at $4\frac{1}{2}$%, March 15. CHARLES HILLMAN.

5. $2260. BALTIMORE, MD., June 19, 1885.

Sixty days from date I promise to pay to the order of John Morrison twenty-two hundred and sixty dollars, value received.

Discounted at $5\frac{1}{2}$%, July 16. FRANK HOWE.

[illegible] July 28, 1885.

[illegible] the order of [illegible] dollars, value [illegible]

[illegible] RICHARD SMITH.

[illegible] Oct. 4, 1884.

[illegible] John Proctor, [illegible] with interest [illegible]

[illegible] JAMES WHITBRIDGE.

[illegible] Feb. 19, 1885.

[illegible] to the order of [illegible] twelve and $\frac{60}{100}$ dol- [illegible]

[illegible] PETER BURKE.

[illegible] Sept. 2, 1885.

[illegible] to the order of [illegible] and $\frac{4}{100}$ dollars, [illegible]

[illegible] PAUL WEST.

[illegible] July 1, 1885.

[illegible] the order of [illegible] hundred and forty dollars, [illegible]

[illegible] JOHN CLEMENT.

[illegible] Denver [illegible] Oct. 14, 1885.

[illegible] pay to the order of [illegible] thousand eighty-nine and $\frac{25}{100}$ dollars, value received.

Discounted at [illegible] Dec. 1. JOHN KELLEY.

231. To determine the face of a note that will yield a given sum when discounted.

For how much must a four-months' note without interest be made that it may yield $1000 when discounted at a bank at 6%?

The discount on $1 for 4 mos. 3 dys. is $0.0205.
Proceeds of $1 is $1 − $0.0205 = $0.9795 = 0.9795 of $1.
Therefore the face required is $ 1000 ÷ 0.9795 = $1020.93.

Ex. 148.

1. Find the face of a note for 30 dys. that will realize $600 when discounted at $6\frac{1}{2}$%.
2. Find the face of a note for 60 dys. that will realize $8000 when discounted at 8%.
3. Find the face of a four-months' note that will realize $800 when discounted at $5\frac{1}{2}$%.
4. Find the face of a note for 90 dys. that will realize $1700 when discounted at 7%.
5. Find the face of a two-months' note that will realize $900 when discounted at $7\frac{3}{10}$%.
6. Find the face of a three-months' note that will realize $2200 when discounted at 7%.

Present Worth.

232. The *present worth* of a sum of money due at the end of a fixed time is the sum that, put at interest for the fixed time, will amount to the given sum.

Thus, $100 will in 2 yrs., at 6%, amount to $112. And $112 to be paid at the end of 2 yrs. is equal in value to $ 100 paid now. Hence $ 100 is regarded as the present worth of $112 to be paid in 2 yrs. (See § 226.)

COMMERCIAL DISCOUNT.

233. Commercial discount is a reduction from the nominal price or value of anything.

234. Price-lists of articles manufactured and sold are issued by manufacturers and wholesale dealers. These prices are subject to many and various discounts.

The following bill will afford a good illustration of this discount:

New York, Feb. 1, 1889.

GEORGE SHATTUCK

Bought of S. L. MORISON & CO.

8 doz. Bolts, $3.00	*$24*	*00*		
(Discount, 40, 5, 25,	*13*	*74*	*$10*	*26*
11 gro. Screws, $2.25	*24*	*75*		
⅔ and 30,	*18*	*97*	*5*	*78*
6 doz. Chest Handles, $1.50	*9*	*00*		
40, 5, 25, 17½,	*5*	*82*	*3*	*18*
			$19	*22*

In the first item, $3.00 is the list price per dozen of the bolts, $24 is the gross price of the 8 dozen bolts, $10.26 is the *net* price. This net price is found by taking 40% from $24, which leaves $14.40; then taking 5% from $14.40, which leaves $13.68, and then taking 25% from $13.68, which leaves $10.26.

In the second item, the gross price is $24.75. The ⅔ means a discount of ⅔ of the gross price, which leaves $8.25, and the 30 means there is a discount of 30% from $8.25, which leaves $5.78.

In the third item, 40% is taken off the gross price, 5% taken from the remainder thus found, then 25% from the second remainder, and then 17½% from the third remainder.

NOTE. In finding 17½%, first find 10%, and then add to it ½ for 5% and ¼ for 2½%.

To combine two discounts so as to form one discount, *from their sum take one per cent of their product.* Thus, 50% and 10% = 50 + 10 − 5 = 55%.

Ex. 149.

Find the net amount of a bill of:

1. $320 subject to a discount of 10% for cash.
2. $1680 subject to discounts of 15 and 10.
3. $980 with 15 off.
4. $1620 with 20 and 15 off.
5. $1440 with 25, 10, and 5 off.
6. $587.50 with 35 and 15 off.
7. $1920 with 25 and 7½ off.
8. $1530 with 25, 10, and 5 off.
9. $500 with 25, 15, and 12½ off.
10. $870.40 with 30, 22½, and 12½ off.
11. Find the net cash amount of a bill of $1088, discounts being 50 and 10, and an additional discount of 5% for cash.
12. Find the difference between a single discount of 55, and successive discounts of 40 and 15.
13. Find the net cash amount of a bill of $136, discounts being 50, 10, and 5. Find a single discount equivalent to these three successive discounts.
14. Find the net cash amount of a bill of $164.50, discounts being ⅔ and 30.
15. Find the net cash amount of a bill of $15, discounts being 40, 5, 25, and 17½. Find a single discount equivalent to these four successive discounts.

PARTIAL PAYMENTS.

235. When settlements of accounts are made at the expiration of a year or less, it is customary to reckon interest on each item from the time it is due to the time of settlement.

And when partial payments are made and indorsed on a note that contains the words **with interest,** provided the note is paid in full within a year, it is usual to compute the interest on the principal, and on each of the payments to the time of settlement.

Samuel Paine buys of Edgar Smith $400 worth of goods at 30 dys. At the end of 3 mos. he pays $200, and the balance 2 mos. later. Find the balance.

The time between the end of 30 dys. and the time of settlement is 4 mos. Therefore interest is reckoned on the $400 for 4 mos., and on the $200 for 2 mos.

$400 + 4 mos. interest =	$408
$200 + 2 mos. interest =	202
Therefore balance due is	$206

A man holds a note for $1000, dated Jan. 1, 1885, on which are indorsed payments as follows: March 1, 1885, $100; Oct. 1, 1885, $50; Nov. 1, 1885, $800. What is due Jan. 1, 1886, interest at 6%?

Amount of $1000 for 1 yr., at 6%, is		$1060.00
Amount of $100 for 10 mos., at 6% =	$105.00	
Amount of $50 for 3 mos., at 6% =	50.75	
Amount of $800 for 2 mos., at 6% =	808.00	
		963.75
Balance due,		$96.25

This method is in accordance with what is called the **Merchants' Rule.**

Ex. 150.

1. A note for \$3000, dated April 1, 1884, payable on demand, with interest at 7%, bears the following indorsements: May 6, \$600; July 5, \$676.11; Oct. 18, \$966. What is due Jan. 1, 1885?
2. A note for \$1237.50, dated April 17, 1884, payable on demand, bears the following indorsements: June 5, \$253; Aug. 20, \$274.50; Nov. 17, \$420. What is due Jan. 1, 1885, reckoning interest at 6%?
3. A note for \$775.50, dated May 15, 1884, payable on demand, bears the following indorsements: July 21, \$150; Oct. 10, \$250; Feb. 24, 1885, \$100. What is due May 15, 1885, reckoning interest at 6%?
4. A note for \$1670.50, dated July 1, 1884, payable on demand, with interest at $6\frac{1}{2}$%, bears the following indorsements: Aug. 20, \$315; Sept. 21, \$360.50; Oct. 5, \$400; Dec. 1, \$160. What is due Jan. 1, 1885?

236. When a note that contains the words "**with interest**" runs longer than a year, and partial payments have been made, the interest is computed by a rule adopted by the Supreme Court of the United States, and therefore called

THE UNITED STATES RULE.

Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest.

From this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal, and proceed as before.

1. [illegible] $1520 dated May 20, 1884, and drawing [illegible] 6% had payments indorsed upon it as [illegible] Oct. 2, 1884, $300; Feb. 26, 1885, $25; [illegible] 1885, $570; Aug. 8, 1885, $600. Find [illegible] amount due Dec. 6, 1885.

$1520	1st principal.
0.022	
$33.44	1st interest.
1520.00	
$1553.44	
300.00	1st payment.
$1253.44	2d principal
0.024	
$30.08	2d interest.
$1253.44	2d principal.
0.006	
$7.52	3d interest.
30.08	2d interest.
1253.44	
$1291.04	
595.00	2d & 3d payments.
$696.04	3d principal.
0.021	
$14.62	4th interest.
696.04	
$710.66	
600.00	4th payment.
$110.66	4th principal.
0.019⅔	
$2.18	5th interest.
110.66	
$112.84	$112.84. *Ans.*

[illegible] first place, find the difference in time between each pair of [illegible] dates. At the right of the result in each case put the [illegible] decimal multiplier for the interest at 6%, and put the [illegible] payment below.

Generally, it can be determined *mentally* whether one or more payments must be taken to make a sum equal to or greater than the interest. If two or more payments are required, the corresponding decimal multipliers may be added, and the result taken for the multiplier. Thus, it is evident that 0.024 of \$1253.44 is more than \$25; therefore, $0.024 + 0.006 = 0.03$ may be taken for the multiplier, which will give for the interest \$37.60. To this the principal is added, and from the amount the sum of the payments is subtracted.

When the rate is greater or less than 6%, the several *interests* must be increased or diminished according to the given rate.

Ex. 151.

1. A note of \$1000, dated Jan. 22, 1884, and drawing interest at 6%, had payments indorsed upon it as follows: May 20, 1884, \$50; July 20, 1884, \$162.50; Dec. 23, 1884, \$72.50. Find the balance due March 1, 1885.

2. A note of \$3325, dated Jan. 15, 1884, and drawing interest at $6\frac{1}{2}$%, had payments indorsed upon it as follows: June 24, 1884, \$100; Sept. 2, 1884, \$1250; Jan. 31, 1885, \$1400. Find the balance due May 12, 1886.

3. A note of \$2280, dated Jan. 22, 1883, and drawing interest at 7%, had payments indorsed upon it as follows: Jan. 10, 1884, \$1000; Aug. 31, 1884, \$250; Jan. 15, 1885, \$600; March 4, 1885, \$430. Find the balance due June 15, 1885.

Compound Interest.

237. When a note contains the words "**with interest annually,**" and the interest is not paid at the time it is due, the interest is usually added to the principal; and new principals are thus formed at regular intervals of time.

238. The interest may be compounded with the principal (that is, made a part of the principal) annually, semi-annually, quarterly, etc., according to agreement.

Ex. Find the compound interest of \$800 for 2 yrs. 3 mos. 15 dys., at 7%.

	\$800	
	0.07	
	\$56	1st interest.
	800	
	\$856	2d principal.
	0.07	
	\$59.92	2d interest.
	856.00	
	\$915.92	3d principal.
3 mos. 15 dys.	0.0175	
	6)16.03	
	2.67	
	\$18.70	3d interest.
	915.92	
	\$934.62	amount.
	800.00	
	\$134.62	interest.

\$134.62. *Ans.*

239. If the given time be not an integral number of years, the amount is found for the number of entire years, and then the amount of this for the fractional part of a year.

Ex. 152.

1. Find the compound interest on \$125 for 3 yrs., at $2\frac{1}{2}$%.
2. Find the amount of \$87.50 for 3 yrs., at 4% per annum, at compound interest.
3. Compare the simple and compound interest on \$21.50, at the end of 4 yrs., at 5%.

4. What will a debt of \$4250 amount to, if left standing for 2 yrs. 6 mos., at 5% per annum, compound interest?

5. Find the compound interest on \$104 for 1 yr. 9 mos., at 5%.

6. Find the compound interest on \$1800 for 2 yrs. 3 mos. 15 dys., at $3\frac{1}{2}$%.

7. Find the compound interest on \$4500 for 3 yrs. 6 mos., at 4%.

If the interest be payable semi-annually, quarterly, etc., the half, quarter, etc., of the rate per cent, must be used, and the amount obtained for each half-year, quarter-year, etc.

8. Find the compound interest on \$4000 for 2 yrs. 6 mos., at 5% per annum, interest payable semi-annually.

9. Find the compound interest on \$1001.50 for 1 yr. 3 mos., at 6%, interest payable semi-annually.

10. Find the compound interest on \$4000 for 1 yr. 3 mos., at 4% per annum, interest payable quarterly.

Ex. What principal will produce in 2 yrs. \$650.14, compound interest at 6%?

Amount of \$1 for 1 yr., at 6%, is $1.06 \times \$1$.
Amount of \$1 for 2 yrs. is $1.06 \times 1.06 \times \$1 = (1.06)^2 \times \$1$.
That is, the amount of \$1 for 2 yrs., at 6%, is \$1.1236.
Interest is $\$1.1236 - \$1 = \$0.1236 = 0.1236$ of \$1.
The principal required is $\$650.14 \div 0.1236 = \$5260$.

\$5260. *Ans.*

11. What principal will amount to \$275.62 in 2 yrs., at 5% compound interest?

12. What principal will amount to \$620.32 in 3 yrs., at 6% compound interest?

ANNUAL INTEREST.

240. Annual Interest is simple interest on the principal and on each year's interest from the time each interest is due until settlement.

(1) Find the interest due Aug. 4, 1885, on a note dated June 4, 1881, for $1700, with interest payable annually, at 6%.

yrs. mos. dys.		
1885 8 4	$1700.00	
1881 6 4	0.25	
4 2	$425.00	Interest for 4 yrs. 2 mos.
yrs. mos.		
3 2	$1700.00	
2 2	0.06	
1 2	$102.00	Annual interest.
2	0.06	
6 8 = $6\frac{2}{3}$ yrs.	$6.12	
	$6\frac{2}{3}$	
	4.08	
	36.72	
	$40.80	Interest on annual int.
	425.00	
	$465.80	Total interest due.

$465.80. *Ans.*

The first year's interest, $102, remains overdue 3 yrs. 2 mos., the second year's 2 yrs. 2 mos., the third year's 1 yr. 2 mos., and the fourth year's 2 mos. Now the interest on $102 for the sum of these periods, $6\frac{2}{3}$ yrs., is $40.80. Hence the total interest is $465.80.

13. Find the amount due May 17, 1885, on a note dated May 17, 1881, for $700, at 6% annual interest.

14. Find the amount due May 27, 1885, on a note dated Jan. 4, 1883, for $431, at $5\frac{1}{2}$% annual interest.

15. Find the amount due May 19, 1885, on a note dated Dec. 26, 1881, for $612.30, at 5% annual interest.

16. Find the amount due Jan. 16, 1885, on a note dated Jan. 8, 1883, for $623.04, at 5% annual interest.

17. Find the amount due Jan. 18, 1885, on a note dated Jan. 8, 1881, for $575, at 6% annual interest.

STOCKS AND BONDS.

241. The name **stock** is applied to the capital of banks, railroads, and other incorporated companies.

The capital of a company is usually divided into **shares**, of which the *original value* is $100, or some other fixed sum; but the *market-value* at any time is the price per share at that time.

When the market-value of stock is equal to its original value, it is said to be **at par**. In quotations of stocks, par is generally represented by 100; and when stock is quoted at above 100, it is said to be at a premium; below 100, at a discount. The premium or discount is the difference between the quotation and 100.

Thus, when the price of a stock on a given day is 91, or, as it is commonly expressed, when the stock is *at* 91, the meaning is, that $100 stock costs on that day $91 money; or that, if 100 be the representative of any quantity of stock, 91 will represent the corresponding value in money. In this case the stock is said to be 9% discount.

The buying and selling of stocks is conducted through the agency of stock-brokers, who receive a brokerage on the stock. The brokerage is generally reckoned at $\frac{1}{8}$ of 1% on the *par value* of the stock. Thus, if a broker buy stock for a person at 91, that person pays $91\frac{1}{8}$.

(1) How much would be received for 52 shares of stock, $100 each, at $89\frac{1}{2}$?

$\$\frac{1}{8}$ per share will represent the commission.

$$\begin{aligned} 52 \times \$89\tfrac{1}{2} &= \$4654. \\ \text{Commission} &= \underline{\quad 6.50} \\ \text{Proceeds} &= \$4647.50 \end{aligned}$$

$4647.50. *Ans.*

(2) What amount of stock, at $84\frac{5}{8}$, including brokerage, may be bought for $9393.37½?

Since $0.84⅝, or 0.84⅝ of $1, buys $1 stock, the amount bought for $9393.37½ will be $\frac{\$9393.37\frac{1}{2}}{0.84\frac{5}{8}} = \$11,100$.

$11,100. *Ans.*

(3) What is the quoted price of stock when $42,464.25 is paid for $46,600 stock?

$46,600 stock costs $42,464.25.

$1 stock costs $\frac{1}{46600}$ of $42,464.25 = $0.91⅛.

91⅛. *Ans.*

Ex. 153.

1. Find the cost of $5000 stock, at 98.
2. Find the cost of $7800 stock, at 78⅛.
3. Find the cost of $20,000 stock, at 109¾.
4. Find the cost of $5000 United States 4% bonds, at 121.
5. Mr. Jones owns 20 United States 4% bonds of $1000 each. The interest on these bonds is paid quarterly. How much interest does Mr. Jones receive every quarter?
6. Find the cost of 20 shares of Boston and Maine Railroad stock, at 174.
7. How much of United States 4% bonds may be bought for $6305, at 121¼?
8. How much of Northern Pacific 6% bonds, selling at 102¾, may be bought for $10,275?
9. How many shares ($100 each) of Old Colony Railroad, at 137½, may be bought for $1650?

10. How many shares of railroad stock, at $91\frac{1}{8}$, may be bought for $\$8474.62\frac{1}{2}$?

11. What must be the price of stock in order that \$9200 stock may be bought for \$8970?

12. What must be the price of stock in order that \$11,600 stock may be bought for \$8729?

13. If \$3000 stock is bought for \$2748.75, what is the price of the stock?

14. What income will be derived from \$15,000 of 5% bonds?

15. Find the income from \$9000 of 6% stock.

16. How much will a person receive from \$18,800 railroad stock, if a dividend of 7% be declared?

17. What income will be derived from \$30,000 of 4% bonds?

Ex. How much 4% stock must be bought to give an income of \$320?

Since \$0.04 is derived from \$1 stock, \$320 will be derived from as many times \$1 as \$0.04 is contained in \$320. $\$320 \div \$0.04 = 8000$.

\$8000. *Ans.*

18. How much 4% stock must be bought to give an income of \$2400?

19. A person receives \$343 as his quarterly dividend from a 7% stock. How much stock does he hold?

20. Find the entire income of a person whose property consists of \$6000 of 6% stock and \$16,400 of 7% stock.

21. Find the rate of dividend paid by a railroad when a holder of 246 shares receives \$1722.

22. Find the rate per cent at which $22,200 will yield a semi-annual return of $999.

Ex. If $5125 is invested in 6% stock, at $102\frac{1}{2}$, what income will be obtained?

$1 stock costs 1.025 of $1.

Hence $5125 will be the cost of $\$5125 \div 1.025 = \$5000$ stock. And 6% of $\$5000 = \$300$.

$300. *Ans.*

23. Find the income on $39,000 invested in 4% stock, at 91.

24. Find the income on $7000 invested in 4% stock, at $103\frac{1}{4}$.

25. Find the income on $13,600 invested in 7% stock, at 130.

26. A person invests $14,280 in railroad stock, at $127\frac{1}{2}$. What will he receive if a dividend of $3\frac{1}{4}$% be declared?

27. Find the income on $14,000 when invested in 8% stock, at $103\frac{1}{4}$.

Ex. If a person buys 5% stock at 120, what rate of interest does he receive on his money invested?

$100 stock costs $120. $100 stock pays $5. Hence the $120 invested yields $5.

Therefore, the rate of interest is $\frac{5}{120} = 0.04\frac{1}{6}$, or $4\frac{1}{6}$%.

$4\frac{1}{6}$%. *Ans.*

28. If an 8% stock is worth 150, what rate of interest will a purchaser receive on his money?

29. If a 10% stock is worth 175, what rate of interest will a purchaser receive on his money?

... 70, what rate of interest will ... his money?

... 70, what rate of interest will ... his money?

...h 65, what rate of interest will ... on his money?

...red for an investment in a 5% stock ... an income of $20 a year

... from $100 stock.

... be received from $\frac{20}{4}$ ... stock = $500.

...s $98¼.

... stock will cost ... = $492...

$492... *Ans.*

...ch money must be invested in 5% stock, at 92, to produce $400 income?

... much money must be invested in a 5% stock, at 87½, to produce an income of $250?

... A person bought some bank stock at 107, and received $265 when a 5% dividend was declared by the bank. How much money had he invested?

6. A person buys some 6% railroad stock at 75, and receives $750 income. How much money has he invested?

Ex. What must be the price of a 5% stock in order that a buyer may receive 6% on his investment?

$100 must be invested to produce $6.
Hence ⅚ of $100 = $83⅓ must be invested to produce $5.
Therefore the price of the 5% stock must be 83⅓.

83⅓. *Ans.*

37. What must be the price of a 6% stock in order that a buyer may receive 7% on his investment?

38. What must be the price of an 8% stock in order that a buyer may receive 6% on his investment?

39. A person invested $5710 in bank stock when the stock was at $142\frac{3}{4}$. What per cent dividend is declared, if he receives $300?

40. A person receives 5% interest on his money by investing in some six per cent stock. At what price did he buy it?

41. What must be the price of a 7% stock in order that a buyer may receive 6% on his investment?

EXCHANGE.

242. A **draft** or **bill of exchange** is a written order directing one person to pay a specified sum of money to another.

243. A **commercial draft** is a draft payable at a specified time after sight (or date).

When the person on whom a commercial draft is drawn accepts a draft, he writes the word "Accepted," with the date, across the *face*, and signs his name. The draft is then called **an acceptance**, and the acceptor is responsible for its payment.

An acceptance is of the nature of a promissory note, the acceptor and maker having respectively the same responsibility for payment as the maker and indorser of a promissory note.

244. The system of paying money to persons at a distance by transmitting bank drafts or bills of exchange instead of money is called **exchange**.

When a draft can be bought for its face, it is said to be *at par*. When the cost is less than the face, it is said to be *at a discount;* and when the cost is more than the face, it is said to be *at a premium*.

Ex. 154.

Ex. Find the cost of a draft on New York for $1000, at $\frac{1}{4}$ of 1% premium.

$\frac{1}{4}$% of $1000 = $2.50 (premium).
$1000 + $2.50 = $1002.50 (cost).

$1002.50. *Ans.*

1. Find the cost of a draft on New York for $1200, at $\frac{1}{4}$ of 1% discount.

2. Find the cost of a draft on St. Louis for $2000, at $\frac{1}{4}$ of 1% premium.

3. Find the cost of a draft on New Orleans for $2400, at $\frac{1}{8}$% premium.

4. Find the cost of a draft on Chicago for $3200, at $\frac{3}{8}$% discount.

Ex. Find the cost of a draft on Cincinnati for $1000, payable in 30 dys. after sight, exchange being $\frac{1}{2}$% premium, and interest 6%.

	$1000.00	
0.0055 of $1000 =	$5.50	discount for 33 dys.
	$994.50	cost of draft at par.
0.005 of $1000 =	5.00	premium.
	$999.50	cost of draft.

5. Find the cost of a draft for $800, payable 30 dys. after sight, when exchange is $\frac{1}{4}$% premium, and interest 6%.

6. Find the cost of a draft for $1900, payable in 30 dys., when exchange is at par, and interest $4\frac{1}{2}$%.

7. Find the cost of a draft for $1450, payable in 60 dys., when exchange is $\frac{1}{4}$% discount, and interest 5%.

8. Find the cost of a draft for $1000, payable 60 dys. after sight, when exchange is $\frac{1}{2}$% discount, and *interest* 7%.

CHAPTER XII.

PROPORTION.

245. The *relative magnitude* of two numbers is called their **ratio,** when expressed by the fraction which has the first number for numerator and the second number for denominator.

Thus the ratio of 2 to 3, commonly written 2 : 3, is expressed by the fraction $\frac{2}{3}$.

246. The first term of a ratio is called the **antecedent,** and the second term the **consequent.**

247. If both terms of a ratio be multiplied or divided by the same number, the ratio is not altered.

Thus, if both terms of the ratio $2\frac{1}{2} : 3\frac{1}{3}$ be multiplied by 6, the resulting ratio is 15 : 20, and the two ratios are equal, for $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{15}{20}$. Since $\frac{15}{20} = \frac{3}{4}$, the simplest expression for $2\frac{1}{2} : 3\frac{1}{3}$ is 3 : 4.

248. If the numerator and denominator are interchanged, the fraction is said to be *inverted;* likewise, if the antecedent and consequent of a ratio are interchanged, the resulting ratio is called the *inverse* of the given ratio.

Thus, if the fraction $\frac{4}{5}$ is inverted, the resulting fraction is $\frac{5}{4}$, and the inverse of the ratio 4 : 5 is 5 : 4.

249. If two *quantities* are expressed in the *same unit,* their ratio will be the same as the ratio of the two *numbers* by which they are expressed.

Thus the quantity \$7 is the same fraction of \$9 as 7 is of 9.

250. Since ratio is simply relative magnitude, two quantities *different in kind* cannot form the terms of a ratio; and two quantities of the same kind must be expressed in a *common unit* before they can form the terms of a ratio.

Thus no ratio exists between $5 and 20 dys.: and the ratio of 3 t. to 5000 lbs. can be expressed only when *both* quantities are written as tons or pounds.

251. When two ratios are equal, the four terms are said to be **in proportion**, and are called **proportionals**.

Thus 6, 3, 18, 9 are proportionals; for $\frac{6}{3} = \frac{18}{9}$.

252. A proportion is written by putting the sign = or a double colon between the ratios.

Thus $6 : 3 = 18 : 9$, or $6 : 3 :: 18 : 9$, means, and is read, the ratio of 6 to 3 is equal to the ratio of 18 to 9.

253. The *first* and *last* terms of a proportion are called the **extremes**, and the two *middle* terms are called the **means**.

254. *Test of a proportion.* When four numbers are proportionals, the product of the extremes is equal to the product of the means.

This is seen to be true by expressing the ratios in the form of fractions, and multiplying both by the product of the denominators.

Thus the proportion $5 : 3 :: 15 : 9$ may be written $\frac{5}{3} = \frac{15}{9}$; and, if both be multiplied by 3×9, the result will be $5 \times 9 = 3 \times 15$.

255. Either extreme, therefore, will be equal to the product of the means divided by the other extreme; and either mean will be equal to the product of the extremes divided by the other mean. Hence, if three terms of a proportion are given, the fourth may be found.

(1) What number is to 4 as 3 is to 6?

This may be written $\frac{\text{What number}}{4} = \frac{3}{6}?$

Multiply both sides of the equation by 4.

The result is, $\text{What number} = \frac{4 \times 3}{6}?$

2. *Ans.*

(2) 20 is to 24 as what number is to 30?

This may be written $\frac{20}{24} = \frac{\text{What number}}{30}?$

Multiply by 30, $\frac{20 \times 30}{24} = \text{What number}?$

25. *Ans.*

(3) 18 is to 32 as 45 is to what number?

This may be written $\frac{18}{32} = \frac{45}{\text{What number}}?$

As these fractions are equal, their reciprocals are equal;

that is, $\frac{32}{18} = \frac{\text{What number}}{45}?$

Multiply by 45, $\frac{32 \times 45}{18} = \text{What number}?$

80. *Ans.*

256. When three terms of a proportion are given, the method of finding the fourth term is called the **Rule of Three.**

It is usual to arrange the quantities (that is, to *state* the question) so that the quantity required for the answer may be the fourth term. Hence the quantity which *corresponds* to that of the required answer must be the third term.

(1) If 5 t. of hay cost \$87.50, what will 21 t. cost?

Since the *cost* of 21 t. is required, \$87.50 is the third term.

Since 21 t. will cost *more* than 5 t., 21 t. is the second term and 5 t. the first term.

That is, 5 t. : 21 t. : : \$87.50 : What quantity?

A difficulty presents itself here, inasmuch as no meaning can be given to the product of the means ($87.50 multiplied by 21 t.). Since, however, the ratio of 5 t. : 21 t. = the ratio of 5 : 21, the ratio 5 : 21 may be substituted for 5 t. : 21 t.

Then 5 : 21 : : $87.50 : What quantity?

That is, What quantity $= \dfrac{21 \times \$87.50}{5}$?

$367.50. *Ans.*

(2) When a post 11.5 ft. high casts a shadow on level ground 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yds. long. How high is the steeple?

Height is required; the height 11.5 ft. is therefore the third term.

Since the *shadow* of the steeple is the longer, the *height* of the steeple must be the greater; therefore the second term must be the greater of the two remaining quantities expressed in the same unit. 63.7 yds. = 191.1 ft.

	Shadow.	Shadow.	Height.	Height.
	17.4 ft. :	191.1 ft. : :	11.5 ft. :	What?
or,	17.4 :	191.1 : :	11.5 ft. :	What?

That is, height of steeple $= \dfrac{191.1 \times 11.5 \text{ ft.}}{17.4} = 126.3$ ft.

126.3 ft. *Ans.*

257. In solving problems by the **Rule of Three,**

Make that quantity which is of the same kind as the required answer the third term.

The *numbers* by which the other two quantities are expressed, when expressed in a *common unit,* will be the first and second terms.

If, from the nature of the question, the answer will be *greater* than the third term, make the *greater* of these two numbers the *second* term; if *less,* make the *smaller* of these numbers the *second* term, and the other the first term.

Divide the product of the second and third terms by the first term, and the quotient will be the answer required.

Ex. 155.

1. An express-train runs 40 mi. in 64 min. At the same rate, how many miles will it run in 24 min.?

2. If 110 A. produce 200 hhds. of sugar, how many hogsheads will 176 A. produce?

3. If 48 reapers cut 20 A. in a given time, how many acres will 156 reapers cut in the same time?

4. If 20 reapers can cut a field in 6 dys., in how many days will 30 reapers do it?

5. The number of copies in the first edition of the "Lady of the Lake" was 2050, and was to the number in the second as 41 to 69. Find the number in the second edition.

6. The length of the steamer-track from Liverpool to Quebec is 2502 mi., and is to that from Liverpool to Boston as 139 is to 155. Find the length of the track from Liverpool to Boston.

7. If a steamer from Liverpool to Portland makes the passage of 2750 mi. in $5\frac{5}{8}$ dys., in how many days, at the same rate, would the passage of 2980 mi. from Liverpool to New York have been made?

8. If a person can walk $8\frac{4}{7}$ mi. in $2\frac{1}{2}$ hrs., how many miles can he walk in $3\frac{1}{4}$ hrs.?

9. If the shadow of a staff 3 ft. 7 in. high is 4 ft. 9 in., find the height of a steeple whose shadow is 158 ft. 4 in.

10 A train, at the rate of $25\frac{3}{8}$ mi. an hour, goes a certain distance in $3\frac{1}{2}$ hrs. In how many hours will one at the rate of $24\frac{1}{2}$ mi. an hour go the same distance?

11. The ratio of the diameter to the circumference of a circle was given by Metius as 113 : 355. Find the circumference of a fly-wheel 10 ft. in diameter.

12. Find the horse-power of an engine that can raise 11,200 lbs. of coal in an hour from a pit whose depth is 396 ft.

NOTE. The labor necessary to raise 1 lb. through 1 ft. is called the **unit of work**; and a horse can do 33,000 units of work a minute. Therefore one horse-power = 33,000 units of work, and $\frac{396 \times 11200}{33000 \times 60}$ = the horse-power required.

13. If 1000 sq. yds. of a field produce a load of hay, how many such loads will 25 A. of the field produce?

14. If a train runs 177 mi. 120 rds. in 3 hrs. $56\frac{1}{2}$ min., what is the rate per hour?

15. If 136 masons can build a fort in 28 dys., how many men will be required to build it in 8 dys.?

16. There are provisions in a fort sufficient to support 4000 soldiers for 3 mos. How many must be sent away to make them last 8 mos.?

17. A coach travels $7\frac{1}{2}$ mi. an hour. How many miles will it go between a quarter past ten A.M. and a quarter to six P.M.?

18. The expense of making the hay on 5 A. 135 sq. rds. is $29.08. What is the expense per acre?

19. If 300 laborers can make an embankment in 48 dys., how many *more* days would be required if the number of men is diminished by 60?

20. If 2.45 tons of straw cost $22.75, how many tons can be bought for $11.70?

COMPOUND PROPORTION.

258. A ratio is said to be *compounded* of two or more given ratios, when it is expressed by a fraction which is the product of the fractions representing the given ratios.

Thus the ratios 2 : 3 and 7 : 11 are represented by the fractions $\frac{2}{3}$ and $\frac{7}{11}$; and the ratio 14 : 33, which is represented by $\frac{14}{33}$ (the product of $\frac{2}{3}$ and $\frac{7}{11}$), is said to be compounded of the ratios 2 : 3 and 7 : 11.

259. A proportion which has one of its ratios a compound ratio is called a **compound proportion.**

In stating problems in compound proportion the quantity which corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required. The process will be understood by the following example:

If 4 men mow 15 A. in 5 dys. of 14 hrs., in how many days of 13 hrs. can 7 men mow $19\frac{1}{2}$ A.?

As the answer is to be in days, make 5 dys. the third term.

I. *Will* it require *more* or *less* days for 7 men to mow 15 A. than it *did* for 4 men? Evidently less.

Therefore make 7 the first term and 4 the second.

II. *Will* it require *more* or *less* days for the same number of men to mow $19\frac{1}{2}$ A. than it *did* to mow 15 A.? Evidently more.

Therefore make 15 the first term and $19\frac{1}{2}$ the second.

III. *Will* it require *more* or *less* days of 13 hrs. to mow the same number of acres than it *did* of 14 hrs.? Evidently more.

Therefore make 13 the first term and 14 the second.

Hence the statement is

$$\begin{array}{l} 7:4 \\ 15:19.5::5 \text{ days . what?} \\ 13:14 \end{array}$$

or
$$\frac{4 \times 19.5 \times 14 \times 5 \text{ days}}{7 \times 15 \times 13}.$$

This, simplified by cancellation, gives 4 days.

Ex. 156.

1. If 13 bu. of oats serve 3 horses for 11 dys., how many bushels will serve 7 horses for 12 dys.?
2. If a traveller walks 140 mi. in 8 dys., walking 7 hrs. a day, how many miles can he walk in 12 dys. of 8 hrs. each?
3. If 4 masons build 27 yds. of wall in 5 dys., working 9 hrs. a day, in how many days will 32 masons build 81 yds. of a similar wall, if they work 10 hrs. a day?
4. A bootmaker who employs 15 men fills an order for 25 doz. pairs of boots in 4 wks. In how many days can he make 45 pairs if he employs 18 men?
5. If a family, by using 2 gas-burners $7\frac{1}{2}$ hrs. a day, pays \$6 a quarter when gas is \$2.40 per 1000 cu. ft., what will a family using 3 burners 4 hrs. a day pay per quarter when gas is \$1.80 per 1000 cu. ft.?
6. If 330 slices $\frac{3}{14}$ of an inch thick are obtained from 12 rounds of beef, how many similar rounds will be required for 495 slices $\frac{1}{6}$ of an inch thick?
7. If 5 horses eat 8 bu. 14 qts. of oats in 9 dys., how many days, at the same rate, will 66 bu. 30 qts. last 17 horses?
8. If a man walks 600 mi. in 25 dys., walking 8 hrs. a day, in how many days will he walk 330 mi., walking 10 hrs. a day?
9. If a pane of glass 18 in. long and $12\frac{1}{2}$ in. wide costs 20 cts., what will be the cost, at the same rate, of a pane $22\frac{1}{2}$ in. long and 15 in. wide?
10. If 18 men can dig a trench 200 yds. long, 3 yds. wide, and 2 yds. deep, in 6 dys. of 10 hrs. each, in how many days of 8 hrs. each will 10 men dig a trench 100 yds. long, 4 yds. wide, and 3 yds. deep?

Proportional Parts.

260. If it be required to divide a quantity into parts proportional to 3, 4, 5, the *numbers* 3, 4, 5 may be taken as *representatives* of the parts, and then the whole quantity will be represented by $3 + 4 + 5$; that is, by 12.

(1) Divide $391 into parts proportional to 5, 7, and 11.

The whole quantity will be represented by $5 + 7 + 11 = 23$.
Therefore the respective parts will be $\frac{5}{23}$, $\frac{7}{23}$, $\frac{11}{23}$ of $391.

$85, $119, $187. *Ans.*

(2) Divide $248 into parts proportional to $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{25}$.

Multiply the fractions by 150, the L.C.M. of their denominators. The results are 15, 10, 6. Hence the parts will be *represented* by the numbers 15, 10, 6, and the whole by 31.

Therefore the respective parts will be $\frac{15}{31}$, $\frac{10}{31}$, $\frac{6}{31}$ of $248.

$120, $80, $48. *Ans.*

Ex. 157.

1. Divide 1200 into parts proportional to 11, 12, 13, 14.
2. Divide 390 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
3. Divide a profit of $689 among 3 partners, of whom the first owns $\frac{2}{13}$, the second $\frac{5}{13}$, and the third $\frac{6}{13}$ of the joint stock.
4. Four men invest $450, $230, $190, $110 respectively in a joint business. Find their respective liabilities in a loss of $313.60.
5. Three partners claim respectively $\frac{1}{3}$, $\frac{11}{18}$, and $\frac{7}{18}$ of $1260. Give to each his proportional share.
6. An analysis of dissolved bones gives the following results for every 100 parts. Water, 13.97; organic matter, 15.71; soluble phosphates, 21.63; insoluble phosphates, 11.43; sulphate of lime, 15.83; sulphuric acid, 15.63; alkaline salts, 1.10; silica, etc., the remainder. Find the number of pounds of each in a ton of dissolved bones.

PARTNERSHIP.

261. Partnership is separated into *simple* and *compound*. In simple partnership the capital of each partner is invested for *the same time*. In compound partnership the time for which the capital of each partner is invested is taken into account, as well as the amount of the capital; and the division of profits and losses is made proportionally to the amount of the capital and the time it is invested.

A and B enter into partnership. A puts in $2000 for 2 yrs., and B puts in $3000 for 1 yr. Their profits are $1400. What is the share of each?

The use of $2000 for 2 yrs. is equivalent to 2 × $2000 for 1 yr. Hence their profits must be divided in the ratio $4000 to $3000; that is, 4 : 3.

Ex. 158.

1. Three drovers rent a field of 9 A., at $5 an acre. A puts in 6 cows for 2 mos; B, 9 cows for 1 mo.; and C, 12 cows for 2 mos. How much should each pay?
2. In a co-partnership A contributed $400 for 9 mos.; B, $350 for 8 mos.; and C, $600 for 2 mos. Divide a gain of $570 among them.
3. At the end of 12 mos. A, B, and C, having a joint capital of $6000, find they have lost $625. A's capital of $2500 has been in the business for 12 mos., B's of $1500 for 8 mos., and C's of $2000 for 4 mos. Divide the loss among them.
4. A and B enter into partnership, A with $1800, and B with $900. At the end of 8 mos. B adds $300 to his capital. Divide a profit of $840 between them, at the end of the year.

AVERAGES.

262. If a dozen eggs weigh 1 lb. 8 oz., what is their average weight?

Since the 12 eggs weigh 1 lb. 8 oz., that is, 24 oz., the average weight of an egg will be the entire weight divided by the number; that is, $\frac{24 \text{ oz.}}{12} = 2$ oz.

Ex. 159.

1. A merchant mixes 3 lbs. of coffee worth 27 cts. a pound, 2 lbs. worth 35 cts., and 1 lb. worth 41 cts. What is the mixture worth a pound?
2. What is the cost of a gallon of a mixture containing 7 gals. worth $1.35 a gallon, 5 gals. worth $1.05 a gallon, and water enough to make the whole mixture 15 gals.?
3. Of 32 candidates for office, 3 were 20 yrs. old, 4 were 21, 12 were 22, 12 were 23, and 1, 24. What was the average age of the candidates?
4. A bankrupt owes A $962.50, B, $3487, and C, $12,686.50. His estate, after paying expenses of settlement, is $3427.20. How much can he pay on a dollar?
5. A grocer buys 106 lbs. of tea, at 80 cts. per pound, 75 lbs., at $1.24 per pound, and 94 lbs., at $1.30 per pound, and mixes the three lots together. At what price per pound must he sell the mixture so as to make 10% on his outlay?
6. In what proportions must oils worth $1.25 a gallon and 80 cts. a gallon be mixed to make a mixture worth $1.00 a gallon?

   HINT. The loss on the $1.25 oil is 25 cts. a gallon. The gain on the 80 ct. oil is 20 cts. a gallon. Therefore there must be *more of the* 80 ct. oil taken than of the $1.25 oil, and in the *ratio of 25* : 20 or 5 : 4.

7. In what proportion must oils worth $1.20 and 60 cts. a gallon be mixed, so that the mixture may be worth 70 cts. a gallon?

8. Solder is composed of tin and lead. If a solder weighs 10.44 times as much as an equal bulk of water, while tin weighs 7.29, and lead 11.35 as much, find the weight of each metal in a pound of solder.

HINT. The losses per unit of volume for the lead and the tin are as 0.91 to 3.15. Therefore their volumes must be as 3.15 to 0.91; and their weights as 3.15 × 11.35 to 0.91 × 7.29. The lead is 13.496 oz., and the tin 2.504 oz.

AVERAGE OF PAYMENTS.

A has given to B notes as follows: $250, due in 3 mos.; $400, due in 6 mos. He wishes to pay them both at one time. In how many months shall the payment be made?

The use of $250 for 3 mos. equals the use of $750 for 1 mo.
The use of $400 for 6 mos. equals the use of $2400 for 1 mo.
$650 $3150 for 1 mo.

The question is, for how many months is the use of $650 equal to the use of $3150 for 1 mo.? The answer is 1350 ÷ 650, or $4\frac{11}{13}$.

9. Find the equated time for the payment of $300 due in 3 mos., $500 due in 6 mos., $200 due in 9 mos.

10. A owes B $50 payable in 6 mos., $60 payable in 8 mos., and $90 payable in 4 mos. Find the equated time of payment.

11. A owes B $1000, payable at the end of 9 mos. He pays $200 at the end of 3 mos. and $300 at the end of 8 mos. When is the balance due?

12. On the first day of January, A purchases of B $200 worth of goods on 3 mos. credit, and $500 worth on 4 mos. credit, and gives one note in payment. When does the note become due?

CHAPTER XIII.

POWERS AND ROOTS.

263. The *square* of a number is the product of *two* factors, each equal to this number.

Thus the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

264. The **square root** of a number is one of the *two equal factors* of the number.

Thus the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

265. The square root of a number is indicated by the *radical sign* $\sqrt{\ }$, or by the fraction $\frac{1}{2}$ written above and to the right of the number.

266. Since $35 = 30 + 5$, the square of 35 may be obtained as follows:

$$\begin{array}{l} 30 + 5 \\ \underline{30 + 5} \\ 30^2 + (30 \times 5) \\ \underline{\qquad\quad (30 \times 5) + 5^2} \\ 30^2 + 2(30 \times 5) + 5^2 \end{array} \qquad \begin{array}{r} 30^2 = 900 \\ 2(30 \times 5) = 300 \\ 5^2 = \underline{\ 25} \\ = 1225 \end{array}$$

267. Hence, since every number consisting of two or more figures may be regarded as composed of tens and units,

The square of a number will contain the square of the tens + twice the tens × the units + the square of the units.

SQUARE ROOT.

268. The first step in extracting the square root of a number is to mark off the figures of the number in groups.

Since $1 = 1^2$, $100 = 10^2$, $10,000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* or *two* figures is a number of *one* figure; expressed by *three* or *four* figures is a number of *two* figures, and so on.

If, therefore, an integral number be divided into groups of two figures each, from the right to the left, the number of figures in the root will be equal to the number of groups of figures. The last group to the left may have one or two figures.

Find the square root of 1225.

```
  12 25(35
   9
65)3 25
   3 25
```

The first group 12, contains the square of the tens' number of the root.

The greatest square in 12 is 9, and the square root of 9 is 3. Hence 3 is the tens' figure of the root.

The square of the tens is subtracted, and the remainder contains twice the tens $\times$ the units + the square of the units. Twice the 3 tens is 6 tens, and 6 tens is contained in the 32 tens of the remainder 5 times. Hence 5 is the units' figure of the root. Since twice the tens $\times$ the units + the square of the units is equal to (twice the tens + the units) $\times$ the units, the 5 units are annexed to the 6 tens, and the result, 65, is multiplied by 5.

269. The same method will apply to numbers of more than two groups of figures, by considering the part of the root already found as so many tens with respect to the next figure.

Extract the square root of 7890481.

```
    7 89 04 81(2809
    4
 48)3 89
    3 84
5609)5 04 81
     5 04 81
```

When the third group, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 56 is not contained in 50. Therefore, 0 is placed both in the root and the divisor, and the next two figures, 81, are brought down.

270. If the square root of a number have decimal places, the number itself will have twice as many.

Thus, if 0.11 be the square root of some number, the number will be $(0.11)^2 = 0.11 \times 0.11 = 0.0121$. Hence, if a given number contain a decimal, we divide it into groups of two figures each, by beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal. We must be careful to have the last group on the right of the decimal point contain *two* figures, annexing a cipher when necessary.

Extract the square root of 52.2729.

```
      52.27 29(7.23
      49
  142)3 27
      2 84
 1443)43 29
      43 29
```

It will be seen from the groups of figures that the root will have one integral and two decimal places.

271. If a number is not a perfect square, ciphers may be annexed, and an *approximate* value of the root found.

Extract to six places of decimals the square root of 19.

```
    19 00 00 00(4.358899
    16
  83)3 00
     2 49
 865) 51 00
      43 25
  8708)7 75 00
       6 96 64
  8716)78 360
       69 728
        8 6320
        7 8444
         78760
```

In this example, after finding four figures of the root, the other three are found by common division. The rule in such cases is, that one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found.

272. The square root of a common fraction is found by extracting the square roots of the numerator and denominator

nator. But, when the denominator is not a perfect square, it is best to reduce the fraction to a decimal and then extract the root.

Ex. 160.

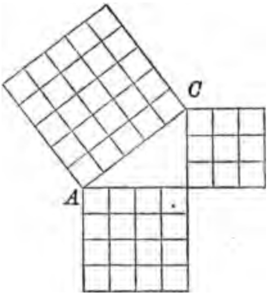
Find the square roots of:

1. 4225.
2. 31.36.
3. 50625.
4. 401956.
5. 15.7609.
6. 0.180625.
7. 0.001296.
8. 0.042849.
9. 0.025.
10. 28.75.
11. 0.009.
12. 0.081.
13. $\frac{64}{169}$.
14. $\frac{225}{361}$.
15. $\frac{3}{4}$.
16. $\frac{1}{2}$.

The side of a square is found by extracting the square root of its area.

17. A rectangle is 972 yds. long and 432 yds. wide. Find the side of a square which has the same area as the rectangle.

18. Find in yards the length of the side of a square field containing 27 A. 12 sq. rds. 1 sq. yd.



In a right triangle, the square on the hypotenuse (AC) is equal to the sum of the squares on the two legs.

Hence hypotenuse = square root of sum of squares on the legs; and one leg = square root of difference of squares on the other two sides.

19. Base = 39, perpendicular = 52; find hypotenuse.

20. Base = 35, hypotenuse = 91; find perpendicular.

21. Perpendicular = 72, hypotenuse = 75; find base.

22. A cord 287 ft. long is stretched from the top of a flagpole 63 ft. high; find the distance of the end in contact with the ground from the base of the pole.

The length of the diagonal of a room is the square root of the sum of the squares of the length, breadth, and height.

23. Find the diagonal of a room 28 ft. long, 21 ft. wide, and 12 ft. high.

24. Find the diagonal of a hall 50 ft. long, 30 ft. wide, and 15 ft. high.

Cube Root.

273. The *cube* of a number is the product of ***three*** factors, each equal to the number.

The cubes of	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
are	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

274. The **cube root** of a number is one of the *three equal factors* of the number.

Thus the cube roots of	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000,
are	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.

275. The cube root of a number is indicated by $\sqrt[3]{\ }$, or by the fraction $\frac{1}{3}$ written above and to the right of the number.

Thus, $\sqrt[3]{343}$, or $343^{\frac{1}{3}}$, means the cube root of 343.

276. Since $35 = 30 + 5$, the cube of 35 may be obtained thus:

$$
\begin{array}{l}
30 + 5 \\
\underline{30 + 5} \\
30^2 + (30 \times 5) \\
\underline{\quad + (30 \times 5) + 5^2} \\
30^2 + 2(30 \times 5) + 5^2 \\
\underline{30 + 5} \\
30^3 + 2(30^2 \times 5) + (30 \times 5^2) \\
\underline{\quad (30^2 \times 5) + 2(30 \times 5^2) + 5^3} \\
30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3
\end{array}
$$

$$
\begin{array}{rr}
30^3 = & 27{,}000 \\
3(30^2 \times 5) = & 13{,}500 \\
3(30 \times 5^2) = & 2{,}250 \\
5^3 = & \underline{125} \\
 & 42{,}875
\end{array}
$$

Hence the cube of any number composed of tens and units contains four parts:

I. *The cube of the tens.*

II. *Three times the product of the square of the tens by the units.*

III. *Three times the product of the tens by the square of the units.*

IV. *The cube of the units.*

277. In extracting the cube root of a number, the first step is to mark off the figures of the number in groups.

Since $1 = 1^3$, $1000 = 10^3$, $1{,}000{,}000 = 100^3$, and so on, it follows that the cube root of any integral number between 1 and 1000, that is, of any integral number that has *one*, *two*, or *three* figures, is a number of *one* figure; and that the cube root of any integral number between 1000 and 1,000,000, that is, of any integral number that has *four*, *five*, or *six* figures, is a number of *two* figures, and so on.

If, therefore, an integral number be divided into groups of three figures each, from right to left, the number of figures in the root will be equal to the number of groups. The last group to the left may consist of one, two, or three figures.

Extract the cube root of 42875.

$$\begin{array}{rr|l}
 & & 42\,875\,(35 \\
 & & 27 \\
\hline
3 \times 30^2 = & 2700 & 15\,875 \\
3 \times (30 \times 5) = & 450 & \\
5^2 = & 25 & \\
\hline
 & 3175 & 15\,875
\end{array}$$

Since 42875 consists of two groups, the cube root will consist of two figures.

The first group, 42, contains the cube of the tens' number of the root.

The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence 3 is the tens' figure of the root.

The remainder, 15875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence the 15875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the units + the square of the units. The larger part of this second factor is three times the square of the tens.

And, if the 158 hundreds of the remainder be divided by the $3 \times 30^2 = 27$ hundreds, the quotient will be the units' number of the root.

The second factor can now be completed by adding to the 2700 $3 \times (30 \times 5) = 450$ and $5^2 = 25$.

278. The same method will apply to numbers of more than two groups of figures, by considering the part of the root already found as so many tens with respect to the next figure of the root.

Extract the cube root of 57512456.

$$
\begin{array}{rr|l}
 & & 57\,512\,456\,(386 \\
 & & 27 \\
\hline
3 \times 30^2 = & 2700 & 30\,512 \\
3 \times (30 \times 8) = & 720 & \\
8^2 = & \underline{64} & \\
 & 3484 & \underline{27\,872} \\
 & & 2\,640\,456 \\
3 \times 380^2 = & 433200 & \\
3 \times (380 \times 6) = & 6840 & \\
6^2 = & \underline{36} & \\
 & 440076 & \underline{2\,640\,456}
\end{array}
$$

279. If the cube root of a number have decimal places, the number itself will have *three times* as many.

Thus, if 0.11 be the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contain a decimal, we divide the figures of the number into groups of three figures each, by beginning at the decimal point and marking toward the left for the integral number, and toward the right for the decimal. We must be careful to have the last group on the right of the decimal point contain *three* figures, annexing ciphers when necessary.

Extract the cube root of 187.149248.

	187.149 248(5.72
	125
$3 \times 50^2 = 7500$	62 149
$3 \times (50 \times 7) = 1050$	
$7^2 = 49$	
8599	60 193
	1 956 248
$3 \times 570^2 = 974700$	
$3 \times (570 \times 2) = 3420$	
$2^2 = 4$	
978124	1 956 248

It will be seen from the groups of figures that the root will have one integral and two decimal places, and therefore the decimal point must be placed in the root as soon as one figure of the root is obtained.

280. If the given number be not a perfect cube, ciphers may be annexed, and a value of the root may be found as near to the *true* value as we please.

Extract the cube root of 1250.6894.

	1 250.689 400(10.77
	1
$3 \times 10^2 = 300$	250

Since 300 is not contained in 200, the next figure of the root will be 0.

$3 \times 100^2 = 30000$	250 689
$3 \times (100 \times 7) = 2100$	
$7^2 = 49$	
32149	225 043
	25 646 400
$3 \times 1070^2 = 3434700$	
$3 \times (1070 \times 7) = 22470$	
$7^2 = 49$	
3457219	24 200 533
	1 445 867

281. The following method very much shortens the work in long examples.

Extract the cube root of 5 to five places of decimals.

	5.000(1.70997
	1
$3 \times 10^2 = 300$	4 000
$3(10 \times 7) = 210$	
$7^2 = 49$	
559	3 913
259	87 000 000
$3 \times 1700^2 = 8670000$	
$3(1700 \times 9) = 45900$	
$9^2 = 81$	
8715981	78 443 829
45981	8 556 1710
$3 \times 1709^2 = 8762043$	7 885 8387
	670 33230
	613 34301

After the first two figures of the root are found, the next trial divisor is obtained by bringing down the sum of the 210 and 49 obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

It is seen at a glance that, when the trial divisor is increased by 3 times the 17 tens of the root, it will be greater than 87000; so that 0 is placed in the root, and 3×1700^2 is obtained by annexing two ciphers to the 86700. Again: the trial divisor is obtained by bringing down the sum of the 45900 and 81, which was obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

The last two figures of the root are found by division. The rule in such cases is, that two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.

282. The cube root of a common fraction is found by taking the cube roots of the numerator and denominator; but, if the denominator be not a perfect cube, it is best to reduce the fraction to a decimal, and then extract the root.

Ex. 161.

Find the cube roots of:

1. 29791.	5. 53157376.	9. 12396.8834.	13. $\frac{64}{729}$.
2. 357911.	6. 62099136.	10. 0.00027.	14. $\frac{125}{343}$.
3. 148877.	7. 41.421736.	11. 0.00008.	15. $\frac{16}{27}$.
4. 103823.	8. 12.812904.	12. 277.2738.	16. $\frac{7}{8}$.

17. The liter contains 61.027 cu. in. Find the side of a cube containing a liter.

18. The edges of a rectangular solid are 154 ft. 11 in., 70 ft. 7 in., 53 ft 1 in. Find the edge of a cube equivalent to it.

The square of $(30+5) = 30^2 + 2(30 \times 5) + 5^2$. § 266.

The 30^2 may be represented by a square (Fig. 1) 30 in. on a side.

The $2(30 \times 5)$ may be represented by two strips 30 in. long and 5 in wide, of Fig. 2, which are added to two adjacent sides of Fig. 1.

The 5^2 may be represented by the small square of Fig. 3 required to make Fig. 2 a complete square.

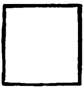


Fig. 1.

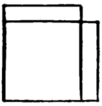


Fig. 2.

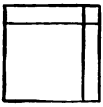


Fig. 3.

In extracting the square root of 1225, the large square, which is 30 in. on a side, is first removed, and a surface of 325 sq. in. remains.

This surface consists of two equal rectangles, each 30 in. long, and a small square whose side is equal to the width of the rectangles.

The width of the rectangles is found by dividing the 325 sq. in. by the sum of their lengths, that is, by 60, which gives 5 in.

Hence the entire length of the surfaces added is 30 in. + 30 in. + 5 in. = 65 in., and the width is 5 in.

Therefore the total area is $(65 \times 5) = 325$ sq. in.

The cube of $(30 + 5) = 30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3$. § 392

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3(30^2 \times 5)$ may be represented by three rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3(30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

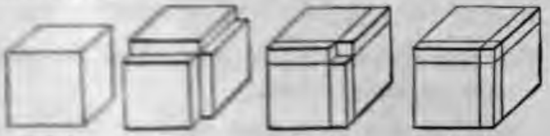


Fig. 1. Fig. 2. Fig. 3. Fig. 4.

In extracting the cube root of 42875, the large cube (Fig. 1), whose edge is 30 in., is first removed.

There remain $(42875 - 27000)$ cu. in. = 15875 cu. in.

The greater part of this is contained in the three rectangular solids which are added to Fig. 1, and which are each 30 in. long and 30 in. wide.

The thickness of these solids is found by dividing the 15875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence the sum of the products of two dimensions of all these solids is

For the larger rectangular solids,	$3(30 \times 30)$ sq. in. =	2700 sq. in.
For the smaller rectangular solids,	$3(30 \times 5)$ sq. in. =	450 sq. in.
For the small cube,	(5×5) sq. in. =	25 sq. in.
		3175 sq. in.

This number multiplied by the third dimension gives (5×3175) cu. in. = 15,875 cu. in.

283. In bodies of the same shape,

Two corresponding lines are in the same ratio as any other two.

The ratio of two corresponding surfaces is the square of the ratio of two corresponding lines.

The ratio of two corresponding volumes is the cube of the ratio of two corresponding lines.

Conversely,

The ratio of two corresponding lines is the square root of the ratio of two corresponding surfaces, and the cube root of the ratio of two corresponding volumes.

Ex. 162.

1. The volume of a rectangular solid is 1728 cu. in. The volume of a similar solid is 3375 cu. in. Find the ratio of two corresponding edges.
2. The surface of a solid is 600 sq. in. What is the surface of a similar solid whose edges are twice as great?
3. If the volumes of two similar solids be 100 cu. in. and 1000 cu. in. respectively, find the ratio of their heights to the nearest thousandth of an inch.
4. If two hills have the same shape, and one is 2700 ft. high, while the other is 3600 ft. high, find the ratio of their surfaces, and also the ratio of their volumes.
5. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.
6. The surfaces of two hills having the same shape are as 25 : 16. Find the ratio of their heights.
7. Of two similar solids, the volume of the larger is $1\frac{61}{64}$ of that of the smaller. Find the ratio of their heights; find also the ratio of their bases.
8. The equatorial diameter of the earth is 7926 mi. Find that of Venus whose volume is 0.953 of the volume of the earth.

CHAPTER XIV.

MENSURATION.

(PRACTICAL RULES.)

284. A **surface** has two dimensions: length and breadth.

285. A **solid** has three dimensions: length, breadth, and thickness.

286. The **area of a surface** is the number of units of surface which it contains, the unit of surface being a square which has a linear unit for each of its dimensions.

287. The **volume of a solid** is the number of units of volume which it contains, the unit of volume being a cube which has a linear unit for each of its three dimensions.

288. In writing the dimensions of surfaces and solids, the sign $\times$ is used for the word *by*, an accent (′) for the word *feet*, and two accents (″) for the word *inches*. Thus, the dimensions of the floor of a room, 15 feet 6 inches long, 13 feet 8 inches wide, are denoted by $15' \, 6'' \times 13' \, 8''$. The dimensions of a brick, 8 inches long, 4 inches wide, $2\frac{1}{2}$ inches thick, are denoted by $8'' \times 4'' \times 2\frac{1}{2}''$.

289. Rectangle. The area of a rectangle equals the product of its length and breadth. (See page 165.)

290. The **perimeter** of a rectangle or of any other surface figure is the sum of the lengths of the lines which bound it.

Ex. 163.

1. The floor of a room is a rectangle $15' 6'' \times 18'$. Find its perimeter and its area.
2. The ceiling of a room is a rectangle $16' \times 20'$. Find its perimeter and its area.
3. A rectangular field is 60 rods × 80 rods. Find its area in acres, and the cost of fencing it at $1.50 a rod.
4. How many boards 12 ft. long will be required to inclose a square field 48 rds. on a side with a fence 4 boards high? How many acres are there in the field?

291. Carpeting. Carpeting is sold by the yard in length. The common widths are a yard, and three-quarters of a yard. It will be remembered that in determining the number of yards of carpet for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of yards in a strip, including the allowance for waste in matching the pattern, multiplied by the number of strips will give the required number of yards. (See page 169.)

5. How many yards of carpeting $\frac{3}{4}$ of a yard wide will be required for a floor $20' \times 17' 6''$, if the strips run lengthwise, and if there is a waste of 9 in. a strip in matching the pattern?
6. How many yards of carpeting 1 yd. wide will be required for a room $18' 4'' \times 17' 8''$, if the strips run lengthwise of the room, and if there is a waste of 8 in. a strip in matching the pattern? Find the cost of carpeting the room if the carpet is worth 85 cents a yard, and 10 cents a yard is paid for making and laying.

[illegible] Find the cost of the carpet for a room 19' 8" × 17' 10", if the carpet is [illegible] yard wide and costs $1.75 a yard, the strips running across the room, and 9 in. a strip being wasted in matching the pattern.

[illegible] **Plastering.** The unit for measuring painting, plastering, and paving is the square yard. The practice in [illegible] is to find the total area within the [illegible] of the walls, to deduct from this amount [illegible] the area of all doors, windows, and other openings, and to take as the net area the nearest whole number of square yards in the remainder.

[illegible] A rectangular room is 15' × 12' 4" × 9'. The baseboard is 1 foot high; there is a door 7' 4" × 4', and two windows of [illegible] each. Find the cost of plastering the walls and ceiling at 18 cents a square yard.

Perimeter of room = [illegible]

Height of room above baseboard = 9' − 1' = 8'.

[illegible] = [illegible] sq. ft.

Area of ceiling = [illegible] = [illegible] sq. ft.

Area of walls and ceiling = [illegible] sq. ft.

Height of door above baseboard = [illegible]

Area of door above baseboard = [illegible] sq. ft.

Area of windows = [illegible] = 48 sq. ft.

Area of door and windows = [illegible] sq. ft.

Half the area of door and windows = [illegible] sq. ft.

Area allowed is

[illegible] sq. ft. − [illegible] sq. ft. = [illegible] sq. ft. = [illegible] = 69 sq. yds.

69 × 18 cents = $12.42. *Ans.*

3. Find the cost of plastering the walls and ceiling of a room [illegible] × [illegible] × 10' 4", at 30 cents per square yard, if [illegible] sq. yds. are deducted for doors, windows, and baseboard.

9. Find the cost of whitening the walls and ceiling of a room $16' 6'' \times 15' 6'' \times 9' 6''$, at five cents per square yard, deducting 12 sq. yds. for doors, windows, and baseboard.

10. Find the cost of plastering a room $18' \times 15' \times 10'$, at 30 cents per square yard, if the room contains one door $7' 6'' \times 4'$, three windows each $6' \times 4'$, and a baseboard one foot high around the room.

293. Wall Paper. Wall paper is 18 in. wide and is sold in single rolls 8 yds. long, or in double rolls 16 yds. long. In estimating the number of rolls of paper required for a room of ordinary height, find the number of feet in the perimeter of the room, leaving out the widths of the doors and windows, and allow a double roll or two single rolls for every 7 ft.

Ex How many double rolls of paper will be required for a room of ordinary height, $18' \times 16'$, with one door, and four windows, each 4 ft. wide?

Perimeter of room $= 2 \times 18' + 2 \times 16' = 68'$
Width of door and windows $= 5 \times 4' = \underline{20'}$
Deducting door and windows $= 48'$
$\frac{48}{7} = 7.$ 7 double rolls. *Ans.*

11. How many double rolls of paper will be required for a room of ordinary height, $18' 4'' \times 16' 6''$, with two doors and three windows, each 4 ft. wide?

12. Find the cost of paper at 25 cents a single roll, and bordering at 8 cents a yard, for a room of ordinary height, $17' 9'' \times 17' 3''$, allowing for one door and four windows, each $4' 2''$. (No allowance for doors and windows is made for the bordering.)

13. Find the cost of paper at 50 cents a single roll for a room of ordinary height, 20′ 8″ × 17′ 6″, with two doors and three windows, each 4′ 2″ wide.

294. Laths. Laths are put up in bundles, 100 pieces, each 4 ft. long, and a bundle is estimated to cover 5 sq. yds. In estimating the number of bundles of laths, deduct the whole area of all openings.

Ex. How many bundles of laths will be required for the ceiling of a room 36 ft. square?

36 ft. = 12 yds.
Hence ceiling contains 12 × 12 = 144 sq. yds.
$\frac{144}{5} = 29.$ 29 bundles. *Ans.*

14. How many bundles of laths will be required for the ceiling and walls of a room 26 ft. square, 14 ft. high, allowing 20 sq. yds for doors, windows, and baseboard?

15. How many bundles of laths are required for the ceiling and walls of a room 28′ × 32′ and 16′ high, allowing for three windows 8′ × 3′ 6″ each, and two doors 8′ × 4′ 2″ each, and a baseboard 1 ft. high?

295. Clapboards. Clapboards are 4 ft. long and are laid $3\frac{1}{2}$ or 4 in. to the weather.

Ex. Find the number of clapboards required for the front of a house 42 ft. long and 22 ft. high, allowing 100 sq. ft. for doors and windows, and adding 10% for waste.

$3\frac{1}{2}$ in. $= \frac{3\frac{1}{2}}{12}$ ft. $= \frac{7}{24}$ ft.
$4 \times \frac{7}{24} = \frac{28}{24} = 1\frac{1}{6}$ sq. ft. for each clapboard.
42 × 22 = 924 sq. ft.
924 sq. ft. − 100 sq. ft. = 824 sq. ft.

$\frac{824}{1\frac{1}{6}} = \frac{6}{7}$ of $824 = 706$.
10% of $706 = 71$.
$706 + 71 = 777$. *Ans.*

16. How many clapboards will be required for the front of a house 40 ft. long and 20 ft. high, allowing 96 sq. ft. for doors and windows, and adding 10% for waste?

296. Roofing and Flooring. The *unit* of measure for roofing and flooring is a *square containing* 100 *sq. ft.* Shingles are 16 in. long, and are estimated to average 4 in. wide, so that a shingle laid 4½ in. to the weather would cover 18 sq. in., and 8 shingles would be required for 1 sq. ft. At this rate 800 shingles would cover a square, but to allow for waste it is usual to reckon 1000 shingles to the square. It is found, however, in practice, that 1000 shingles of the best quality, laid 4½ in. to the weather, will cover about 120 sq. ft.

17. Allowing 1000 shingles for 120 sq. ft., how many thousand would be required to cover the pitched roof of a house 44 ft. long, if the width of each side of the roof is 24 ft.?

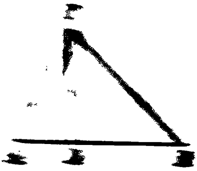
18. Allowing 1000 shingles for 110 sq. ft., how many thousand would be required to cover the pitched roof of a building 54 ft. long, if the width of each side of the roof is 28 ft.?

19. How many slates at 3 to the square foot will be required to cover 28 squares of roof?

20. The floor of a gymnasium is $100' \times 60'$. Find the cost of birch for the floor at $40 a thousand, adding 20% for waste.

[illegible]

[illegible] by [illegible] the figure [illegible] The side AB [illegible] C opp- [illegible] and the per- [illegible] C to AB, [illegible]



[illegible]

[illegible]

[illegible] the product of [illegible]

[illegible] a triangle are given, [illegible]

[illegible] each side sepa- [illegible] product of the half-sum and [illegible] product is [illegible]

[illegible] base 16 ft. and [illegible]

[illegible] sq. ft. Ans.

[illegible] sides are 6, 8, and [illegible]

[illegible] sq. ft. Ans.

[illegible] having given:

[illegible]

[illegible] altitude [illegible]

[illegible] altitude 8 chains 50 links.

[illegible]

[illegible] 120 ft.

299. *The area of any surface figure bounded by straight lines can be found by dividing the figure into triangles, computing the areas of these triangles, and taking the sum of these areas.*

300. To find the area of a circle.

Multiply the square of the radius by 3.1416. (See page 168.)

Find the area of a circle, having given:

26. Radius 14 ft.
27. Radius 7 yds.
28. Diameter 32 yds.
29. Diameter 40 ft.
30. Diameter 100 rds.

301. To find the volume of a rectangular solid.

Multiply the area of the base by the altitude; that is, *take the product of its three dimensions.* (See page 174.)

Find the volume of:

31. A cube whose edge is 3 in.
32. A cube whose edge is 14 in.
33. A cube whose edge is 2 ft.
34. A rectangular solid $10'' \times 8'' \times 6''$.
35. A rectangular solid $6' \times 5' \times 4'$.
36. A rectangular solid $4'\,8'' \times 3'\,10'' \times 3'\,6''$.
37. If a cellar which measures $32' \times 28'$ is flooded to a depth of 4 in., what is the weight of the water, allowing 1 cu. ft. of water to weigh 1000 oz.?

302. To find the number of gallons that a cistern of given dimensions will hold.

Find the number of cubic inches in the cistern, and divide this number by 231.

Ex. If a rectangular cistern is $4' \times 3' 4'' \times 3'$, how many gallons of water will it hold?

4 ft. = 48 in.   3 ft. 4 in. = 40 in.   3 ft. = 36 in.
Since a gallon is 231 cu. in., the number of gallons is

$$\frac{48 \times 40 \times 36}{231} = 299.2. \qquad 299.2 \text{ gals. } \textit{Ans.}$$

303. Since a cubic foot contains $\frac{1728}{231}$ gals. = 7.48 gals., we may find the number of gallons of water a cistern will hold as follows:

Express the dimensions in feet, and multiply the continued product of these dimensions by $7\frac{1}{2}$, and take from the result $\frac{1}{2}$ of 1% of it.

304. To find the number of barrels of water a cistern will hold.

Divide the number of cubic feet the cistern contains by 4.21.

Ex. Find the number of gallons in a cistern 6 ft. square and 4 ft. deep.

$$\begin{array}{rr}
6 \times 6 \times 4 = & 144 \\
 & 7\frac{1}{2} \\
\hline
 & 1080 \\
\frac{1}{2} \text{ of } 1\% = & 5.4 \\
\hline
 & 1074.6 \text{ gals. } \textit{Ans.}
\end{array}$$

Ex. Find the number of gallons that a round cistern 7 ft. in diameter and 7 ft. deep will hold.

$$\begin{array}{rr}
\text{Area of base} = 3.1416 \times 3.5 \times 3.5 = & 38.4846 \text{ sq. ft.} \\
 & 7 \\
\hline
 & 269.3922 \text{ cu. ft.} \\
 & 7.5 \\
\hline
 & 2020.44150 \\
\frac{1}{2} \text{ of } 1\% = & 10.10 \\
\hline
 & 2010. \text{ gals. } \textit{Ans.}
\end{array}$$

38. How many gallons of water will a cistern hold that is $5\frac{1}{2}$ ft. long, $3\frac{3}{4}$ ft. wide, and 4 ft. deep?

39. How many barrels of water will a cistern hold that is 13 ft. long, 8 ft. wide, and 7 ft. deep?

40. How many barrels of water will a cistern hold that is 12 ft. long, 9 ft. wide, and 6 ft. deep?

41. Find the number of gallons that a round cistern will hold, 8 ft. in diameter and 7 ft. deep.

42. Find the number of barrels in a round cistern 21 ft. in diameter and 10 ft. deep.

305. To find the number of bushels of grain in a bin.

A bushel = 2150.42 cu. in., and 0.8 of this equals 1720.336 cu. in. If we add to 1720 $\frac{1}{2}$ of 1% of 1720, rejecting the decimals, we obtain 1728 cu. in.

Hence, *take* 0.8 *of the number of cubic feet in the bin, and add to the result* $\frac{1}{2}$ *of* 1% *of it.*

Ex. Find the number of bushels in a bin 12 ft. long, 8 ft. wide, and 5 ft. high.

$$
\begin{array}{lr}
12 \times 8 \times 5 = & 480 \\
 & \underline{0.8} \\
 & 384.0 \\
\frac{1}{2} \text{ of } 1\% = & \underline{1.92} \\
 & 385.92 \text{ bu. } \textit{Ans.}
\end{array}
$$

43. Find the number of bushels in a bin 20 ft. long, 6 ft. wide, and 4 ft. high.

44. Find the number of bushels in a bin $8\frac{1}{2}$ ft. long, $5\frac{1}{2}$ ft. wide, and 4 ft. high.

45. Find the number of bushels in a bin 8 ft. long, $6\frac{3}{4}$ ft. wide, and $4\frac{1}{2}$ ft. high.

306. To express in cubic feet a given number of bushels.

To the number of bushels add $\frac{1}{4}$ *of the number, and subtract from the sum* $\frac{1}{2}$ *of* 1% of it.

Ex. To find the number of cubic feet required for 1200 bu.

$$\begin{array}{rr} & 4)\underline{1200} \\ & \underline{300} \\ & 1500 \\ \frac{1}{2} \text{ of } 1\% = & \underline{7.5} \\ & 1492.5 \text{ cu. ft. } \textit{Ans.} \end{array}$$

46. How many cubic feet in a bin that will hold 400 bu.?

47. How many cubic feet in a bin that will hold 372 bu.?

48. How many cubic feet in a bin that will hold 1326 bu.?

307. To find the number of bushels in a load of charcoal.

Multiply the continued product of the length, width, and height, expressed in feet, by 0.8 *and add to the result* ½ *of* 1% *of it.*

49. How many bushels of charcoal in a load 8 ft. long, 4 ft. wide, and 6 ft. high?

50. Find the number of bushels in a load of charcoal that is 8 ft. long, 4½ ft. wide, and 6 ft. high.

308. To measure wood.

Find the product of the length, width, and height, expressed in feet, and divide this product by $8 \times 4 \times 4$. *The result is the number of cords.*

Ex. Find the number of cords in a pile of wood 24 ft. long, 4 ft. wide, and 6 ft. high.

$$\frac{\overset{3}{\cancel{24}} \times \cancel{4} \times \overset{3}{\cancel{6}}}{\cancel{8} \times \cancel{4} \times \underset{2}{\cancel{4}}} = 4\tfrac{1}{2} \text{ cords. } \textit{Ans.}$$

51. What part of a cord does a load of wood contain which is 8 ft. long, 4 ft. wide, 3½ ft. high?

52. What part of a cord does a load of wood contain which is 8 ft. long, 3 ft. 8 in. high, if the average length of the sticks is only 3 ft. 8 in.?

53. Find the number of cords in a pile of wood 120 ft. long, 4 ft. wide, and 6 ft. high.

54. How much should be paid for a pile of 4-foot wood, 100 ft. long, and averaging 5 ft. high, at $5 a cord?

309. To measure coal.

A short ton of anthracite coal measures about 37 *cu. ft. A long ton of soft coal measures about* 48 *cu. ft., a short ton about* 42 *cu. ft. A bushel of hard coal weighs* 80 *lbs. A bushel of soft coal weighs* 70 *lbs.*

55. How many short tons of hard coal will a rectangular bin hold, 9 ft. long, 6 ft. 6 in. wide, and 6 ft. high?

56. How many short tons of hard coal can be put into a rectangular bin 8 ft. long, 7 ft. wide, and 6 ft. high?

57. How many long tons of soft coal can be put into a rectangular bin 12 ft. long, 9 ft. wide, and 7 ft. high?

310. To measure sand, gravel, and earth.

A cubic yard of earth is called a load.

58. How many loads are there in a rectangular embankment 200 ft. long, 15 ft. wide, and 10 ft. high?

59. How many loads in an embankment 150 ft. long, 20 ft. wide, and 5 ft. high?

311. To measure brickwork.

Brickwork is estimated by the thousand, reckoning 22 *bricks laid in mortar to the cubic foot.*

60. How many bricks will be required to build a wall 84 ft. long, 32 ft. high, and 1 ft. thick?

61. How many bricks will be required for the walls of a house 42 ft. long, 32 ft. wide, and 21 ft. high, if the

walls are 1 ft. thick, and there are deducted 2 doors 7′ 6″ × 4′ each, and 16 windows 5′ × 4′ each?

NOTE. In finding the perimeter of the building, measure the exterior.

312. To measure stone masonry.

Stone masonry is reckoned by the cubic foot, or by the perch of 25 cu. ft.

62. How many cubic feet of stone masonry in the foundation of a house 40′ × 30′, if the foundation is to be 4 ft. high and 2 ft. thick?

63. How many perches of stone are required for the foundation of a building 100′ × 60′, if the foundation is 6 ft. high and $2\frac{1}{2}$ ft. thick?

313. To measure boards and dimension lumber.

Boards one inch or less in thickness are sold by the square foot. Boards more than one inch in thickness and all squared lumber are sold by the number of square feet of boards one inch in thickness to which they are equivalent.

Thus, a board 12 ft. long, 1 ft. wide, and 1 in. thick, contains 12 ft. board measure. If only $\frac{3}{4}$ or $\frac{1}{2}$ or $\frac{1}{4}$ of an inch thick, it still contains 12 ft. board measure; but if $1\frac{1}{4}$ in. thick, it contains $1\frac{1}{4} \times 12 = 15$ ft. board measure. Hence, for boards more than an inch thick and squared lumber:

Express the length and width in feet, and the thickness in inches. The product of these three numbers will be the number of feet board measure.

In practice, the width of a board, unless sawed to order, is reckoned only to the next smaller half-inch. Thus, a width of $11\frac{3}{8}$ inches is reckoned 11 inches; of $13\frac{5}{8}$ or $13\frac{3}{4}$ inches, is reckoned $13\frac{1}{2}$ inches.

Ex. How many feet in a 2-inch plank, 18 ft. long and 14 in. wide?

14 in. = $1\frac{1}{6}$ ft. $\quad 2 \times 1\frac{1}{6} \times 18 = 42$ ft. board measure. *Ans.*

64. How many feet board measure in 8 planks, 4 in. thick, 18 ft. long and 16 in. wide?

65. How many feet board measure in a stick of timber 1 ft. square and 20 ft. long?

66. How many feet board measure in 40 joists $10'' \times 2''$ and 12 ft. long?

314. To measure round logs. Round logs are sold by the amount of square lumber that can be cut from them, according to calipers now in use. When logs do not exceed 16 ft. in length, the length and the diameter of the small end are taken, and a table stamped upon the calipers gives the number of feet board measure. This table may be calculated as follows:

Express the diameter in inches, subtract twice the diameter from the square of the diameter, and $\frac{21}{40}$ *of the remainder will be the number of feet board measure in a log* 10 *ft. long.*

The formula is $\frac{21}{40}(d^2 - 2d)$, in which d stands for the diameter of the log in inches.

Ex. Find the number of feet board measure in a log 16 ft. long and 20 inches in diameter.

$20^2 - 2 \times 20 = 400 - 40 = 360.$

$\frac{21}{40}$ of $360 = 189.$

$\frac{16}{10}$ of $189 = 302.4$ ft. board measure. *Ans.*

By this rule find the number of feet board measure in:

67. A log 12 ft. long and 16 in. in diameter.

68. A log 13 ft. long and 12 in. in diameter.

69. A log 14 ft. long and 20 in. in diameter.

70. A log 15 ft. long and 15 in. in diameter.

315. Oak and other heavy timber.

Large heavy timber of hard wood is generally sold by the ton, signifying 50 *cu. ft. or* 600 *ft. board measure.*

316. To find the contents of a cask.

Subtract the diameter of one of the heads from the bung diameter expressed in inches, and multiply the difference by 0.65; *to the product add the head diameter, and this will give the mean diameter.*

Square the mean diameter and multiply it by the length in inches. Divide this product by 294. *The quotient is the number of gallons the cask will hold.*

71. Find the number of gallons contained in a cask of which the bung diameter is 24 in., head diameter 20 in., and the length 36 in.

72. Find the number of gallons contained in a cask of which the bung diameter is 30 in., head diameter 26 in., and the length 38 in.

317. To find the volume of an irregular body.

Immerse the body in a vessel full of water. Remove the body and calculate the amount of water displaced.

318. To find the surface of a sphere.

Multiply the square of the diameter by 3.1416.

73. How many square inches on the surface of a ball 4 in. in diameter?

74. How many square inches on the surface of a globe 18 in. in diameter?

319. To find the volume of a sphere.

Multiply the cube of the diameter by 0.5236 (that is, $\frac{1}{6}$ of 3.1416).

75. Find the volume of a globe 2 ft. in diameter.

CHAPTER XV.

Ex. 164.

MISCELLANEOUS PROBLEMS.

1. Fifteen men and eight boys together earn \$342 a week. If a boy's pay is half a man's pay, what are the daily wages of a man, and also of a boy?

2. A man divides \$1622.50 among four persons so that the first has \$40 more than the second, the second \$60 more than the third, and the third \$87.50 more than the fourth. Find the part of the fourth.

3. A family of six persons makes \$8.75 a day, and works 304 days in the year. At the end of the year each member of the family puts \$80 in a savings bank. Find the daily expense of the family.

4. A man bought 5.5 yds. of cloth for \$35. In having a suit made from it, he found that he lacked 1.75 yds., which he procured at the price per yard of his first purchase. What is the cost of the suit if the trimmings cost \$6.50 and the making \$15?

5. A man has 76.25 yds. of linen, worth 44 cts. a yard, made into shirts. It takes 3.05 yds. for a shirt, and the price for making is 50 cts. a shirt. Find the cost of a shirt, and the number he has made.

6. A man's expenses from the first of January to the end of October 17 are \$1845.50. How much must he diminish his daily expense in order that the total expense for the year shall not exceed \$2200?

7. A quart contains 1000 beans of average size, and a field is planted with 22 rows of 800 hills each, with 6 beans in a hill. The increase is tenfold. What is the value of the crop at $3 a bushel? (There are 32 quarts in a bushel.)

8. For making 25 gallons of ordinary beer 60 pounds of barley and .65 of a pound of hops are needed. If the barley costs $1.50 for 60 pounds, and the hops cost 18 cents a pound, what is the profit to the brewer on a cask of 42 gallons if he sells it for $5 and reckons his labor $1.50?

9. A person receives his income quarterly. The first quarter he receives $[illegible], the second $1535.20, the third $856.44, the fourth $725.19. His expenses for these quarters are respectively $686.60, $734.25, $589.15, $849.65. How much does he save for the year?

10. A hen lays on an average 120 eggs a year worth 24 cents a dozen. She eats a quart of barley every 5 days. The barley is worth 56 cents a bushel. What is the annual profit from this hen?

11. A square garden measuring on each side 40.50 yards is enclosed by three lines of galvanized iron wire. Eight yards of this wire weigh a pound, and it is worth 7.5 cents per pound. What is the cost of the wire?

12. A family composed of five persons consumes daily one pound of stale bread for each person, or 1.15 pounds of fresh bread. If bread is worth 5 cents a pound, find the annual saving which this family will make if it eats stale bread altogether.

13. It is estimated that in France 240,000 women and girls are employed in making lace. The annual production has a value of $13,000,000, and the value of the raw material is 0.27 of the value of the lace. Find the average daily wages of these women and girls, supposing that each works 240 days in the year.

14. The salt water which is obtained from the bottom of a mine of rock salt contains 0.09 of its weight of pure salt. What weight of salt water is it necessary to evaporate in order to obtain 4734 pounds of salt?

15. The weight of ashes from the burning of oak wood is 0.03 of the weight of the wood, and the weight of carbonate of potash contained in the ashes is 0.065 of the weight of the ashes. Find the weight of carbonate of potash from 1170 pounds of wood.

16. The weight of sugar from the sugar beet is nearly 0.06 of the weight of the beet. If an acre produces 30,000 pounds of beets that are sold at the rate of $2 a thousand pounds, how many acres of land is it necessary to sow to furnish beets to a sugar factory which produces 150,000 pounds of sugar a year, and what will the be value of the crop obtained?

17. If a workman has taken every day for the last 12 years two glasses of beer at 5 cents a glass, how much could he have saved if he had not indulged this habit, reckoning 365 days each year?

18. A woman has three children. She pays for each $15 a year for having their clothes made, $1.50 a month for mending, and $0.35 a week for washing. How much could she save in a year if she knew how to wash, make clothes, and mend?

19. A sheep raiser shears his sheep at an expense of 11 cts. a head. The sheep average 8 lbs. of wool which he sells for 23 cts. a pound. He finds that his net profit after paying for the shearing is $1297.50. How many sheep has he?

Common Fractions.

20. Find the prime factors of 41,580.

21. Find the G.C.M. of 144, 126, 108.

22. Find the L.C.M. of 18, 90, 60, 24.

23. Find the L.C.M. of 14, 35, 343.

24. At $16\frac{1}{4}$ cts. a yard. what will $3\frac{1}{2}$ yds. of cloth cost?

25. A man has $376\frac{2}{3}$ quarts of berries, which he wishes to put into boxes holding $2\frac{1}{4}$ qts. each. How many boxes will be required, and what part of a box will be left over?

26. If a man earns $\$2\frac{3}{4}$ a day, how many days will it take him to earn $100?

27. A lady has $37\frac{1}{2}$ qts. of berries to can. If each can holds $2\frac{3}{4}$ qts., how many cans of berries will she have, and what part of another can will there be over?

28. If a man walks $4\frac{2}{3}$ miles an hour, how many hours will it take him to walk $40\frac{3}{4}$ miles?

29. Some boys wanted a long rope to use on the ice. They made the rope by taking off their sled-ropes and tying them together. The first sled-rope was $2\frac{3}{4}$ yds. long, the second $3\frac{1}{2}$ yds., the third $2\frac{7}{8}$ yds., the fourth $5\frac{3}{4}$ yds., and the fifth $3\frac{2}{15}$ yds. If the whole

shortened $1\frac{1}{8}$ yds. by the knots, from

tying the sled-ropes together, how long was the rope?

30. A lady bought $3\frac{7}{8}$ yds. of cotton cloth, $4\frac{1}{3}$ yds. of calico, $16\frac{2}{5}$ yds. of flannel, and $12\frac{1}{8}$ yds. of gingham. How many yards did she buy in all?

31. A boy went to a store with $5.75 in his purse. He bought $3\frac{1}{4}$ lbs. of butter at 28 cts. a pound, $13\frac{1}{2}$ lbs. of sugar at 11 cts. a pound, and $1\frac{1}{2}$ lbs. of coffee at 35 cts. a pound. How much money did he have left?

32. Four boys went fishing, and caught 40 trout; the first caught $\frac{2}{5}$ of the whole, the second $\frac{1}{8}$, and the third $\frac{1}{4}$. How many did the fourth boy catch?

33. George has his choice to be one of 3 boys to receive 8 oranges, or one of 4 boys to receive 11 oranges. Which shall he choose?

34. Five girls pick blueberries together; the first picks $7\frac{2}{3}$ qts., the second $5\frac{9}{10}$ qts., the third $12\frac{3}{4}$ qts., the fourth $8\frac{1}{6}$ qts., and the fifth $3\frac{7}{8}$ qts. How much will they all together get for their berries, at $12\frac{1}{2}$ cts. a quart?

35. A farmer puts the following lots of apples into 6 bins: namely, $6\frac{3}{4}$ bu., $18\frac{1}{2}$ bu., $25\frac{7}{8}$ bu., $19\frac{3}{8}$ bu., $143\frac{3}{4}$ bu., $976\frac{1}{4}$ bu., $25\frac{1}{2}$ bu. How many bushels will there be for each bin?

Compound Quantities.

36. How many rods are there in 4379 ft.?

37. Reduce 9,627,834 ft. to yards, rods, etc.

38. Reduce 96,284 sq. in. to square feet.

39. Reduce 15 sq. rds. 3 sq. yds. 18 sq. ft. 3 sq. in. to square inches.

40. What will 1000 sq. ft. of land cost at $67 an acre?

41. What will 20 sq. yds. of land cost at 75 cts. a square foot?

42. How much less will 15 acres of land cost, at $16 an acre, than 96,342.42 sq. ft. at 5 cts. a foot?

43. How many acres in a rectangular piece of land 9634 ft. long and 3840 ft. wide?

44. A pile of four-foot wood is 4 ft. high and 75 ft. long. How many cords of wood are there in the pile?

45. In a woodshed there is a pile of wood 12 ft. long and 10 ft. high. If the sticks average a foot in length, what part of a cord is there in the pile?

46. What will 7 bu. 3 pks. of blueberries bring at 9 cts. a quart?

47. How many gallons of milk, at 8 cts. a quart, can be bought for $7.37?

48. How many quarts of water will a tin box hold that is 13 in. long, 6 in. wide, and 7 in. deep?

49. The total net weight of several loads of hay is 63,782 lbs. How many tons in all the loads of hay?

50. If an ounce of candy is worth 5 cts., what will 5 lbs. cost at the same rate?

51. Reduce 9 dys. 5 hrs. 16 min. to seconds.

52. Reduce 948,741 min. to higher denominations.

53. How many weeks between Jan. 1 and Nov. 1?

54. A boy has 10 mi. to go. After he has gone 6 mi. 48 rds. 12 ft., how much of his journey has he still to go?

55. A lady bought 4 remnants of cloth; the first contained $9\frac{1}{4}$ yds., the second 4 yds. 11 in., the third $6\frac{1}{3}$ yds., and the fourth $5\frac{1}{2}$ yds. How much cloth did she buy in all?

56. A certain basket holds 1 bu. 3 pks. 7 qts. A farmer raises enough of yellow-eyed beans to fill this basket 7 times. How many bushels does he raise?

57. A farmer cuts 26 loads of hay, which average 1 t. 436 lbs. How many tons does he cut in all?

58. What is $\frac{1}{25}$ of 9 mi. 5 rds. 13 ft.?

59. Three men in company buy 175 t. 19 cwt. 36 lbs. of hay. What is each man's share?

60. Seven boys together pick 4 bu. 3 pks. 7 qts. of berries. What is each boy's share?

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61. Bought 9 lbs. of sugar at 13 cts. a pound, 18 yds. of cloth at 33 cts. a yard, 4 doz. eggs at 29 cts. a dozen, and 5 lbs. of butter at 32 cts. a pound. What change should I receive from a ten-dollar bill given in payment?

62. How many quarts of berries, at 12 cts. a quart, will it take to pay for 8 yds. of cloth, at $16\frac{1}{2}$ cts. a yard?

63. A basket of peaches is half a bushel; how many bushels are there in 250 car-loads of 500 baskets each?

64. A fast railway train in England went 186 mi. 240 rds. in 3 hrs. What was the rate per hour?

65. If a man could proceed to the moon at the same rate per hour as the train went in example 64, how many hours would it take him, reckoning the distance 239,000 miles?

66. In one bin there are 23 bu. 2.48 pks. of wheat, and in another 141 bu. 2 pks. If $\frac{3}{4}$ of the wheat in the first bin is put into the second, how much wheat will there be in the second bin?

67. A load of four-foot wood is $3\frac{1}{2}$ ft. high and 7 ft. long. What is it worth at the rate of $6.40 a cord?

68. In one field there are $17\frac{1}{4}$ A., in a second there are 49 sq. rds., and a third field is 1740 ft. long and 927 ft. wide. What is the area of the three fields together?

69. A bin contains 164 bu. 3 pks. 2 qts. of oats. How long will these oats last if there are taken out 3 qts. of oats three times a day?

70. From a barrel containing $27\frac{3}{4}$ gals. of oil, 3 qts. a day were taken out for 3 weeks. How many gallons were left in the barrel at the end of that time?

71. If from a barrel of oil holding 27 gals. 2 qts. 1 pt. there is drawn out a can full, holding 1 gal. 2 qts. 1 pt., every day, how many days will the oil last?

72. Reduce $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{14}{19}$ of a mile to rods.

73. Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $3\frac{1}{2}$ in. to the fraction of a yard.

Special Problems.

If a man can do a piece of work in 5 dys., in one day he can do $\frac{1}{5}$ of the work; and if another man can do the same work in 4 dys., in one day he can do $\frac{1}{4}$ of it.

Therefore, both men together can do $\frac{1}{5} + \frac{1}{4} = \frac{9}{20}$ in one day.

Hence they will do $\frac{1}{20}$ in $\frac{1}{9}$ of a day, and therefore the whole work in $\frac{20}{9}$ days, that is, in $2\frac{2}{9}$ days.

74. If A can do a piece of work in 4 dys., B in 5 dys., and C in 7 dys., in how many days will they do it, all working together?

75. A can do a piece of work in 2 hrs., B in $2\frac{1}{2}$ hrs., and C in $3\frac{1}{2}$ hrs. How much of the work can they do in 20 min., all working together?

76. If A and B can do a piece of work in 18 dys., A and C in 12 dys., and B and C in 9 dys., find the number of days that it will take them, all working together.

77. A can do a piece of work in 6 dys., B in 8 dys., and C in 10 dys. How much of it can they do in 2 dys. together?

78. A cistern can be filled by means of a water-pipe in 30 min., and can be emptied by a waste-pipe in 20 min. If the cistern is full, and both pipes are open, in what time will it be emptied?

79. From Paris to Berlin by railway it is 1308^{km}. A kilometer is 1093.63 yds. Express the distance between Paris and Berlin in miles and yards.

80. Mercury revolves around the sun in 87.9692580 dys. Express the period of revolution in days, hours, minutes, and seconds.

81. The Roman foot was 0.97075 of our foot. The Greek foot was $\frac{25}{24}$ of the Roman foot. Find the length in inches of the Greek foot.

82. The radius of a circle is 0.1591549 of its circumference, which contains 360°. Find the angle at the centre whose arc is equal to the radius.

83. Find the L.C.M. of all the multiples of 3, from 6 to 27, inclusive.

84. Arrange $\frac{2}{3}$, $\frac{25}{39}$, and $\frac{53}{80}$ in order of magnitude.

85. Subtract the sum of $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{14}{15}$ from 5.

86. Find the decimal which, when added to the difference of $\frac{9}{200}$ and 0.002775, produces the square of 0.215.

87. A, at the rate of $4\frac{1}{4}$ miles an hour, walks a certain distance in $3\frac{1}{10}$ hrs. In what time will B walk the same distance at the rate of $\frac{3}{4}$ of $5\frac{1}{6}$ miles an hour?

Percentage.

88. A house worth $15,000 sustains injury from fire to the amount of $3840. What is the rate per cent of loss?

89. A and B have each $350; A spends 16% and B spends 20%. A's expenditure is what per cent of B's?

90. A gentleman having a court 20 ft. by 40 ft. enlarged it 10% in each dimension. Find the per cent of increase in area.

91. A young man buys a farm for $5200, which sum is 30% more than a legacy received from his grandfather, Required the amount of the legacy.

92. A lady gave to her daughter 25% and to her son 20% of her estate. The difference between the shares of the son and daughter was $1500. What is the value of the estate?

93. If a quart of Jersey milk is worth 10 cts., and produces 1 gi. of cream worth 25 cts. a pint, what per cent of the value of the milk is the value of the cream?

94. A farmer raised 360 bu. of potatoes, and the crop was 2400% of the seed. How many bushels did he plant?

95. A man received from a bankrupt $937.50, which was $37\frac{1}{2}$% of the sum due. What was the sum due?

96. What per cent of $\frac{3}{4}$ is $\frac{1}{2}$?

97. If 200% of a number is $\frac{1}{5}$% of 70, what is the number?

98. A grocer sold 10% of his stock of sugar, and then 25% of the remainder, after which he had 3 t. 1560 lbs. How much sugar had he at first?

99. A man lost $37\frac{1}{2}$% of his money. He then earned $50, and had 125% of what he had at first. How much did he have at first?

100. A merchant bought a cask of molasses from which 20% of the molasses had been drawn. He sold $30\frac{1}{4}$ gals., and then the cask was one-quarter full. Find the capacity of the cask in gallons.

101. What per cent of a common year is the time from July 1 to November 23, both days included?

102. A horse and chaise together are valued at $225; the horse is worth 25% more than the chaise. Find the value of the horse.

103. A man owning 30% of a mine sold 50% of his share for $3000. What was the value of the mine?

104. For what price per pair must shoes be sold to gain 25%, if 15% is lost when they are sold at $1.275 per pair?

105. If $\frac{1}{5}$ of goods valued at $1500 are sold at a loss of 10%, what must the remainder bring to gain 20% on the whole?

106. A fruit dealer bought 200 apples at the rate of 4 for a cent, and 200 at 5 for a cent. He sold them all at 5 for 3 cents. What per cent did he gain on his investment?

107. If 75% of the price of a bushel of corn is 50% of the price of a bushel of wheat, how many bushels of corn can be bought for $24 when wheat is worth $1.20 a bushel?

108. A horse dealer sold a horse for $90, and lost 25% of the cost of the horse. He sold another horse at an advance of 20% on the cost, and gained as much as he lost on the first horse. What was the selling price of the second horse?

109. If 20 men can build a wall in 9 dys., what per cent of the number of men could build the wall in 12 dys.?

110. If 7% of a ton of butter costs $42, what per cent of a ton can be bought for $57?

111. Five hundred barrels of flour were sold for $4125, at a profit of 10%. Find the cost per barrel.

112. An agent makes 20% by selling a book for 72 cts. If he had sold it for $1, what per cent would he have made?

113. A merchant bought from a shoe dealer 12 cases of shoes, each containing 60 pairs, at $87\frac{1}{2}$ cts. per pair, and sold the whole for $756. Find his gain per cent.

114. If 196 sq. rds. are 40% of the area of a field 30 rds. in length, what is the width of the field?

115. When brooms are $5.50 a dozen, what will be paid for $18\frac{3}{4}$ gross, if a discount of 10% is allowed on the bill for cash?

116. A grocer bought, at 60 cts. per gallon, 16 hhds. of molasses of 63 gals. each, and sold it at a profit of $120.96. What was his gain per cent?

117. A merchant in his first year of business increased his capital 40%, and increased his capital the second year 30%. He lost $33\frac{1}{3}$% of his capital the third year, and had \$18,200 left. What was his capital at first?

118. A contractor engaged to build a railroad at \$31,200 a mile. The work actually cost \$90 per rod. What was his gain per cent?

119. A merchant sold goods at 25% discount and 4% off from the selling price for cash. What was the whole per cent discount?

120. At $1\frac{1}{2}$% commission an agent receives \$97.29 for selling goods. Find the amount of the sale.

121. A merchant sent \$30,750 to his agent in New Orleans, for the purchase of cotton. Find the sum spent for cotton, if the agent charges $2\frac{1}{2}$% commission for buying.

122. Find the sum paid for insurance, at $\frac{1}{2}$%, on a house worth \$8000, and at $\frac{3}{4}$% on furniture worth \$2000, if the insurance is on $\frac{7}{8}$ of the value of the property insured.

123. A sea captain paid \$345, at $1\frac{1}{2}$%, for insuring $\frac{5}{8}$ of the value of a ship. Find the value of the ship.

124. A town has to raise \$192,000 for expenses. If 4% is allowed for collecting, how much money must be raised?

125. A merchant sends \$24,600 to his agent at St. Louis, for the purchase of flour at \$5 a barrel. How many barrels can be bought if the agent charges $2\frac{1}{2}$% commission for buying?

126. A paper-mill worth \$30,000 was insured for an annual premium of $1\frac{3}{8}\%$ on 90% of its value. In the second year it was injured by fire to the amount of \$1780. How much did the mill owner save by insuring?

127. A city voted a tax of \$74,500; the poll-tax was \$1.25 on 2000 polls; the assessed value of city property was \$6,000,000. What was the tax on \$1000?

128. What insurance must be placed upon a store and its contents, valued at \$20,085, that the entire value of the goods and store and of a premium of $2\frac{1}{2}\%$ may be recovered in case of loss by fire?

129. A premium of \$88.14 is paid upon a cargo of wheat insured at $2\frac{3}{5}\%$ on $\frac{3}{4}$ of its value. Find the number of bushels shipped, if the average price is 80 cts. a bushel.

130. A 30% duty of \$5594.40 was paid on 252 watches. What was the invoice price of each watch?

131. Eleven and one-half yards of cloth $1\frac{1}{8}$ yds. wide are required for a dress. How many yards must be bought if the shrinkage in sponging is 10% in length and 8% in width?

132. If 30% of a merchant's sales is profit, what is his gain per cent?

133. A merchant insured a ship and cargo at $4\frac{3}{8}\%$. If \$158,650 cover both property and premium, what is the value of the ship and cargo?

134. How much money must be sent to purchase 10,000 bbls. of sugar, at \$8.50 per barrel, if the commission for buying is 3%, and the sum prepaid for freight is \$315?

INTEREST.

Find the interest of:

135. \$1000 for 2 yrs. 7 mos. 18 dys., at 6%.

136. \$1496 for 7 mos. 21 dys., at 6%.

137. \$582 for 1 yr. 7 mos. 15 dys., at 6%.

138. \$168 for 1 yr. 5 mos. 12 dys., at 2⅔%.

139. \$548 for 7 mos. 18 dys., at 6½%.

140. \$1272 from July 12, 1880, to Feb. 24, 1882, at 3¼%.

141. \$1975.30 for 60 dys., at 6%.

142. \$1675 for 90 dys., at 6%.

143. \$976 for 3 yrs. 6 mos., at 1% a month.

Find the rate per cent:

144. When the interest on \$3000 for 3 yrs. is \$630.

145. When the interest on \$1500 for 2 yrs. is \$172.50.

146. When the interest on \$1278.50 for 3 yrs. 6 mos. is \$178.99.

147. When a sum of money is doubled in 8 yrs.

148. When \$1758 amount to \$1869.34 in 8 mos.

Find the time:

149. When the interest on \$278.40, at 7½%, is \$100.92.

150. When \$600, at 3⅓%, amount to \$660.

151. When the interest on \$78, at 1½% a month, is \$28.08.

152. When the principal, at 5%, is doubled.

Find the principal that will:

153. Produce \$424.94 interest in 3 yrs., at $5\frac{1}{2}\%$.

154. Produce \$285.60 interest, at 7%, in 1 yr. 8 mos. 12 dys.

155. Produce \$81.37 interest, at $3\frac{2}{3}\%$, in 2 yrs. 9 mos. 18 dys.

Find the principal that will amount to:

156. \$88.80, at 6%, in 3 yrs. 4 mos.

157. \$308.10, at $5\frac{2}{5}\%$, in 6 mos.

158. \$570.475, at 6%, in 3 yrs. 4 mos. 6 dys.

159. \$661.32, at $\frac{4}{5}\%$ a month, in 3 yrs. 6 mos.

160. Find the interest on \$1825 from Jan. 1 to June 25, at $5\frac{1}{2}\%$, counting the exact number of days, and allowing 365 dys. for a year.

BANK DISCOUNT

Find the proceeds of the following notes:

161. \$300. SPRINGFIELD, Ill., Aug. 12. 1884.

Sixty days after date I promise to pay Nicholas Welsh, or order, \$300, value received.

Discounted at 6%, Sept. 1. JOHN BRYCE.

162. \$700. BOSTON, Nov. 13, 1880.

Ninety days after date I promise to pay to the order of David Morrison seven hundred dollars, value received.

Discounted at 7%, Jan. 1, 1881. GEORGE BROWN.

163. \$217.40. MINNEAPOLIS, July 30, 1884.

Ninety days after date I promise to pay to the order of Seth Jay two hundred seventeen and $\frac{40}{100}$ dollars, value received.

Discounted at 6%, Aug. 10, 1884. JAMES BENT.

164. $500. CHICAGO, July 9, 1883.

Ninety days from date, for value received, I promise to pay to the order of John Hogan five hundred dollars, with interest at 9%.

Discount at 6%, July 9, 1883. JOHN FOSTER.

165. $5897.50. MILWAUKEE, June 24, 1881.

Four months from date, for value received, I promise to pay to the order of Aaron Reed five thousand eight hundred ninety-seven and $\frac{50}{100}$ dollars, with interest at 6%.

Discounted at 5%, Aug. 15. JAMES CAREY.

Find the face of a note which:

166. Discounted at 6% for 90 dys. yields $344.57.

167. Discounted at 9% for 46 dys. yields $493.87.

168. Discounted for 6% for 3 mos. yields $984.50.

PARTIAL PAYMENTS.

169. A note for $680, dated June 15, 1884, payable on demand, with interest at 6%, bears the following endorsement: May 15, 1885, $425. What is due June 15, 1885?

170. On a note of $1400, dated March 1, 1880, there was received Oct. 19, 1880, $700; Jan. 1, 1881, $400. What is due March 1, 1881, reckoning interest at 6%?

171. A note of $900, dated Jan. 1, 1884, and bearing interest at 5%, has the following endorsements: May 13, $240; Aug. 19, $300; Oct. 25, $180. Required the balance due Jan. 1, 1885.

172. A note of $1800, dated Jan. 1, 1880, and bearing interest at 5%, has the following endorsement: June 1, 1881, $400. Find the balance due June 1, 1884.

173. A note of $600, dated Aug. 13, 1881, and bearing interest at 6%, has the following endorsements: Jan. 1, 1882, $200; April 1, 1882, $110. Find the balance due Aug. 13, 1883.

174. A note of $1150, dated June 30, 1878, and bearing interest at 6%, has the following endorsements: Jan. 30, 1879, $15; April 30, 1880, $570; July 30, 1881, $420. Find the balance due Dec. 30, 1882.

Compound Interest.

Find the compound interest of:

175. $300, at 6%, for 3 yrs. 4 mos. 18 dys.

176. $350, at 6%, for 3 yrs. 5 mos. 24 dys.

177. $840, at 8%, from June 13, 1880, to Aug. 1, 1881, interest being payable quarterly.

178. $400, at $4\frac{1}{2}$%, from Jan. 1, 1881, to Feb. 13, 1884.

179. $1100, at 6%, for 2 yrs. 7 mos. 6 dys., interest being payable semi-annually.

180. $1000, at 8%, for 2 yrs. 3 mos. 18 dys., interest payable quarterly.

Stocks.

181. Find the cost of $2400 stock, at $97\frac{1}{2}$.

182. Find the cost of $2785 stock, at $105\frac{1}{8}$.

183. Find the cost of $5680 stock, at $103\frac{1}{4}$.

184. How much stock, at $85\frac{3}{8}$, including brokerage, can be bought for \$2376.84?

185. How much 6% stock will produce an income of \$840?

186. Find the price of stock, when \$4647.50 will pay for \$5200 worth of stock.

187. How many hundred-dollar shares of 7% stock will yield a yearly income of \$686?

188. A gentleman gave his daughter \$25,700 of $4\frac{1}{2}$% bonds. What yearly income from them will she receive?

189. What amount of 8% stock will yield a yearly income of \$8000?

190. What is the rate of dividend when the sum of \$300 is received from \$7500 stock?

191. Find the rate of dividend when the sum of \$1603.80 is received from \$35,640.

192. How much stock, at $121\frac{1}{2}$, can be bought for \$6318?

193. How much stock, at $97\frac{1}{2}$, can be bought for \$1755?

194. Find the sum paid for \$5600 stock, at $112\frac{3}{4}$, and brokerage $\frac{1}{8}$.

195. What income will be obtained from \$5125, invested in 6% stock, at $102\frac{1}{2}$?

196. Find the income from \$8190, invested in 5% stock, at 91.

197. Find the income on \$1935, invested in 8% stock, at $107\frac{1}{2}$.

198. Find the income from \$6750 invested in $4\frac{1}{8}$% stock, at 75.

199. If \$7656 be invested in stock, at $63\frac{4}{5}$, and the stock pays a dividend of $3\frac{1}{4}\%$, how much will be received on the money invested?

200. If \$7000 be invested in stock, at $87\frac{1}{2}$, and the stock pays a dividend of $7\frac{1}{2}\%$, how much will be received?

201. If 9% stock is bought at 150, what rate of interest will be received on the investment?

202. What rate of interest will be received on 5% stock at 75?

203. What rate of interest will be received on 4% stock, at $62\frac{1}{2}$?

204. How much money must be invested in 5% stock, at 80, to produce \$400 income?

205. How much money must be invested in 4% stock, at 90, to produce \$320 income?

206. How much money must be invested in 6% stock, at 75, to produce \$200 income?

207. A man received \$240 from his 6% dividend, on stock bought at 105. How much money did he have invested in the stock?

208. What should be paid for a 4% stock, that 5% interest may be realized on the investment?

209. What should be paid for a 6% stock, that 8% interest may be realized on the investment?

210. If 4% stock, which produces an income of \$180, is sold at 90, what sum will be realized from the sale?

211. What increase of income will there be, if \$3600 of 4% stock is sold at 90, and the proceeds invested in 7% stock, at 108?

212. Find the increase in income if $[illegible] in [illegible]% stock is sold at [illegible] and the proceeds invested in [illegible]% stock at par.

213. Find the increase of income if $[illegible] in [illegible]% stock is sold at [illegible], and the proceeds invested in [illegible]% stock at [illegible].

[illegible]

214. If 15 bbls. of apples cost $[illegible], what will [illegible] bbls. cost?

215. If [illegible] workmen can do a piece of work in [illegible] dys., in how many days can [illegible] men do the same work?

216. If [illegible] horses eat a certain quantity of hay in [illegible] mos., how long will the same quantity last [illegible] horses?

217. A meadow can be mowed by [illegible] men in [illegible] dys. How many days will it take [illegible] men to mow it?

218. If [illegible] men can build a wall in [illegible] dys., how many men will be required to build it in [illegible] dys.?

219. A bankrupt owes $[illegible], and his assets amount to $[illegible]. How much on a dollar will his creditors receive?

220. What does a bankrupt pay on a dollar, if his creditors receive $[illegible] on $[illegible]?

221. A bankrupt's effects amounted to $2675.40, and his debts to $3057.60. What did his creditors receive on a dollar?

222. If 4 men reap 5 A. 159 sq. rds. in 1 week, how many men at the same rate will reap [illegible] A. 104 sq. rds.?

993. A wall whose length is 1.875 [illegible] 10.5 [illegible] Find the length of the [illegible] 18 [illegible] high.

994. A cistern can be filled in 54 min. by a pipe running 3¾ gals. a minute. In how many minutes can it be filled by another pipe, running [illegible] gals. a minute?

995. A watch set on Saturday, at half-past eight in the evening, loses 1¼ min. in 30 hrs. What time does it show the next Thursday, at 4 o'clock in the afternoon?

996. When do the hour and minute-hands of a watch coincide between 5 and 6 o'clock?

NOTE. Since the hour hand moves through 5 minute-spaces while the minute hand traverses 60, the minute hand moves 12 times as fast as the hour hand. The minute-hand, therefore, in moving through 12 minute-spaces, traverses 11 minute-spaces more than the hour hand.

When the hour hand is at V., the minute hand, being at XII., is 25 minute-spaces behind it. The question, therefore, is, if the minute hand to gain 11 spaces, must move through 12 spaces, how many spaces must it move through to gain 25 spaces?

11 : 25 :: 12 : ?

997. When do the hour and minute hands of a watch coincide between 8 and 9 o'clock?

998. When do the hour and minute-hands of a watch coincide between 3 and 4 o'clock?

999. When do the hour and minute hands of a watch coincide between 10 and 11 o'clock?

[illegible] weight of a body weighed successively in the [illegible] of a false balance is the square root of the product of [illegible] weights.

230. A body appears to weigh [illegible] lbs. in one scale, and 5[illegible] lbs. in the other scale, of a false balance. Find its true weight.

The times in which bodies fall are proportional to the square roots of the distances traversed. Since a body falls 16.1 ft. the first second, to find the time a body is falling, divide the distance by 16.1 and extract the square root of the quotient.

231. In how many seconds will a stone fall to the bottom of a coal-pit [illegible] ft. deep?

Compound Proportion.

232. If 60 bu. of corn feed [illegible] horses for [illegible] dys., in how many days will [illegible] horses consume [illegible] bu.?

233. If 20 cwt. are carried [illegible] miles for $[illegible], how much will be the cost of carrying [illegible] cwt. [illegible] miles?

234. If 20 men can perform a piece of work in [illegible] dys., required the number of men who can perform another piece of work three times as great in [illegible] of the time.

235. If 12 horses, in 5 dys., draw 44 loads of stone, how many horses will draw 132 loads the same distance in 18 dys.?

236. If a footman travels 130 mi. in 3 dys., of 14 hrs. each, in how many days, of 7 hrs. each, will he travel 390 mi.?

237. If 50 men dig a cellar in 7 dys., working 11 hrs. a day, how many days will 24 men require, working 8 hrs. a day?

238. A garrison of 1500 men has provisions for 12 wks., at the rate of 20 oz. per day to each man. How many men will the same provisions maintain for 20 wks., allowing each man only 8 oz. per day?

239. If 12 candles, of which 8 weigh a pound, serve 4 winter evenings, from five to eleven, how many candles, of which 6 weigh a pound, will serve 3 spring evenings, from seven to eleven?

240. A contractor, having engaged to lay 10 mi. of railway in 150 dys., finds that 90 men have finished 3 mi. in 80 dys. How many more men must be engage to finish the work in the given time?

241. If 200 men in 12 dys., of 8 hrs. each, can dig a trench 160 yds. long, 6 yds. wide, and 4 yds. deep, in how many days, of 10 hrs. each, will 90 men dig a trench 450 yds. long, 4 yds. wide, and 3 yds. deep?

242. If 120 men make an embankment [illegible] of a mile long, 30 yds. wide, and 7 yds. high, in 42 dys., how many men will it take to make an embankment 1000 yds. long, 36 yds. wide, and 22 ft. high, in 30 dys.?

POWERS AND ROOTS.

Find the square root of:

243. 80976.

244. 106929.

245. 622521.

246. 1234321.

247. 2052.09.

248. 4795.25731.

249. 24674.1264.

250. [illegible]

Find the cube root of :

251. 373248.	**256.** 52734375.
252. 54872.	**257.** 7834.87438.
253. 389017.	**258.** 0.053157376.
254. 1092727.	**259.** $\frac{4}{7}$.
255. 84604519.	**260.** $7\frac{1}{8}$.

MENSURATION.

261. What is the total surface of a cube, the edge of which measures $4\frac{1}{2}$ in.?

262. How many planks, each 15 ft. long, and 10 in. wide, will be required for the flooring of a room 30 ft. in length and $22\frac{1}{2}$ ft. in width?

263. A square court, whose side is 42 yds., is paved with 28,224 square tiles. Find the dimensions of each tile.

264. Find the area of a triangle whose base is 9 ft. 8 in., and whose altitude is 5 ft. 3 in.

265. How many yards of carpeting, 1 yd. wide, will be required for a room 27 ft. long, and 21 ft. 3 in. wide, if the strips run across the room?

266. How many yards in the side of a square field containing 3 A. 44 sq. rds. 25 sq. yds.?

267. The weight of a cubic inch of water is 253.17 grs.; that of a cubic inch of air, 0.31 grs. Find to three places of decimals the number of cubic inches of water equal in weight to 1 cu. ft. of air.

268. How many cubic feet in a piece of timber 18 ft. long, 15 in. wide, and 10 in. thick?

238. A garrison of 1500 men has provisions for 12 wks., at the rate of 20 oz. per day to each man. How many men will the same provisions maintain for 20 wks., allowing each man only 8 oz. per day?

239. If 12 candles, of which 8 weigh a pound, serve 4 winter evenings, from five to eleven, how many candles, of which 6 weigh a pound, will serve 3 spring evenings, from seven to eleven?

240. A contractor, having engaged to lay 10 mi. of railway in 150 dys., finds that 90 men have finished 3 mi. in 80 dys. How many more men must he engage to finish the work in the given time?

241. If 200 men in 12 dys., of 8 hrs. each, can dig a trench 160 yds. long, 6 yds. wide, and 4 yds. deep, in how many days, of 10 hrs. each, will 90 men dig a trench 450 yds. long, 4 yds. wide, and 3 yds. deep?

242. If 120 men make an embankment $\frac{3}{4}$ of a mile long, 30 yds. wide, and 7 yds. high, in 42 dys., how many men will it take to make an embankment 1000 yds. long, 36 yds. wide, and 22 ft. high, in 30 dys.?

POWERS AND ROOTS.

Find the square root of:

243. 30976.

244. 106929.

245. 622521.

246. 1234321.

247. 2052.09.

248. 4795.25731.

249. 24674.1264.

250. $\frac{196}{1369}$.

CHAPTER XVI.

METRIC MEASURES.

320. The **Metric System** is a system of weights and measures expressed in the decimal scale.

321. The **standard meter,** as defined by law, is the length of a bar of very hard metal carefully preserved at Paris, accurate copies of which are furnished to the governments of all civilized nations.

322. The principal units of the metric system are:

The meter (^{m}) for lengths;
The square meter (^{qm}) for surfaces;
The cubic meter (^{cbm}) for large volumes;
The liter (^{l}) (*lee'-ter*) for smaller volumes;
The gram (^{g}) for weights.

323. All these units are divided and multiplied decimally, and the size of the measures thus produced is shown by one of seven prefixes; namely, *deka*, meaning 10; *hekto*, meaning 100; *kilo*, meaning 1000; *myria*, meaning 10,000; and *deci*, meaning 0.1; *centi*, meaning 0.01; *milli*, meaning 0.001.

324. But, as in United States money we seldom speak of anything else than dollars and cents, so in other measures it is only those printed in **black letter** in this chapter that are in common use.

NOTE. A meter is a trifle more than 39.37 inches, and all the units of the system are derived from the meter. All the compound names are accented on the first syllable; thus, *mil'limeter*. The teacher *should be supplied* with a meter stick, a liter, and a cubic centimeter.

UNITS OF LENGTH.

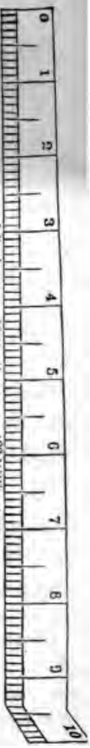
325.

A millimeter (^{mm})	= 0.001	of a meter.
A centimeter (^{cm})	= 0.01	" "
A decimeter	= 0.1	" "
A meter (^{m}).	Principal Unit.	
A dekameter	= 10	meters.
A hektometer	= 100	"
A kilometer (^{km})	= 00	"
A myriameter	= 00	"

326. A length given in any of these meas**ures may** be expressed in terms nother measure **by simply** moving the deci t to the right **or left.**

Thus, 17,856,342^{mm} may be ten as **kilo**-meters by observing that **milli**-mete are changed to meters by moving the point three places to the left; and these meters into **kilo**-meters by carrying it three places further, making, in all, six places. Therefore, 17,856,342^{mm} = 17.856342^{km}.

Again, 4.876326^{km} may be written as centimeters, by observing that **kilo**-meters are changed to meters by moving the point three places to the right, and meters to **centi**-meters by moving it two places further, making, in all, five places. Therefore, 4.876326^{km} = 487,632.6^{cm}.


1 Decimeter = 10 Centimeters = 100 Millimeters.

327. The rule, therefore, for this conversion is:

First change the point so as to convert the **given measures** into terms of the **principal unit;** then change the point so as to convert the principal into the **required** units.

328. Remember that, before *adding* or *subtracting*, the quantities must be written in the ***same units*** of measure.

Ex. 165. (*Oral.*)

1. How many meters in a dekameter? hectometer? kilometer? How many dekameters in a hectometer? kilometer?
2. What part of a meter is a decimeter? centimeter? millimeter? What part of a centimeter is a millimeter?
3. Read 32.3^{m}; 12.6^{cm}; 15.4^{km}; 59.8^{mm}.
4. Express 3256^{m} as kilometers; as centimeters.
5. Express 5368^{mm} as centimeters; as meters.
6. Express 12.4^{km} as meters; as centimeters.

Ex. 166.

Find the value of each of the following expressions in meters:

1. $0.435^{m} + 852^{cm} + 4263^{mm} + 0.1595^{km}$.
2. $0.927^{km} - 6495^{cm}$; $4.37^{cm} - 42.87^{mm}$.
3. 8×0.0457^{km}; 3.04×60.93^{cm}; 5.43×67.2^{mm}.
4. $38{,}019^{mm} \div 0.097$; $0.41^{km} \div 25.625$.
5. A book is 2.1^{cm} thick; if the average thickness of the leaves is 0.05^{mm}, find the number of pages in the book.
6. The expense of building a certain railroad is $25,000 on the average per kilometer. What is the whole cost of the road, if its length is 72^{km} and 53^{m}?
7. The wheels of a locomotive that makes 45^{km} an hour are 7.5^{m} in circumference. How many revolutions will they make a minute?
8. A locomotive runs 1284^{m} in 1½ min. How many kilometers will it go in 1 hr. 35 min. 15 sec.?
9. The top of a monument is 143.9^{m}, and the base 67.19^{m} above the level of the sea; the steps which lead from the base to the top of the monument are each 19^{cm} high. How many steps are there?

Measures of Surface.

329. The principal unit of surface is a **square meter** (^{qm}).

330. In square measure the multiplication and division of units is by hundreds and hundredths, instead of by tens and tenths. Suppose the square in the margin to represent a **square meter.** It is divided into ten equal horizontal bands, and each band is one-tenth of the square meter. Each band can be divided, as the upper one is, into ten little squares measuring one-tenth of a meter on a side. Each of these squares will be 0.1 of the band, or 0.01 of the whole square. The **square meter,** therefore, contains 10 × 10 or 100 **square decimeters.**

If the square meter were divided into 100 equal horizontal bands, each band would be 0.01 of the square; and if each of the 100 bands were divided into 100 squares, that is, into 100 square centimeters, the whole square would contain 100 × 100 or 10,000 square centimeters. A **square meter,** therefore, contains 10,000 **square centimeters.**

In like manner, a **square meter** contains 1,000,000 **square millimeters.**

331. Units of Surface.

A square millimeter (^{qmm})	= 0.000001 of a square meter.			
A **square centimeter** (^{qcm})	= 0.0001	"	"	"
A square decimeter	= 0.01	"	"	"
A **square meter** (^{qm})	**Principal Unit.**			
A square dekameter	= 100 square meters.			
A square hektometer	= 10,000	"	"	
A **square kilometer** (^{qkm})	= 1,000,000	"	"	

332. It will be observed that while centimeters are in the second, and millimeters in the third decimal place from meters, **square centimeters** are in the **fourth** and **square millimeters** in the **sixth** decimal place from **square meters.**

Land Measure.

333. In measuring land the square meter is called a **centar** (^{ca}), the square dekameter is called an **ar** (^{a}), and the square hektometer a **hektar** (^{ha}).

Ex. 167. (*Oral.*)

1. How many square meters in a square dekameter? square hectometer? square kilometer?
2. How many centars in an ar? in a hektar?
3. What part of a hektar is an ar? a centar?
4. What part of a square meter is a square decimeter? square millimeter?
5. Read 56.4^{qm}; 2.05^{qkm}; 531.6^{qcm}.
6. Read 53^{a}; 36.03^{ha}; 56^{ca}; 56^{qm}.

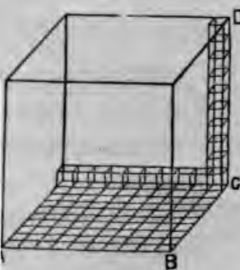
Ex. 168.

1. Convert $1,854,276^{qm}$ into hektars; into square kilometers.
2. How many hektars in 2.7856^{qkm}?
3. Write 1.7431^{qm} as square centimeters; as square millimeters.
4. How many square kilometers in $17,467.5^{ha}$?
5. How many square meters in 1.3614^{qkm}?
6. How many square meters in 2.25^{ha}?
7. How many square centimeters in 0.0137 of a square meter?
8. Write 3.571^{qcm} as square millimeters.
9. A man bought 3^{ha} of land at $200 per hektar, and sold it for $2.50 per ar. How much did he gain?
10. If 6^{ha} are divided into 64 equal lots, how many square meters will there be in each lot?

MEASURES OF VOLUME.

334. The principal unit of capacity or volume is a cubic **meter (cbm).**

335. The cubic meter can be divided into 10 layers, **each a meter** square and a decimeter thick. Each layer will, therefore, be 0.1 of a cubic meter.



Again, each layer divided into 10 equal Each part will, therefore, 0.1 of the layer, or 0.01 of the meter, and will be a decimeter square and a meter long.

Also, each one of these pa can be divided into 10 equal each of which will be a cubic decimeter, and will be of 0.01, that is, 0.001 of the cubic meter.

The **cubic meter,** therefore, contains 1000 **cubic decimeters.**

In like manner, each cubic decimeter can be divided into 1000 cubic centimeters, and each **cubic centimeter into** 1000 cubic millimeters.

336. UNITS OF VOLUME.

A cubic millimeter (^{cmm})	= 0.000000001	**of a cubic meter.**
A cubic centimeter (^{ccm})	= 0.000001	" " "
A cubic decimeter	= 0.001	" " "
A cubic meter (^{cbm})	**Principal Unit.**	

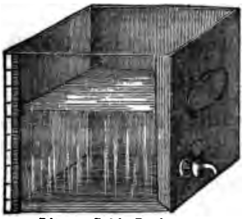
337. It will be seen that **cubic centimeters** are in the **sixth** and **cubic millimeters** are in the **ninth** decimal place from **cubic meters.**

338. In measuring wood, the **cubic meter is called a ster** (st); and in measuring liquids, grain, etc., the **cubic decimeter** is always called a **liter.**

When the liter is the unit the numeral **prefixes have the** *same value* as in linear r

Ex. 169. (*Oral.*)

1. In a cubic meter, how many cubic decimeters? cubic centimeters?
2. What part of a cubic meter is a cubic decimeter?
3. What part of a cubic meter is a cubic centimeter?
4. What part of a cubic decimeter is a cubic centimeter?
5. How many liters in a hektoliter?
6. How many liters in a cubic meter?
7. How many cubic centimeters in a liter?
8. Express 7685.25^{l} as cubic meters; as hektoliters.
9. If the water in the liter represented in the margin stands 6^{cm} high, how many *cubic centimeters* of water are there in the measure? How many will be required to fill it? If the faucet be turned, and the water allowed to run out until the measure is only a quarter full, how many cubic centimeters will run out? How many will still remain?



Liter = Cubic Decimeter.

Ex. 170.

1. How many cubic meters in a rectangular box 125^{cm} long, 112^{cm} wide, and 80^{cm} deep? how many liters?
2. How many cubic meters of earth must be removed to dig a ditch 90^{m} long, 85^{cm} wide, and 50^{cm} deep?
3. How deep must a cistern be to hold 6000^{l}, if the bottom is a square measuring 2.25^{m} on a side?
4. How many hektoliters in a bin 4^{m} long, 2^{m} wide, and 1^{m} high?
5. How high must a box be to hold 30^{l}, if it is 50^{cm} long and 20^{cm} wide?

[illegible]

[illegible]

2. Wha[illegible] milligram?
3. [illegible] grams?
4. [illegible] water weighs how many kilograms? [illegible]
5. What is the weight in grams [illegible] of water?
6. Change [illegible] into grams.
7. Give the weight in kilograms of [illegible] of water.
8. Change [illegible] to grams [illegible] to milligrams.
9. Change [illegible] of a ton to kilograms.

Ex. 272.

1. What is the weight of water required to fill a vat 98 [illegible] long, [illegible] wide, and 38 [illegible] deep?
2. A mass of 21.7 [illegible] is divided into 70 pills. What is the weight of each pill?
3. At 2 cts. a kilogram, what will 2.25 tons of hay cost?
4. At $6 a ton for coal, what will it cost to heat a building [illegible] days, if it takes 400 [illegible] of coal a day?

SPECIFIC GRAVITY.

341. *The* **specific gravity** *of any substance is* **the number** *found by dividing the weight of the substance by the weight of an equal bulk of water.*

342. Therefore the **specific gravity** of a substance is the **number** that expresses the **weight** of a **cubic centimeter** of it in **grams**; or of a **liter** in **kilograms**; or of a **cubic meter** in **tons.**

343. The volume of a body is found by dividing its weight by its specific gravity.

Ex. 173.

1. A bar of iron 50^{cm} long, 4^{cm} wide, 1^{cm} thick has a specific gravity of 7.8. Find its weight in kilograms.
2. A piece of iron weighing 117^{kg} is made into a bar 6^{cm} wide and 2^{cm} thick. What is its length, if the specific gravity of the iron is 7.8?
3. A cubical vessel, 40^{cm} on an edge, is full of water. If 14^{l} are drawn off, and replaced by a liquid of which the specific gravity is $\frac{7}{8}$ of that of water, what is the weight of the mixture?
4. What will be the weight in air, in water, and in olive oil, of which the specific gravity is 0.915, of a cube of iron 6^{cm} on an edge, if the specific gravity of the iron is 7.8?

   NOTE. A body weighed in a liquid weighs less than in air by the weight of the volume of the liquid which it displaces.
5. What is the specific gravity of a substance of which 7.3^{ccm} weighs 31.5^{g}?
6. A block of granite 60^{cm} long, 50^{cm} wide, 15^{cm} thick weighs 130.5^{kg}. Find its specific gravity.

7. What is the weight of a load of 250 paving stones, 25cm square and 8cm thick, if the specific gravity of the stone is 2.50?

8. A stick of timber 8m long, 25cm wide, and 23cm thick is carried by 4 men. What weight does each carry, if the specific gravity of the timber is 0.58 of that of water?

9. Allowing that water in freezing is increased by $\frac{1}{11}$ of its volume, find the volume and the weight of a block of ice 80cm long, 50cm wide, 30cm thick, and find how much water it will give on melting.

10. A hall has a capacity of 450cbm. Compute the weight of oxygen in the air that fills the hall, knowing that air contains 23% of its weight of oxygen, and weighs $\frac{1}{773}$ as much as water.

11. A bottle empty weighs 650g; full of oil, it weighs [illegible]g. What part of a liter will the bottle hold, if the specific gravity of the oil is 0.905?

12. A vessel full of water weighs 9.68kg, and, full of oil, of which the specific gravity is 0.91, weighs 9.266kg. Find its capacity and its weight when empty.

   NOTE. To find the capacity, divide the difference of the two weights by the difference of the weights of a liter of water and a liter of oil.

13. In a barometer tube full of water is placed a chain of iron, of which the specific gravity is 7.8. When the chain is removed, the height of the water is just 5cm. Find the volume and weight of the chain.

14. If a body weighs 3.71kg in air, and 2.38kg in water, what is its specific gravity?

15. From 106.25g of rhubarb are made 125 powders. What is the volume of water of which the weight is equal to the weight of a powder?

Ex. 174.

Miscellaneous Problems.

1. A well is 18.2^{m} deep and the wheel is 1.4^{m} round. How many turns of the wheel will be required to raise the bucket?

2. Find the number of liters in a vat 2^{m} by 75^{cm} by 50^{cm}.

3. Into how many pills of 325^{mg} each can a mass of 23.4^{g} be made?

4. How many liters will a box hold which is 75^{cm} long, 15^{cm} wide, and 12^{cm} deep?

5. Add 3.473^{m}, 50^{cm}, 83^{mm}, 4.5^{m}, and 16^{cm}.

6. There are 4 measuring lines: the first is 7.5^{m}, the second is $3^{m}\ 75^{cm}$, the third is $4^{m}\ 80^{cm}$, and the fourth is $8^{m}\ 6^{cm}$. Express in meters the total length of the four lines.

7. On the same railroad are four stations, between which the consecutive distances are as follows: $7^{km}\ 249^{m}$, $3^{km}\ 200^{m}$, and 5.007^{km}. Find in kilometers the distance between the first and fourth stations.

8. A goldsmith has sold jewels of the following weights respectively: $27^{g}\ 9^{mg}$, $30^{g}\ 70^{cg}$, $7^{g}\ 4^{cg}$, and $19^{g}\ 34^{cg}\ 7^{mg}$. Find the total weight in grams.

9. From $17^{km}\ 6^{m}$ take $243^{m}\ 691^{mm}$.

10. From a farm containing $340^{ha}\ 7^{a}$ there are sold $119^{ha}\ 29.03^{a}$; how many hektars are left?

11. A liter of mercury weighs $13^{kg}\ 598^{g}$. Find the weight in kilograms of 3.69 liters.

12. If 16.94^{l} of olive oil weigh $15^{kg}\ 500^{g}$, find the weight of one liter.

13. Into how many lots of 3.75^{a} may $8^{ha}\ 40^{a}$ be divided?

14. If 122.6^{g} of chlorate of potassa yield 48^{g} of oxygen, what weight of oxygen may be obtained from 1^{kg} of the chlorate?

15. A field of wheat containing $8\frac{1}{2}^{ha}$ furnished 600 sheaves per hektar; 2 sheaves of wheat furnished a bundle of straw weighing 5^{kg}. What will the whole straw bring, at $15 a ton?

16. A farmer shears 620 sheep and 180 lambs; the sheep give on an average 4^{kg} of wool each, and the lambs $\frac{1}{4}$ of a kilogram each. The wool is sold for 50 cts. a kilogram. How much money does the farmer receive?

17. A vessel full of water weighs 5.25^{kg}; the weight of the vessel when empty is 250^{g}. How many liters will the vessel hold?

18. Wheat weighs 80^{kg} a hektoliter. A field of 4.6^{ha} has produced 9.2 t. of wheat. If a sheaf of the wheat on the average gives 4^{l} of grain, find the average number of sheaves produced per hektar.

19. A piece of zinc weighs in the air 343^{g}, and in water 293^{g} only. What is its volume?

20. A piece of zinc weighs in the air 343^{g}, and in water 293^{g}. What is its specific gravity?

21. A jug empty weighs 1.02^{kg}; full of water it weighs 3.8^{kg}. Find the capacity of the jug in liters.

22. The price of 8 casks of olive oil, containing each 9.05^{hl}, is $1072. What will be the price of 20^{l}?

SPANISH LAND MEASURES.

In those sections of the United States that formerly belonged to Mexico, the Spanish Measures are much used in Land Surveying.

The vara is taken as the unit of measure.

The length of the vara varies in Mexico and in different sections of the United States.

In Mexico the length of the vara used is 32.9927 inches, in California 33 inches, in Texas $33\frac{1}{3}$ inches.

In the problems given below its length will be taken as $33\frac{1}{3}$ inches.

A Labor is a square, each side of which is 1000 varas.

A Square League is a square, each side of which is 5000 varas.

TABLE I.

1 vara = $33\frac{1}{3}$ inches.
3 varas = 100 inches.
36 varas = 100 feet.
108 varas = 100 yards.
1900.8 varas = 1 mile.

TABLE II.

1,000,000 square varas are 1 Labor = 177.136 acres.
25 Labors or 25,000,000 square varas are 1 Square League = 4428.4 acres.
5645.376 square varas are 1 acre.

Ex. 175.

1. Reduce 432 varas to yards, 918 varas to yards, 3726 varas to feet.

2. Reduce 375 yards to varas, 875 feet to varas, 17,286 inches to varas.

3. Reduce 7865 varas to miles and integers of lower denomination.

4. Reduce 1 mile, 5 rods, 3 yards, 1 foot, 6 inches to varas.

5. If a chain is 10 varas long how many chains between two trees that are 3275 yards apart?

6. The sides of a tract of land in the shape of a triangle are 250, 198, 350 varas. How many pounds of wire will be required to fence the land with seven wires, allowing 15 feet for 1 lb. of wire?

7. How many varas in one side of a public square that contains 1 acre?

8. How many acres in a rectangular tract of land, the adjacent sides of which are 1700 and 2200 varas?

9. What is the length of a tract of land of rectangular shape that contains a League and a Labor, the width being 3275 varas?

10. A horse is tied to a stake with a rope long enough for him to graze just 60 feet from the stake, over how many square varas can he graze?

11. Two towns are situated in the diagonally opposite corners of a township. How many varas between the two towns?

12. How many varas in the perimeter of a rectangular garden that is twice as long as wide and contains 10 acres?

APPENDIX.

PRINCIPLES AND RULES FOR REFERENCE.

Any standard by which we count or measure is called **a unit.**

A number that does not specify the objects counted is called an **abstract number**, as 4, 17, 25.

A number that does specify the objects counted is called a **concrete number**, as 4 horses, 7 men.

Hence an abstract number signifies the repetitions of some unit; while a concrete number is really a quantity consisting of the number proper and the name of the things counted.

Numbers that have the same unit are called **like numbers.**

Numbers that have different units are called **unlike numbers.**

NOTATION AND NUMERATION.

Principle 1. Ten units of any order make one unit of the next higher order.

Principle 2. One unit of any order is equal to ten units of the next lower order.

Rule for writing numbers. Beginning at the left, write the figures of each period in their order, filling the vacant places with zeros.

Rule for reading numbers. Beginning at the right, mark off the number by commas into as many periods as possible of three figures each.

Begin at the left hand and read each period as if it stood alone, adding the unit-name of each period.

ADDITION OF INTEGRAL NUMBERS

Principle 1. Only like numbers can be added.
Principle 2. The result is the same in whatever order the numbers are added.

Rule for addition. Write the numbers so that units of the same order shall stand in the same column.

Add the right-hand column; write the units of the sum beneath, and add the tens, if any, to the next column. So proceed with each column.

Write the entire sum of the last column.

Proof. Add each column in the reverse order. If the results agree, the work may be assumed to be correct.

SUBTRACTION OF INTEGRAL NUMBERS.

Principle 1. Only like numbers can be subtracted.
Principle 2. The sum of the subtrahend and the remainder is equal to the minuend.

Rule for subtraction. Write the subtrahend under the minuend, placing units of the same order in the same column.

Begin at the right and subtract each order of units of the subtrahend from the corresponding order of the minuend, and write the result beneath, step by step.

If any order of the minuend has fewer units than the same order of the subtrahend, increase the units of the minuend by 10 and subtract; then diminish by one the units of the next higher order of the minuend and subtract.

Proof. Add together the remainder and subtrahend. If the sum is equal to the minuend, the work is correct.

MULTIPLICATION OF INTEGRAL NUMBERS.

Principle 1. The multiplier is an abstract number.
Principle 2. The product and the multiplicand are like numbers.
Principle 3. The product is the same whatever the order of the factors.

Rule for multiplication. Write the multiplier under the multiplicand, units under units, tens under tens, etc., and draw a line beneath.

Begin at the right and multiply each order of units of the multiplicand by each figure of the multiplier.

Place the right-hand figure of each product under the figure of the multiplier used to obtain it, and add the partial products.

Proof. Interchange the multiplier and multiplicand and multiply. If the results agree, the work may be assumed to be correct.

DIVISION OF INTEGRAL NUMBERS.

Principle 1. The dividend is the product of the divisor and quotient.

Principle 2. If the divisor and the dividend are like numbers, the quotient is an abstract number.

Principle 3. If the divisor is an abstract number, the dividend and the quotient are like numbers.

Principle 4. If the dividend and divisor are both multiplied or both divided by the same number, the quotient is not altered.

Rule for division. Write the divisor to the left of the dividend with a curved line between them.

Take for the first partial dividend the fewest left-hand figures that will contain the divisor, and write the quotient over the right-hand figure of this partial dividend (*under* the right-hand figure in short division).

Multiply the divisor by this quotient and place the product under the partial dividend used.

Subtract this product and to the remainder annex the next figure of the dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

NOTE. If there is a remainder after the last division, it may be written with the divisor under it as a part of the complete quotient.

Proof. Find the product of the divisor and quotient and to this product add the remainder, if any. If the work is correct, the result will be equal to the dividend.

DECIMAL FRACTIONS.

Principle 1. Zeros at the end of a decimal do not affect its value.

Principle 2. Every zero prefixed to a decimal diminishes the value of the decimal tenfold.

NOTATION AND NUMERATION OF DECIMALS.

Rule for writing decimals. Write the decimal, prefixing ciphers, if necessary, to indicate the denomination, and place the decimal point before the tenths.

If the number has no integral part, indicate that by placing a zero before the decimal point.

Rule for reading decimals. Read the decimal as if it were an integral number, and add the *name of the lowest decimal place.*

In reading the integral and decimal parts of a number, pronounce the word *and* at the decimal point and omit it in all other places.

ADDITION OF DECIMALS.

Rule for addition of decimals. Write the numbers so that the decimal points shall stand in a column.

Add as in integral numbers and place the decimal point in the result directly under the column of decimal points.

SUBTRACTION OF DECIMALS.

Rule for subtraction of decimals. Write the subtrahend under the minuend so that the decimal points shall stand in a column.

Subtract as in integral numbers, and place the decimal point in the result directly under the column of decimal points.

NOTE. If the number of decimal places in the subtrahend is greater than the number in the minuend, annex zeros to the minuend.

MULTIPLICATION OF DECIMALS.

Rule for multiplication of decimals. Multiply as if the numbers were integral numbers, and point off from the right of the product as many decimal places as there are decimal places in the multiplicand and multiplier taken together.

DIVISION OF DECIMALS.

Rule for division of decimals. *If the divisor is a whole number*, divide as in integral numbers, and put the decimal point in the quotient when the decimal point in the dividend is reached.

If the divisor contains decimal places, remove the decimal point from the divisor, and move the decimal point in the dividend as many places to the right as there are decimal places in the divisor, before dividing.

MULTIPLES AND MEASURES.

Principle 1. The greatest common measure of two or more numbers is the product of all their common factors.

Principle 2. The least common multiple of two or more numbers contains each of the prime factors of those numbers, and no other factors.

Rule for finding the prime factors of a number. Divide the given number by any prime number that is contained in it without remainder; then this quotient by any prime number that is contained in it without remainder; and so on until the quotient is itself a prime number.

The several divisors and the last quotient are the prime factors.

NOTE. If no prime factor is found before the quotient becomes equal to, or less than, the divisor, the number is prime.

Rule for finding the greatest common measure of two or more numbers.

First method. Separate the numbers into their prime factors.

Select the lowest power of each factor that is common to the given numbers, and find the product of these powers.

Second method. Divide the greater number by the less, and then the divisor by the remainder, and so on until there is no remainder.

The last divisor will be the greatest common measure required.

Rule for finding the least common multiple of two or more numbers. Separate each number into its prime factor.

Select from these the highest power of each factor and find the product of these powers.

NOTE. When the prime factors of the given numbers cannot be found by inspection, they may be found by the method of finding the greatest common measure in such cases.

Rule for cancellation. Remove from the dividend and divisor all factors common to them; divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

COMMON FRACTIONS.

Principle 1. Multiplying both terms of a fraction by the same number does not alter the value of the fraction.

Principle 2. Dividing both terms of a fraction by the same number does not alter the value of the fraction.

REDUCTION OF FRACTIONS.

Rule for reducing an improper fraction to a whole or mixed number. Divide the numeration by the denominator.

Rule for reducing a whole number to an improper fraction Multiply the whole number by the denominator of the required fraction; under this product write the denominator.

Rule for reducing a mixed number to an improper fraction. Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

Rule for reducing a fraction to lower terms. Divide the numerator and denominator by any common factor.

Rule for changing a fraction to higher terms. Multiply both terms of the fraction by the number that will change the given denominator to the required denominator.

NOTE. The required multiplier is found by dividing the required denominator by the denominator of the given fraction.

MULTIPLICATION OF FRACTIONS.

Rule for finding the product of a whole number and a fraction. Find the product of the numerator and whole number and divide the result by the denominator.

Rule for finding the product of two or more fractions. Find the product of the numerators for the required numerator, and of the denominators for the required denominator.

NOTE. Whole numbers and mixed numbers must be first expressed as improper fractions. Any factor common to a numerator and denominator should be cancelled before multiplying.

DIVISION OF FRACTIONS.

Rule for dividing by a fraction. Multiply by the reciprocal of the fraction.

REDUCTION TO SIMILAR FRACTIONS.

Rule for reducing dissimilar fractions to similar fractions. Find the least common multiple of the denominators; this will be the required denominator.

Divide this denominator by the denominator of the first fraction; and multiply the quotient by the numerator of the first fraction.

This product will be the numerator of the first required fraction. So proceed with each fraction.

ADDITION OF FRACTIONS.

Rule for adding fractions. Reduce the fractions to similar fractions and write the sum of the numerators over the common denominator.

SUBTRACTION OF FRACTIONS.

Rule for subtracting fractions. Reduce the fractions to similar fractions and subtract the numerator of the subtrahend from the numerator of the minuend.

Write the difference over the common denominator.

NOTE. If the numbers are mixed numbers, subtract the fractions and the whole numbers separately and combine the results.

REDUCTION OF COMPLEX FRACTIONS.

Rule for reducing a complex fraction to a simple fraction. Multiply both terms of the complex fraction by the least common denominator of the fractions contained in them, and reduce the result to its lowest terms.

CONVERSION OF FRACTIONS.

Rule for reducing a decimal to a common fraction. Write the decimal as an integral number for the numerator, and for the denominator 1 followed by as many zeros as there are figures in the decimal; and reduce this fraction to its lowest terms.

Rule for reducing a common fraction to a decimal. Annex ciphers to the numerator and divide by the denominator.

COMPOUND QUANTITIES.

Definition. A compound quantity is a quantity expressed in two or more units.

REDUCTION OF COMPOUND QUANTITIES.

Rule for reduction descending. Multiply the highest denomination by the number of units of the next lower denomination it takes to make one unit of this higher. Add the product to the units of the same denomination in the given number, if any.

Reduce this sum in like manner to the next lower denomination, and so proceed until the given number is changed to units of the required denomination.

Rule for reduction ascending. Divide the given number by the number of units of that denomination it takes to make one of the next higher; and reserve the remainder, if any.

Divide the quotient in like manner, and so proceed until the required denomination is reached.

The last quotient and the several remainders arranged in order will be the answer sought.

Rule for reduction of denominate fractions to integers. Multiply the numerator of the fraction by the number of units it takes to make one of the next lower denomination; and reduce the result to a whole or mixed number.

If there is a fraction in the result, proceed in like manner with this fraction, and so continue to the lowest denomination required.

Rule for reduction of integers to a denominate fraction. Change the given number of the lowest denomination to a fraction of the next higher denomination.

Write this fraction in its lowest terms as a part of the number of the next higher denomination.

Change the number thus found to a fraction of the next higher denomination, and so proceed to the denomination required.

Rule for finding the fraction that one compound quantity is of another. Reduce the quantities to the same denomination and divide the result *denoting the part* by the result denoting the whole.

ADDITION OF COMPOUND QUANTITIES.

Rule for addition of compound quantities. Write the numbers so that units of the same denomination shall be in the same column.

Beginning with the numbers of the lowest denomination, add together these numbers.

Divide this sum by the number required to make one unit of the next higher denomination. Reserve the quotient to be added to the next higher denomination, and write the remainder under the column added. Write the entire sum of the units of the highest denomination.

SUBTRACTION OF COMPOUND QUANTITIES.

Rule for subtraction of compound quantities. Write the numbers so that units of the same denomination shall be in the same column.

Beginning at the lowest denomination, subtract, and write the remainder under the column; and so proceed to the highest denomination.

If in any case the number in the minuend is smaller than the corresponding number in the subtrahend, take one unit from the next higher denomination in the minuend, reduce it to units of the next lower denomination, add the result to the number in the minuend, and then subtract.

MULTIPLICATION OF COMPOUND QUANTITIES.

Rule for multiplication of compound quantities. Multiply the number in the lowest denomination and divide the result by the number required to make one of the next higher denomination.

Write the remainder under the column multiplied and add the quotient to the product of the next higher denomination; and so proceed, writing the entire product of the last denomination.

DIVISION OF COMPOUND QUANTITIES.

Rule for division of compound quantities. 1. When the divisor is an abstract number. Divide the number of the highest denomination. Reduce the remainder to units of the next lower denomination and add to this product the units, if any, of that denomination in the given quantity.

Divide the result, and so proceed to the lowest denomination.

The several quotients written in order will be the answer required.

2. **When the divisor and dividend are both compound quantities.** Reduce both quantities to the same denomination; then divide as in simple division.

LONGITUDE AND TIME.

Rule for reducing longitude to time. Divide the number of degrees, minutes, and seconds by 15. The quotient is the equivalent number of hours, minutes, and seconds.

Rule for reducing time to longitude. Multiply the number of hours, minutes, and seconds by 15. The product is the equivalent number of degrees, minutes, and seconds.

Rule for finding the time of a place east. Add to the given time the difference of time between the two places.

Rule for finding the time of a place west. Subtract from the given time the difference of time between the two places.

PERCENTAGE.

Rule for finding the percentage of a number. Multiply the given number by the given rate per cent.

Rule for finding the rate per cent when a number and its percentage are given. Divide the percentage by the given number, carrying the division to hundredths.

Rule for finding a number when the rate per cent and percentage are given. Divide the percentage by the rate per cent.

INTEREST.

General rule for finding interest. Multiply the principal by the rate per cent, and this product by the time expressed in years.

To find the amount, add the principal and the interest.

Rule for finding the interest at 6 per cent for a given number of months. Move the decimal point in the principal two places to the left and multiply by one-half the number of months.

Rule for finding the interest at 6 per cent for a given number of days. Move the decimal point in the principal three places to the left and multiply by one-sixth the number of days.

NOTE. This rule will apply to years, months, and days by reducing the years and months to days, reckoning 360 days for a year, and 30 days for a month.

Rule for obtaining any other rate than 6 per cent. Find the interest at 6 per cent. Divide this interest by 6 and multiply the quotient by the number that denotes the required rate per cent.

Rule for finding the exact interest. Multiply the interest of the principal for one year at the given rate per cent by the exact number of days in the time, and divide by 365.

Rule for finding the rate per cent when the principal, interest, and time are given. Divide the given interest by the interest of the principal for the given time at 1 per cent.

Rule for finding the time when the interest, principal, and rate per cent are given. Divide the given interest by the interest of the principal for one year. The quotient will be the time in years.

Rule for finding the principal when the time, interest, and rate per cent are given. Divide the given interest by the interest of $1 at the given rate per cent for the given time.

NOTE. If the amount is given, divide the given amount by the amount of $1 at the given rate per cent for the given time.

BANK DISCOUNT.

Rule for finding bank discount. Find the simple interest on the face of the note from the day of discount to the day of maturity.

If the note bears interest, find the amount of the note to the day of maturity, and then find the simple interest on this amount from the day of discount to the day of maturity.

Rule for finding the proceeds of a note. Subtract the bank discount from the amount of the note.

Rule for finding the face of a note required to yield a given sum. Divide the given sum by the proceeds of $1 for the given rate per cent and term of discount.

PARTIAL PAYMENTS.

The merchants' rule for partial payments. Find the amount of the note or debt from the time of its beginning to draw interest to the time of settlement; also find the amount of each payment from its date to the time of settlement, and then subtract the sum of the amounts of the payments from the amount of the note or debt.

The United States rule for partial payments. Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest.

From this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal, and proceed as before.

COMPOUND INTEREST.

Rule for finding compound interest. Find the amount of the given principal for the first period of time.

Using this amount as a principal, find its amount for the second period, and so on for the entire time.

The last amount, less the given principal, will be the compound interest.

NOTE. When the interest is compounded semi-annually, the rate must be considered one-half the annual rate, and when quarterly, one-fourth the annual rate.

ANNUAL INTEREST.

Rule for finding annual interest. Find the interest annually and the simple interest on each annual interest for the time it remains unpaid.

PROPORTION.

Rule for simple proportion. Make that quantity which is of the same kind as the required answer the third term.

Make the numbers by which the other two quantities are expressed in a common unit the first and second terms, making the greater of these two numbers the second term, if from the nature of the question the answer will be greater than the third term, but making the smaller of these two numbers the second term if the answer will be smaller than the third term.

Divide the product of the second and third terms by the first term. The quotient will be the answer required.

Rule for compound proportion. Make that quantity which is of the same kind as the required answer the third term.

Consider each pair of the remaining quantities separately, and arrange the numbers by which they are expressed, as in simple proportion.

Divide the continued product of the second and third terms by the product of the first terms. The quotient will be the answer required.

Rule for partitive proportion. Divide the quantity by the sum of the numbers representing the parts, and multiply the quotient by the several representative numbers.

The several products will be the parts required.

NOTE. In compound partnership each partner's share of the profit or loss is represented by the product of his amount invested by the length of time that it is invested.

SQUARE AND CUBE ROOTS.

Principle 1. The square of a number is expressed by twice as many figures as the number itself, or by one less than twice as many.

Principle 2. The orders of units in the square root of a number correspond to the number of groups of two figures each into which the number can be divided, beginning at the right.

Rule for square root. Separate the number into groups of two figures each, beginning at the right.

Find the greatest square in the left-hand group and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand group, and to the remainder annex the next group for a dividend.

For a partial divisor, double the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished) will be the second part of the root.

To this partial divisor add the second part of the root for a complete divisor. Multiply this complete divisor by the last figure of the root, subtract the product from the dividend, and to the remainder annex the next group for a new dividend.

Proceed in this manner until all the groups have been thus annexed. The result will be the square root required.

NOTE 1. When the number is not a perfect square annex groups of zeros and continue the process.

NOTE 2. If a given number contains a decimal, divide it into groups of two figures each, by beginning at the decimal point and marking toward the left for the integral number and toward the right for the decimal number.

Be careful to have the last group on the right of the decimal point contain two figures, annexing a zero when necessary.

NOTE 3. The square root of a common fraction is found by extracting the square roots of the numerator and denominator.

If the denominator is not a perfect square, reduce the fraction to a decimal, and extract the square root of the decimal.

Principle 1. The cube of a number is expressed by three times as many figures as the number itself, or by one or two less than *three times* as many.

Rule for measuring round logs. Express the diameter of the small end in inches, subtract twice the diameter from the square of the diameter, and twenty-one fortieths of the remainder will be the number of feet board measure in a log ten feet long.

NOTE. For a log more than ten feet long, divide the number of feet board measure by 10 and multiply the quotient by the length of the log expressed in feet.

VOLUMES.

The unit of volume is a cube whose edge is a unit of length.

The volume of a solid is the number of units of volume it contains.

Rule for finding the volume of a rectangular solid. Express the length, breadth, and height in the same unit of length. Find the product of these numbers. This product is the number of cubic units of the same name as the unit of length.

Rule for measuring wood. Find the product of the length, width, and height expressed in feet, and divide this product by 128. The result is the number of cords.

Rule for finding the volume of a sphere. Multiply the cube of the diameter by 0.5236.

Rule for finding the volume of a cylinder. Multiply the area of the base by the height of the cylinder, expressed in units of the same name.

Rule for finding the number of gallons that a cistern of given dimensions will hold. Find the number of cubic inches in the cistern and divide this number by 231.

Rule for finding the number of gallons that a cask will hold. Subtract the diameter of one of the heads from the bung diameter expressed in inches, and multiply the difference by 0.65. To the product add the head diameter, and this will give the mean diameter.

Square the mean diameter and multiply it by the length of the cask expressed in inches. Divide this product by 294. The quotient is the number of gallons the cask will hold.

Rule for finding the number of bushels of grain a bin will hold. Take eight-tenths of the number of cubic feet in the bin, and add to the result one-half of one per cent of it.

Rule for expressing in cubic feet a given number of bushels. To the number of bushels add one-fourth of the number, and subtract from the sum one-half of one per cent of it.

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